Math 142 - Week in Review #3

1. Simplify each of the following if possible.

(a) $\left(\frac{4e^{-7x}}{e^{x-4}}\right)^3 = \frac{4^3(e^{-7x})^3}{(e^{x-4})^3} = \frac{64e^{-21x}}{e^{3x-12}} = 64e^{-21x-(3x-12)}$

$= 64e^{-21x-3x+12}$

$= \boxed{64e^{-24x+12}}$

(b) $9^{3x+1} \cdot 27^{2-x}$

$= (3^2)^{3x+1} \cdot (3^3)^{2-x}$

$= 3^{6x+2} \cdot 3^{6-3x} = 3^{6x+2+6-3x} = \boxed{3^{3x+8}}$

(c) $5^x + 5^{2x}$

cannot simplify

(d) $2^{\log_2 7} = \boxed{7}$ since 2^x and $\log_2 x$ are inverses of each other. (property 4 on pg. 109)

(e) $9^{\log_3 x}$

$(3^2)^{\log_3 x} = 3^{2\log_3 x} = 3^{\log_3 x^2} = \boxed{x^2}$

(f) $e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = \boxed{\frac{1}{x^2}}$

2. Solve each of the following for x.

(a) $5^{11x-4} = 125^{x^2+2}$

$5^{11x-4} = (5^3)^{x^2+2}$

$5^{11x-4} = 5^{3x^2+6}$

$11x-4 = 3x^2+6$

$0 = 3x^2 - 11x + 10$

$0 = (3x-5)(x-2)$

$3x-5=0$ $\quad$ $x=2$

$3x=5$

$x = \frac{5}{3}$

$\boxed{x = \frac{5}{3} \text{ and } x = 2}$

(b) $2^x \cdot 7x + 2^{x+1} = 0$

$2^x \cdot 7x + 2 \cdot 2^x = 0$

$2^x(7x+2) = 0$

$2^x = 0$ No solution $\quad$ $7x+2=0$

$7x=-2$

$\boxed{x = -\frac{2}{7}}$

(c) $2^x \cdot 8^{x+3} = 32$

$2^x \cdot (2^3)^{x+3} = 2^5$

$2^x \cdot 2^{3x+9} = 2^5$

$2^{4x+9} = 2^5$ $\quad$ so $4x+9=5$

$4x=-4$

$\boxed{x=-1}$

(d) $e^{2x} + 3e^x = 40$

$(e^x)^2 + 3e^x - 40 = 0$

$(e^x - 5)(e^x+8) = 0$ $\quad$ so $e^x - 5 = 0$ $\quad$ or $\quad$ $e^x + 8 = 0$

$e^x = 5$ $\quad$ $e^x = -8$

$\boxed{x = \ln 5}$ $\quad$ $x = \ln(-8) \leftarrow$ DNE

(e) $7^x + 49^x = 20$

$(7^2)^x + 7^x - 20 = 0$

$(7^x)^2 + 7^x - 20 = 0$

$(7^x+5)(7^x-4) = 0$

so $7^x + 5 = 0$ $\quad$ and $\quad$ $7^x - 4 = 0$

$7^x = -5$ $\quad$ $7^x = 4$

No solution

$\ln 7^x = \ln 4$

$x \ln 7 = \ln 4$

$x = \frac{\ln 4}{\ln 7} \approx \boxed{0.7124}$

(f) $3^{x^2+2x} = \frac{1}{27^x}$

$3^{x^2+2x} = 27^{-x}$

$3^{x^2+2x} = 3^{-3x}$

$x^2+2x = -3x$

$x^2+5x = 0$

$x(x+5) = 0$

$\boxed{x=0 \text{ and } x=-5}$

3. Find the domain of each of the following.

(a) $f(x) = 7 \cdot 3^{2x-5}$

Exponential function – Domain is all real numbers

$\mathbb{R}$ or $(-\infty, \infty)$

(b) $g(x) = \log_5(x+2)$

$x + 2 > 0$

$x > -2$

Domain: $\boxed{(-2, \infty)}$

(c) $m(x) = \ln|x|$

$|x| > 0$

This is true for all x, except $x = 0$

Domain: All real numbers except 0

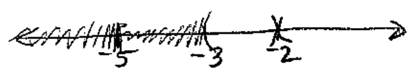
$\boxed{(-\infty, 0) \cup (0, \infty)}$

(d) $k(x) = \dfrac{\sqrt{2x+10}}{\log_2(x+3)}$ (Hint: $\log_b 1 = 0$)

$2x + 10 \geq 0$ AND $x + 3 > 0$ AND $x + 3 \neq 1$

$2x \geq -10$ $\quad$ $x > -3$ $\quad$ $x \neq -2$

$x \geq -5$



Domain: $\boxed{(-3, -2) \cup (-2, \infty)}$

4. Bob invested \$4,100 in an account paying 3.25% per year compounded quarterly.

(a) How much money will Bob have in his account in 10 years?

$A = P\left(1 + \frac{r}{m}\right)^{mt}$

$A = 4100\left(1 + \frac{0.0325}{4}\right)^{4 * 10}$

$A = \$5667.08$

(b) How long will it take for Bob's account to reach \$10,000?

$10000 = 4100\left(1 + \frac{0.0325}{4}\right)^{4t}$

$\ln\left(\frac{100}{41}\right) = \ln(1.008125)^{4t}$ ← Divide both sides by 4100, then take natural log of both sides.

$\ln\left(\frac{100}{41}\right) = 4t \ln(1.008125)$

$\dfrac{\ln(100/41)}{4\ln(1.008125)} = t$

$\boxed{t = 27.5451 \text{ years}}$

5. Rewrite each of the following in exponential form and solve for x, y, or b as indicated.

(a) $\log_7 x = -2$

$7^{-2} = x$ → $x = \frac{1}{7^2} = \boxed{\frac{1}{49}}$

(b) $\log(x-3) = 3$

$10^3 = x-3$

$1000 + 3 = x$ → $\boxed{x = 1003}$

(c) $\log_{16}\left(\frac{1}{4}\right) = y$

$16^y = \frac{1}{4}$ $\quad 16^{1/2} = 4 \quad$ so $16^{-\frac{1}{2}} = \frac{1}{4} \quad$ so $\boxed{y = -\frac{1}{2}}$

(d) $\log_b 125 = 3$

$b^3 = 125 \rightarrow \boxed{b = 5}$

(e) $\log_b 9 = \frac{2}{3}$

$b^{2/3} = 9$ → $\left(b^{2/3}\right)^3 = 9^3$

$b^2 = 729$

$\boxed{b = 27}$ ← only take the positive root since bases cannot be negative.

6. The table below shows the relationship between dollars spent on advertising and the number of units of an item sold.

	5	10	25	30	40	48 ← x in 1000's of dollars
Dollars spent	5,000	10,000	25,000	30,000	40,000	48,000
Millions of units sold	8.534	8.757	9.050	9.109	9.201	9.260

(a) Use your calculator to view a scatter plot of this data set. What types of functions should NOT be considered as a model for this data set? Justify your answer.

Not linear – scatter plot is curved

Not exponential – data do not seem to have the x-axis as a horizontal asymptote

Not quartic – quadratic and cubic are possible, and these are simpler models

(b) If your goal is to find a model that can be use for predicting the number of units sold when more and more money is used for advertising, which type of function seems most appropriate to model the data? Find this function and write it as a model. Model's purpose: Extrapolation

After considering a quadratic and a cubic model, the logarithmic model seems best for extrapolation:

$f(x) = 8.0179 + 0.3208 \ln x$ million units sold, where x is the amount of money spent on advertising in thousands of dollars.

(c) Use the model to estimate the number of units sold if \$82,000 is spent on advertising.

$f(82) \cong 8.0179 + 0.3208 \ln(82)$

$= \boxed{9.4316 \text{ million units sold}}$

or $Y_3(82) = 9.431446499 \rightarrow 9{,}431{,}446$ units sold

7. Write each of the following as a sum or difference of logarithms.

(a) $\log_5 \frac{25x^2y}{w^3z}$ $= \log_5 25x^2y - \log_5 w^3 z$

$= \log_5 25 + \log_5 x^2 + \log_5 y - (\log_5 w^3 + \log_5 z)$

$= \log_5 5^2 + 2\log_5 x + \log_5 y - 3\log_5 w - \log_5 z$

$= \boxed{2 + 2\log_5 x + \log_5 y - 3\log_5 w - \log_5 z}$

(b) $\ln \frac{\sqrt{3x^2+7}}{x^5(x-7)^2}$

$= \ln(3x^2+7)^{1/2} - \ln(x^5(x-7)^2)$

$= \frac{1}{2}\ln(3x^2+7) - (\ln x^5 + \ln(x-7)^2)$

$= \boxed{\frac{1}{2}\ln(3x^2+7) - 5\ln x - 2\ln(x-7)}$

8. Solve for x in each of the following.

(a) $\log x + \log(x+3) = \log 18$

$\log(x(x+3)) = \log 18$

$x^2 + 3x = 18$

$x^2 + 3x - 18 = 0$

$(x+6)(x-3) = 0$

$x = -6$ and $x = 3$

With logs, you must check your answers!

$\log(-6) + \log(-6+3)$ DNE

so $x = -6$ is NOT a solution.

$\log 3 + \log(3+3)$ is ok

$\boxed{x = 3}$

(b) $\log_7 x = 4\log_7 2 - \frac{1}{3}\log_7 8$

$\log_7 x = \log_7 2^4 - \log_7 8^{1/3}$

$\log_7 x = \log_7 \frac{16}{2}$

$\boxed{x = 8}$

9. Fred invested some inheritance money in a mutual fund that pays 7.3% per year compounded continuously. How long will it take for his investment to double?

$A = Pe^{rt}$

$2P = Pe^{0.073t}$

$2 = e^{0.073t}$

$\ln 2 = 0.073t$

$\frac{\ln 2}{0.073} = t$

$\boxed{t = 9.4952 \text{ years}}$

10. The number of bacteria in a culture after t days can be modeled by $A(t) = 13e^{0.043t}$ thousand bacteria.

(a) What is the size of the initial population?

← $t=0$

$A(0)=13$ so $\boxed{13{,}000 \text{ bacteria}}$

(b) How many bacterial will be in the culture after 18 hours?

$18 \text{ hrs} = \frac{18}{24} \text{ day} = \frac{3}{4} \text{ day}$

$A(\frac{3}{4}) = 13e^{0.043(3/4)}$

$= 13.42608367$ so $\boxed{\text{Approx. } 13{,}426 \text{ bacteria}}$

(c) How long will it take for the population of bacteria to grow to 125,000?

Find t when $A(t)=125$

$125 = 13e^{0.043t}$

$\frac{125}{13} = e^{0.043t}$

$\ln(\frac{125}{13}) = 0.043t$

$t = \dfrac{\ln(\frac{125}{13})}{0.043}$

$\boxed{t = 52.6364 \text{ days}}$

11. A student was given a vitamin pill. The concentration of the vitamin in the student's body at different times is given in the table below.

	2	4	5.5	8.25	9	10.5
Time	8am	10am	11:30am	2:15pm	3pm	4:30pm
Concentration ($\frac{\mu g}{mL}$)	5.2	3.0	2.0	0.9	0.7	0.5

(a) Find the best model for the data where t is the number of hours since 6am.

We expect the concentration to level off to 0, so try exponential:

$f(t) = 9.1592 \cdot 0.7555^t$ μg/mL, where t is the number of hours since 6Am.

(b) According to your model, when will the concentration reach 0.25 $\frac{\mu g}{mL}$?

↑ Find t ↑ y-value

By hand:

$0.25 = 9.1592 \cdot 0.7555^t$

$\frac{0.25}{9.1592} = 0.7555^t$

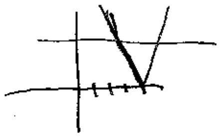
$\ln(\frac{0.25}{9.1592}) = \ln 0.7555^t$

$-3.6010532 = t \ln 0.7555$

$t = \dfrac{-3.6010532}{\ln(0.7555)} = 12.8437$ hours since 6Am so approx. $\boxed{6{:}51\text{pm}}$

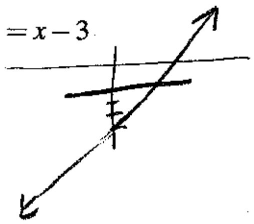
12. Which of the following functions are one-to-one? ← Must pass the Horizontal Line Test.

(a) $f(x) = 3\,|x-4|$



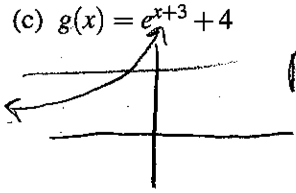
$\boxed{\text{Not one-to-one}}$

(b) $h(x) = x - 3$



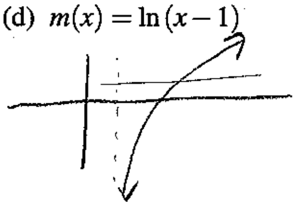
Passes - $\boxed{h(x) \text{ is one-to-one}}$

(c) $g(x) = e^{x+3} + 4$



Passes - $\boxed{g(x) \text{ is one-to-one}}$

(d) $m(x) = \ln(x-1)$



passes - $\boxed{m(x) \text{ is one-to-one}}$

13. Find the inverse of each of the following functions.

(a) $f(x) = \ln(x+3)$

$y = \ln(x+3)$

Inverse: $x = \ln(y+3)$

$e^x = y+3$

$e^x - 3 = y$ so $\boxed{f^{-1}(x) = e^x - 3}$

Another method

$\ln(x+3) = y$ so $e^y = x+3$

Inverse: $e^x = y+3$

$e^x - 3 = y$

so

$\boxed{f^{-1}(x) = e^x - 3}$

(b) $g(x) = 5 \cdot 7^x$

$y = 5 \cdot 7^x$

Inverse: $x = 5 \cdot 7^y$

$\frac{x}{5} = 7^y$

$\log_7\left(\frac{x}{5}\right) = \log_7 7^y$

$\log_7\left(\frac{x}{5}\right) = y$

so $\boxed{g^{-1}(x) = \log_7\left(\frac{x}{5}\right)}$