- Row-Reduced Form of a Matrix

    1. Each row consisting entirely of zeros lies below any other row having nonzero entries.
    2. The first nonzero entry in each row is 1 (called a **leading 1**).
    3. In any two successive (nonzero) rows, the leading 1 in the lower row lies to the right of the leading 1 in the upper row.
    4. If a column contains a leading 1, then the other entries in that column are zeros.

    NOTE: We only consider the coefficient side (the left side) of an augmented matrix when determining whether the matrix is in row reduced form.

- <u>Gauss-Jordan Row Operations</u>
    1. Interchange any two rows ($R_i \leftrightarrow R_j$).
    2. Replace any row by a nonzero constant multiple of itself (cR_i).
    3. Replace any row by the sum of that row and a constant multiple of any other row ($R_i + aR_j$).
- <u>Gauss-Jordan Elimination Method</u>
    1. Write the given linear system in augmented matrix form.
    2. Interchange rows, if necessary, to obtain an augmented matrix in which the first entry of the first row (the a_{11} entry) is nonzero. The a_{11} entry is the first "pivot element."
    3. Pivot about a_{11}, i.e., transform the a_{11} entry to a 1 and then transform the other elements in the first column into zeros using the 3 row operations.
    4. Now interchange the second row with any row below it, if necessary, to obtain an augmented matrix in which the second entry in the second row (the a_{22} entry) is nonzero. This is the second pivot element.
    5. Pivot about a_{22}.
    6. Continue in a similar manner until the the left side of the augmented matrix is in row-reduced form.

- *Pivoting* on an entry in a matrix means transform the pivot element into a 1 and then transform all other entries in the same column into 0's.

- Using RREF to Solve Systems of Equations
  STEP 1: Check the final matrix to see if there is no solution. (If the system has no solution, state so and stop here.
  Otherwise, go on to Step 2.)
  STEP 2: Circle the leading 1's.
  a) If each variable has a leading 1 in its column, then there is a unique solution.
  b) Otherwise, there are infinitely many solutions and each variable not having a leading one in its column is
  a parameter.

- Overdetermined systems have more equations than unknowns. These systems can have a unique solution, infinitely many solutions, or no solution.

- Underdetermined systems have fewer equations than unknowns. These systems can only have infinitely many solutions or no solution.

- If a system of equations has infinitely many solutions, you should represent the solutions in parametric form.

1. Find the equation of the line that passes through the origin and is perpendicular to the line $-3x+5y=7$.

(0,0)

$5y = 3x + 7$

$y = \frac{3}{5}x + \frac{7}{5}$

$m = \frac{3}{5}$

$m_2 = -\frac{5}{3}$

$y = mx + b$

$\boxed{y = -\frac{5}{3}x}$

2. Find the equation of the line that passes through $(-6,3)$ and is parallel to the line $-3x+5y=7$.

$m = \frac{3}{5}$

$m_2 = \frac{3}{5}$

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{3}{5}(x+6)$

$y = \frac{3}{5}x + \frac{18}{5} + 3$

$\boxed{y = \frac{3}{5}x + \frac{33}{5}}$

3. Find the equation of the line with an x-intercept of 8 and a y-intercept of -4.

(8,0)

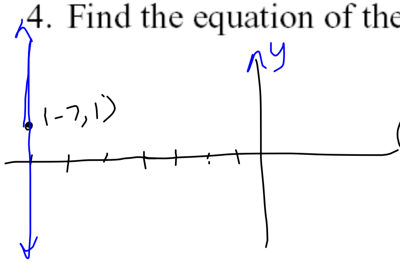
(0,-4) ← b

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-0}{0-8} = \frac{1}{2}$

$y = mx + b$

$\boxed{y = \frac{1}{2}x - 4}$

4. Find the equation of the line that passes through $(-7, 1)$ and is parallel to the y-axis.



$$x = -7$$

5. Entomologists have discovered that there is a linear relationship between the number of chirps of a certain species of crickets and the air temperature. When the temperature is 70^oF, they chirp 120 times per minute and when the temperature is 80^oF, 160 times per minute. Find an equation that gives the number of chirps as a function of the air temperature.

y is a ... x

$y = f(x)$

y is a function of x

$(70, 120)$

$(80, 160)$

$$m = \frac{160-120}{80-70} = 4$$

$$y - y_1 = m(x - x_1)$$

$$y - 120 = 4(x - 70)$$

$$y = 4x - 280 + 120$$

$$\boxed{y = 4x - 160}$$

$m_1 m_2 = -\frac{1}{m_2} m_2 = -1$

$m_1 m_2 = -1$

6. True/False

TRUE FALSE a) If two nonvertical lines with slopes m_1 and m_2 are perpendicular, then $m_1 m_2 = -1$.

TRUE FALSE b) A company will break even by supplying the same number of units as the market demands. $C(x) = R(x)$

TRUE FALSE c) $I_n A = A I_n = A$ for all matrices A.

E: $A = [1 \ \ 2]$ 1×2 $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 2×2

TRUE FALSE d) A nonsingular matrix has no inverse.

$AI_2 = I_2 A$

TRUE FALSE e) To be able to compute the matrix product AB, the number of columns of A must equal the number of rows of B.

$2\times 2 \cdot 1\times 2$

$2 \neq 1$

TRUE FALSE f) If B is a 2×2 matrix, then $B + I_2 = B$.

TRUE FALSE g) When solving the matrix equation $AX = B$ by computing $A^{-1}B$ in the calculator, a message of "ERR: SINGULAR MAT" implies that the system of equations has no solution.

rref

TRUE FALSE h) Every square matrix has an inverse.

TRUE FALSE i) An equation of a vertical line cannot be found because the slope of a vertical line is undefined.

$(1, 0)$

$x = 1$

TRUE FALSE j) If the parametric solution to a system of equations is $(3t+2, -t-3, t)$, then $(-7, 0, -3)$ is a particular solution.

$(3t+2, \ -t-3, \ t)$

$(-7, \ 0, \ -3)$

$t = -3$

$3t+2 = 3(-3)+2$

$= -9+2 = -7$

$-(-3)-3 = 3-3$

$= 0$

$$B = \begin{bmatrix} 5 & 0 \\ 8 & 7 \end{bmatrix}$$

$$B + I_2$$

$$\neq B$$

$$\begin{bmatrix} 5 & 0 \\ 8 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 8 & 8 \end{bmatrix}$$

$BI_2 = B$ ← True.

Size of A	Size of B
$m \times n$	$n \times p$

$=$

7. The demand equation for a particular brand of tennis shoes is given by $31x + 11y - 825 = 0$ and the corresponding supply equation is given by $-14x + 11y - 330 = 0$, where x represents the number of items demanded or supplied and y represents the price . Determine the market equilibrium quantity and price of the tennis shoes. Show your work or explain how you got your answer.

Demand Eqn

$31x + 11y - 825 = 0$

$y =$

Supply Eqn

$-14x + 11y - 330 = 0$

$y =$

Demand = Supply.

Finding an intersection point.

(the same as solving a system of equations.)

$31x + 11y = 825$

$-14x + 11y = 330$

Augmented Matrix:

$$\begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \left[\begin{array}{cc|c} 31 & 11 & 825 \\ -14 & 11 & 330 \end{array}\right] \end{array} \xrightarrow{rref} \begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \left[\begin{array}{cc|c} 1 & 0 & 11 \\ 0 & 1 & 44 \end{array}\right] \end{array}$$

$$\boxed{\begin{array}{l} x = 11 = \text{equil. quantity} \\ y = \$44 = \text{equil. price} \end{array}}$$

8. Fred and George have started a small business in which they sell Skiving Snackboxes for 2 Galleons (similar to dollars) each. Fred and George have a monthly fixed cost of 15 Galleons (a fee they must pay to Pansy Parkinson so that she doesn't inform Professor Snape about their unauthorized business).

1.5

$C(x) = 40$

(a) What is the production cost per unit if the twins experience a total cost of 40 Galleons when 100 Skiving Snackboxes are produced in a month?

$C(x) = cx + F$ where c = cost per unit, F = Fixed cost

$F = 15$

$C(x) = cx + 15$

.25

$40 = c(100) + 15$

$25 = 100c$

$\boxed{c = .25 \text{ Galleons}}$

(b) Find the linear cost function.

$\boxed{C(x) = .25x + 15}$

(c) Find the linear revenue function.

$R(x) = sx$ s = selling price

$\boxed{R(x) = 1.5x}$

(d) Find the profit function.

$P(x) = R(x) - C(x) = 1.5x - (.25x + 15)$

$= 1.5x - .25x - 15$

$\boxed{P(x) = 1.25x - 15}$

(e) What is the break-even point for Fred and George's Skiving Snackbox business?

$C(x) = R(x)$ or $P(x) = 0$

$0 = 1.25x - 15$

$15 = 1.25x$

$12 = x$ = break-even quantity

Break-even revenue:

$R(12) = 1.5(12) = 18$ Galleons

$\boxed{(12, 18)}$

9. Find the value of k so that the following system has no solution: $\begin{aligned} 5x+2y &= -7 \\ -3x+ky &= 8 \end{aligned}$

parallel lines → same slope

$5x + 2y = -7$

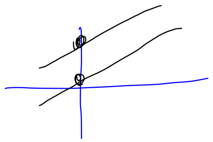
$(0, -\frac{7}{2})$

↑ y-int.

$2y = -5x - 7$

$y = -\frac{5}{2}x - \frac{7}{2}$

$m = -\frac{5}{2}$, $b = -\frac{7}{2}$



$-3x + ky = 8$

$ky = 3x + 8$

$y = \frac{3}{k}x + \frac{8}{k}$

$m = \frac{3}{k}$, $b_2 = \frac{8}{k}$

$-\frac{5}{2} = \frac{3}{k}$

slopes equal

$-5k = 6$

$\boxed{k = -6/5}$

We know $k = -6/5$

Check y-intercepts

$b_1 = -\frac{7}{2}$, $b_2 = \frac{8}{k} = \frac{8}{-6/5} = \frac{8}{1} \cdot \left(-\frac{5}{6}\right)$

$= \frac{-40}{6}$

$= \frac{-20}{3}$

$b_2 \neq -\frac{7}{2}$

10. A study was conducted to determine the relationship between household income and car ownership. The data are summarized in the table below, where household income is given (**in thousands of dollars**):

Household Income (x)	20	30	35	40	45	50
Percentage of Households Owning a Car at this Income Level (y)	61	57	68	74	83	86

(a) (4 points) Find the least squares regression equation for the data, assuming it is linear. *Round to* ***four*** *decimal places.*

$y = 0.9857x + 35.3571$

(b) (6 points) Use the result of part (a) to find the income level if 70% of households at this income level own a car. Round to the nearest dollar. Show your work or explain how you got your answer.

Find x when $y = 70$

$x = \$35$

$70 = 0.9857x + 35.3571$

$34.6429 = 0.9857x$

$35.14548\ldots = x$

$\boxed{\$35,145}$

(c) (3 points) What is the value of the correlation coefficient? Interpret its meaning.

$r = .9120$

There is a strong linear relationship in the data

11. For each of the following, if the matrix is in row reduced form, interpret its meaning as a solution to a system of equations. If the matrix is not in row reduced form, explain why.

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & \cancel{4} \\ 0 & 1 & 3 & \cancel{-5} \\ 0 & 0 & 1 & \cancel{0} \end{array}\right]$

Not in row-red. form.
Col. 3 has a leading 1 and not all other entries in that col. are 0.

(b) $\begin{array}{cc} x & y \end{array}$
$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$

Yes, $\left.\begin{array}{l} x=5 \\ y=3 \end{array}\right\}$ unique soln

$(5,3)$

(c) $\begin{array}{ccc} x & y & z \end{array}$
$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & -7 \end{array}\right]$

Yes. Let $y=t$ where t is any real #.

$x+3y=5$

$x=-3y+5$

$\boxed{x=-3t+5}$

$z=-7$

Parametric Soln

$\boxed{(-3t+5,\ t,\ -7)}$

(d) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 8 \end{array}\right]$

No!

The 1st nonzero # in row 2 is not 1.

(e) $\left[\begin{array}{ccc|c} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{array}\right]$

Yes.

No solution

$0x+0y+0z=1$

$0=1$ False

(f) $\left[\begin{array}{ccc|c} 0 & 1 & 0 & 5 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & -5 \end{array}\right]$

No. Leading 1's are not down & to the right of ea. other.

(g) $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 1 & 8 \end{array}\right]$

No. Row of all 0's should be below all other nonzero rows.

12. For the next two word problems do the following:
I) Define the variables that are used in setting up the system of equations.
II) Set up the system of equations that represents this problem. III) Solve for the solution.
IV) If the solution is parametric, then tell what restrictions should be placed on the parameter(s). Also give three specic solutions.

(a) The management of a private investment club has a fund of $300,000 earmarked for investment in stocks. To arrive at an acceptable overall level of risk, the stocks that the management is considering have been classied into three categories: high-risk, medium-risk, and low-risk. Management estimates that high-risk stocks will have a rate of return of 16 percent per year; medium-risk stocks, 10 percent per year; and low-risk stocks, 4 percent per year. The investment in medium-risk stocks is to be twice the investment in stocks of the other two categories combined. If the investment goal is to have an average rate of return of 11 percent per year on the total investment, determine how much the club should invest in each type of stock.

Let x = the amount of money invested in high-risk stocks.
Let y = the amount of money invested in medium-risk stocks.
Let z = the amount of money invested in low-risk stocks.

$$x + y + z = 300000$$

$$y = 2(x+z)$$

$$(y = 2x + 2z)$$

$$.16x + .1y + .04z = .11(300000) = 33000$$

$$x + y + z = 300000$$
$$-2x + y - 2z = 0$$
$$.16x + .1y + .04z = 33000$$

$$\begin{array}{ccc} x & y & z \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 300000 \\ -2 & 1 & -2 & 0 \\ .16 & .1 & .04 & 33000 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 75000 \\ 0 & 1 & 0 & 200000 \\ 0 & 0 & 1 & 25000 \end{array}\right]$$

$$\boxed{\begin{aligned} x &= \$75000 \\ y &= \$200000 \\ z &= \$25000 \end{aligned}}$$

Title: Feb 6 - 6:09 PM (14 of 26)

(b) A convenience store sold 23 sodas one summer afternoon in 12-, 16-, and 20-ounce cups (small, medium, and large). The total volume of soda sold was 376 ounces, and the total revenue was $48. If the prices for small, medium, and large sodas are $1, $2, and $3 respectively, how many of each size did the store sell that day?

(pg. 70-72, *Finite Mathematics* by Lial, et. al.)

Let x = the number of small sodas sold that day.
Let y = the number of medium sodas sold that day.
Let z = the number of large sodas sold that day.

$$x + y + z = 23$$
$$12x + 16y + 20z = 376$$
$$x + 2y + 3z = 48$$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 1 & 23 \\ 12 & 16 & 20 & 376 \\ 1 & 2 & 3 & 48 \end{array}\right] \end{array} \xrightarrow{\text{rref}} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 25 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Let $z = t$ where t is an integer greater than or equal to 0.

Find x and y:

$$x - z = -2$$
$$x = z - 2$$
$$x = t - 2 \geq 0$$
$x \geq 0$ so $t - 2 \geq 0$
$$t \geq 2$$

$$y + 2z = 25$$
$$y = -2z + 25$$
$$y = -2t + 25$$

Find y:

$$y = -2t + 25 \geq 0$$
$$-2t + 25 \geq 0$$
$$-2t \geq -25$$
$$t \leq \frac{25}{2} = 12.5$$

Parametric solution →

$$x = t - 2$$
$$y = -2t + 25$$
$$z = t$$

$(t-2, -2t+25, t)$ where $t = 2, 3, 4, \cdots, 12$

1st – ~~$t \geq 0$~~ (integer) and $t \geq 2$

$t \leq 12.5$

t can range from 2 to 12 (t is an integer)

Parametric Solution: $(t-2, -2t+25, t)$

The particular solution when $t=2$ is
(specific)

$(2-2, -2(2)+25, 2)$

$(0, 21, 2)$

13. Perform the first pivot in the Gauss-Jordan Elimination Method. Indicate all row operations used.

only row that is changing

$$\left[\begin{array}{ccc|c} 3 & -6 & 0 & 9 \\ 7 & 4 & -8 & 6 \\ 2 & -3 & 1 & 9 \end{array}\right] \xrightarrow{\frac{1}{3}R_1} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 3 \\ 7 & 4 & -8 & 6 \\ 2 & -3 & 1 & 9 \end{array}\right]$$

$$\xrightarrow{R_2 - 7R_1} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 3 \\ 0 & 18 & -8 & -15 \\ 2 & -3 & 1 & 9 \end{array}\right] \xrightarrow{R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 3 \\ 0 & 18 & -8 & -15 \\ 0 & 1 & 1 & 3 \end{array}\right]$$

$R_2 - 7R_1$

$[7 \; 4 \; -8 \; 6] - 7[1 \; -2 \; 0 \; 3]$

$[7 \; 4 \; -8 \; 6] + \cancel{[-7 \; 14 \; 0 \; -21]}$

$+ [-7 \; 14 \; 0 \; -21]$

$[0 \; 18 \; -8 \; -15]$

$R_3 - 2R_1$

$[2 \; -3 \; 1 \; 9] - 2[1 \; -2 \; 0 \; 3]$

$= [2 \; -3 \; 1 \; 9] + [-2 \; 4 \; 0 \; -6]$

$= [0 \; 1 \; 1 \; 3]$

14. Solve for the variables x, y, z, and u. If this is not possible, explain why.

$$\underbrace{\begin{bmatrix} -1 & 0 & 5 \\ 7 & 3 & 0 \end{bmatrix}\begin{bmatrix} 7x & -2 \\ 4 & -3z \\ x & 8 \end{bmatrix}}_{2\times 2} \underbrace{-4\begin{bmatrix} y & 10 \\ 10x & -2y \end{bmatrix}}_{2\times 2} = \underbrace{\begin{bmatrix} 9 & 3u \\ -6 & 0 \end{bmatrix}}_{2\times 2}$$

$$= \begin{bmatrix} -1 & 0 & 5 \\ 7 & 3 & 0 \end{bmatrix}\begin{bmatrix} 7x & -2 \\ 4 & -3z \\ x & 8 \end{bmatrix} - 4\begin{bmatrix} y & 10 \\ 10x & -2y \end{bmatrix}$$

$$\begin{array}{ccc} & C_1 & C_2 \\ = R_1 & -1(7x)+0\cdot 4+5x & 2+0+40 \\ R_2 & 49x+12+0 & -14+(-9z)+0 \end{array} + \begin{bmatrix} -4y & -40 \\ -40x & 8y \end{bmatrix}$$

$$= \begin{bmatrix} -2x & 42 \\ 49x+12 & -14-9z \end{bmatrix} + \begin{bmatrix} -4y & -40 \\ -40x & 8y \end{bmatrix} = ''$$

$$= \begin{bmatrix} -2x-4y & 2 \\ 49x+12-40x & -14-9z+8y \end{bmatrix} = ''$$

$$\begin{bmatrix} -2x-4y & 2 \\ 9x+12 & -14-9z+8y \end{bmatrix} = \begin{bmatrix} 9 & 3u \\ -6 & 0 \end{bmatrix}$$

$-2x-4y=9$

$-2(-2)-4y=9$

$4-4y=9$

$-4y=5 \rightarrow \boxed{y=-\frac{5}{4}}$

$9x+12=-6$

$9x=-18$

$\boxed{x=-2}$

$2=3u$

$\boxed{\frac{2}{3}=u}$

$-14-9z+8y=0$

$-9z+8\left(-\frac{5}{4}\right)=14$

$-9z-10=14$

$-9z=24$

$\cancel{z=-\frac{24}{9}}$

$\boxed{z=\frac{-8}{3}}$

15. Find the matrix A that makes the following equation true:

$$\begin{bmatrix} -5 & 3 \\ 8 & 7 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -7 & 5 \end{bmatrix} A = I_2$$

$$\underline{B} + \frac{1}{3}\underline{C}A = I_2$$

$$\frac{1}{3}\begin{bmatrix} 2 & -1 \\ -7 & 5 \end{bmatrix} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -5 & 3 \\ 8 & 7 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 2/3 & -1/3 \\ -7/3 & 5/3 \end{bmatrix}} A = \underbrace{\begin{bmatrix} 6 & -3 \\ -8 & -6 \end{bmatrix}}$$

$$\underbrace{Q^{-1}Q}A = Q^{-1}M$$

$$I_2 A = Q^{-1}M$$

$$A = Q^{-1}M$$

$$A = \begin{bmatrix} 2/3 & -1/3 \\ -7/3 & 5/3 \end{bmatrix}^{-1} \begin{bmatrix} 6 & -3 \\ -8 & -6 \end{bmatrix} \leftarrow \text{in calc}$$

$$A = \begin{bmatrix} 22 & -21 \\ 24 & -33 \end{bmatrix}$$

16. Use the given matrices to compute each of the following. If an operation is not possible, explain why.

$$A = \begin{bmatrix} -5 & 3 \\ 7 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 5 & -8 \\ 3 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & 4 & 7 \\ 6 & -3 & 1 \end{bmatrix},$$

$D = 3 \times 3$ nonsingular matrix.

$E = 4 \times 4$ singular matrix.

(a) $B + C$
(b) BC
(c) $D^{-1}C$
(d) $AB^T - 5C$
(e) DD^{-1}
(f) $E^{-1}E$
(g) CA
(h) C^{-1}

$$DD^{-1} = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

17. Work #41 on page 130 of Section 2.5 in your textbook by Tan. This is a problem about understanding the meaning of the entries in the product of two matrices. There is also another example in Week in Review #2.

18. First write the following system as a matrix equation and then solve by using a matrix inverse.

$$\begin{aligned} -3x &= 4y+z \\ y-7 &= -x+5z \\ 2x+z &= 14-y \end{aligned}$$

$$X = A^{-1}B$$

19. **This problem is from Section 3.1. Not all instructors are including this section on Exam 1.**

Graph the solution set for the following system of inequalities. Label all corner points. Is the solution set bounded or unbounded?

L_1 $x - y \leq 0$ → $x - y = 0$

L_2 $2x + y \leq 6$ $\quad -y = -x$

$4x - y \geq -6$ $\quad y = x$

Test a point $(0,1)$

$x - y \leq 0$

$0 - 1 \leq 0$?

$-1 \leq 0$? TRUE

L_2

$2x + y = 6$

When $x = 0$ $(0,6)$

$y = 6$

When $y = 0$, $2x = 6$ $(3,0)$

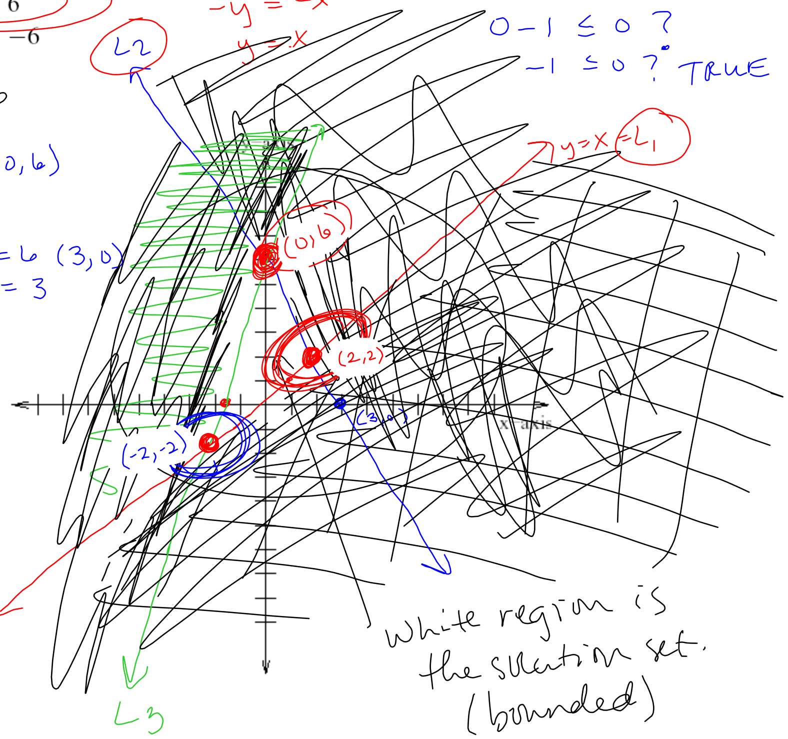
$x = 3$

Test $(0,0)$

$2x + y \leq 6$

$0 + 0 \leq 6$?

TRUE



white region is the solution set. (bounded)

L_3 $4x - y = -6$

When $x = 0$, $-y = -6$ $(0,6)$

$y = 6$

When $y = 0$, $4x = -6$ $(-\frac{3}{2}, 0)$

$x = -\frac{3}{2}$

Test $(0,0)$

$4x - y \geq -6$

$0 - 0 \geq -6$?

TRUE.

$\left.\begin{matrix} L_1 \\ L_3 \end{matrix}\right\}$ crossing

$$x - y = 0$$
$$4x - y = -6$$

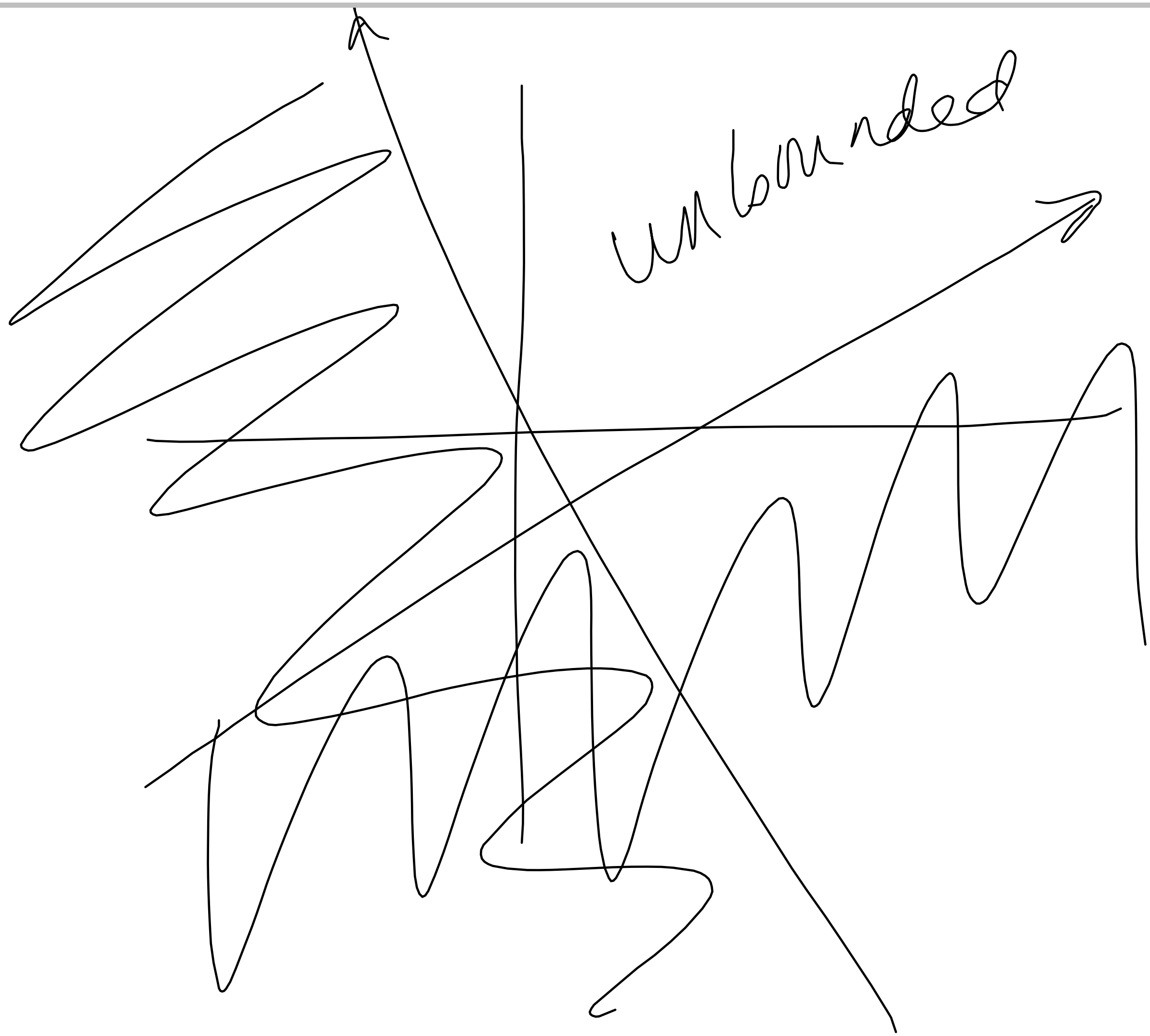
$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 4 & -1 & -6 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & -2 \end{array}\right] \quad (-2, -2)$$

L_2 + L_1 are crossing :

$$x - y = 0$$

$$2x + y = 6$$

$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 2 & 1 & 6 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 2 \end{array}\right] \quad (2,2)$$


unbounded

20. **This problem is from Section 3.2. Not all instructors are including this section on Exam 1.**

Ruff, Inc. makes dog food out of chicken and grain. Chicken has 10 grams of protein and 5 grams of fat per ounce, and grain has 2 grams of protein and 2 grams of fat per ounce. A bag of dog food must contain at least 200 grams of protein and at least 150 grams of fat. If chicken costs 10 cents per ounce and grain costs 1 cent per ounce, how many ounces of each should Ruff use in each bag of dog food in order to minimize cost? **Formulate as a linear programming problem, but do not solve.**

(#23, pg. 217 of *Finite Mathematics* by Waner and Costenoble)

1) Define variables
(2) State the objective & the objective function.
(3) State the constraints.

① Let x equal the number of ounces of chicken.
Let y equal the number of ounces of grain.

② Minimize Cost $\boxed{C = .1x + .01y}$
Objective
or $C = 10x + y$ Objective function

③

	Chicken	Grain	Min. Requirement
protein	10g/oz	2g/oz	200 grams
fat	5g/oz	2g/oz	150 grams
Cost per ounce	10¢	1¢	

$$\left.\begin{aligned} 10x + 2y &\geq 200 \\ 5x + 2y &\geq 150 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}\right\} \text{Constraints}$$