Math 166 - Exam 1 Review

NOTE: For reviews of the other sections on Exam 1, refer to the first page of WIR #1 and #2.

Section 1.5 - Rules for Probability

- **Elementary Rules for Probability** - For any event E in a sample space S, we have

  Rule 1: $0 \leq P(E) \leq 1$

  Rule 2: $P(S) = 1$

  Rule 3: $P(\emptyset) = 0$

- **Union Rule for Probability** - If E and F are any two events of a sample space S, then $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

- **Union Rule for Mutually Exclusive Events** - If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$.

- If $E_1, E_2, \ldots, E_n$ are mutually exclusive events, then $P(E_1 \cup E_2 \cup \cdots \cup E_n) = P(E_1) + P(E_2) + \cdots + P(E_n)$.

- **Complement Rule for Probability** - If E is an event of an experiment and E^c denotes the event that E does not occur, then $P(E^c) = 1 - P(E)$ and $P(E) = 1 - P(E^c)$.

- **Odds** - Odds are another way of representing the probability of an event and are given as a ratio of the number of times the event will occur to the number of times the event will not occur in the long run. Mathematically, the **odds in favor** of an event E are define to be a ratio of $P(E)$ to $P(E^c)$: odds in favor of $E = \dfrac{P(E)}{P(E^c)}$. NOTE: We reduce the ratio $\dfrac{P(E)}{P(E^c)}$ to lowest terms, $\dfrac{a}{b}$, and then we say that the odds in favor of E are a to b or $a : b$.

- **Obtaining Probability from Odds** - If the odds in favor of an event E occurring are a to b, then the probability of E occurring is $P(E) = \dfrac{a}{a+b}$.

Section 1.6 - Conditional Probability

- **Conditional Probability** - the probability of an event occurring given that another event has already occurred.

- We denote "the probability of the event A given that the event B has already occurred" by $P(A|B)$.

- **Conditional Probability of an Event** - If A and B are events in an experiment and $P(B) \neq 0$, then the conditional probability that the event A will occur given that the event B has already occurred is $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$.

- **Independent Events** - Two events A and B are independent if the outcome of one does not affect the outcome of the other.

- If A and B are independent events, then $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

- **Independent Events Theorem** - Let A and B be two events with $P(A) > 0$ and $P(B) > 0$. Then A and B are independent if and only if $P(A \cap B) = P(A)P(B)$.

- NOTE: To determine if two events are independent, you must compute the three probabilities $P(A \cap B)$, $P(A)$, and $P(B)$ *separately*, and then substitute these three numbers into the equation $P(A \cap B) = P(A)P(B)$. If the equality holds, then A and B are independent. If after substituting you find that $P(A \cap B) \neq P(A)P(B)$, then A and B are NOT independent (i.e., A and B are dependent and somehow affect each other).

- **Independence of More Than Two Events** - If $E_1, E_2, \ldots, E_n$ are independent events, then

$$P(E_1 \cap E_2 \cap \cdots \cap E_n) = P(E_1)P(E_2)\cdots P(E_n)$$

Section 1.7 - Bayes' Theorem

- There is a huge formula in the book for Bayes' Theorem, but you don't have to memorize it! Just make sure you know the conditional probability formula $P(A|D) = \frac{P(A \cap D)}{P(D)}$, and know how to use a tree diagram (or Venn diagram in some cases) to find these probabilities.

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1. Consider the propositions

   p: Bob will have a hamburger for lunch.
   q: Bob will have pizza for lunch.
   r: Fred will have a hamburger for lunch.

   (a) Write the proposition $r \wedge (p \veebar q)$ in words.

   Fred will have a hamburger for lunch, and Bob will either have a hamburger or pizza for lunch (but not both).

   (b) Write the proposition $(q \vee \sim p) \wedge r$ in words.

   Bob will have pizza for lunch or he will not have a hamburger for lunch, but (and) Fred will have a hamburger for lunch.

   (c) Write the proposition "Bob and Fred will both have a hamburger for lunch, or Bob will have pizza for lunch," symbolically.

   $(p \wedge r) \vee q$

   (d) Write the proposition "Bob will not have a hamburger or pizza for lunch, but Fred will have a hamburger for lunch," symbolically.

   $\sim(p \vee q) \wedge r$

2. Write a truth table for each of the following.

   (a) $\sim(\sim p \vee q) \vee (p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$\sim(\sim p \vee q)$	$p \wedge \sim q$	$\sim(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	F	F	T	F	F	F
T	F	F	T	F	T	T	T
F	T	T	F	T	F	F	F
F	F	T	T	T	F	F	F

(b) $\sim r \wedge (q \vee \sim p)$

p	q	r	$\sim p$	$\sim r$	$q \vee \sim p$	$\sim r \wedge (q \vee \sim p)$
T	T	T	F	F	T	F
T	T	F	F	T	T	T
T	F	T	F	F	F	F
T	F	F	F	T	F	F
F	T	T	T	F	T	F
F	T	F	T	T	T	T
F	F	T	T	F	T	F
F	F	F	T	T	T	T

Problems 3, 4, 6, and 8 are courtesy of Joe Kahlig.

3. True or False. $U = \{0,1,2,3,4,5,6,7,8,9\}$ and $A = \{0,1,2,3,4,5\}$

T F	$\emptyset \in A$		T F	$n(A) = 5$		T F	$n(\{3,4\}) = 2$
T F	$\emptyset \subseteq A$		T F	$\{1,3,5\} \in A$		T F	$n(\emptyset) = 1$
T F	$\{1,2,3\} \subseteq A$		T F	$2 \in A$		T F	$3 \in A^C$
T F	$2 \subseteq A$		T F	$\{\emptyset\} = \emptyset$		T F	$0 = \emptyset$

4. A = $\{a,b,c\}$

(a) List all subsets of A.

$\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$, $\{a,b,c\}$

(b) List all of the proper subsets of A.

all of the above except $\{a,b,c\}$

(c) Give an example of two subsets of A that are disjoint. If this is not possible, then explain why.

$\{a\}$ and $\{b\}$ are disjoint since $\{a\} \cap \{b\} = \emptyset$

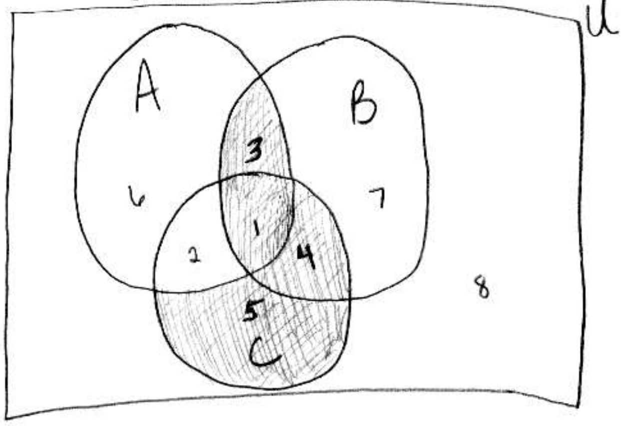
5. Shade the part of the Venn diagram that is represented by

(a) $(A^C \cup B) \cap (C \cup A)$

A^c - 5,4,7,8 } $A^c \cup B$ - $\boxed{5,4,7,8,3,1}$
B - 3,1,4,7

Intersect

C - 2,1,4,5 } $C \cup A$ - $\boxed{2,1,4,5,6,3}$
A - 6,3,2,1

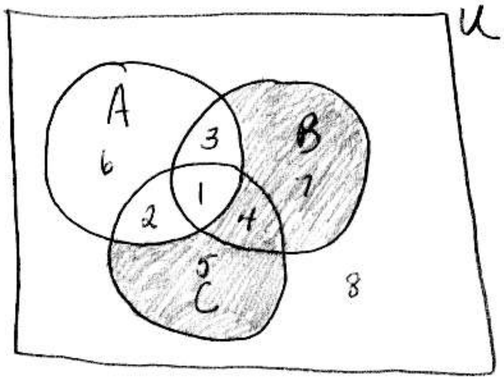


(b) $(B \cup C) \cap A^C$

$B - 1,3,4,7$
$C - 1,2,4,5$ $\Big\}$ $B \cup C - 1,3,4,7,2,5$

$\boxed{\text{Intersect}}$

$A^c - 5,4,7,8$



6. $U = \{0,1,2,3,4,5,6,7,8,9\}$, $A = \{1,3,5,7,9\}$, $B = \{1,2,4,7,8\}$, and $C = \{2,4,6,8\}$. Compute the following.

(a) $(A \cap B) \cup C$

$A \cap B = \{1,7\}$

$\boxed{(A \cap B) \cup C = \{1,7,2,4,6,8\}}$

(b) $A^C \cap B$

$A^c = \{0,2,4,6,8\}$

$\boxed{A^c \cap B = \{2,4,8\}}$

(c) $A \cap (B \cup C)^C$

$B \cup C = \{1,2,4,7,8,6\}$

$(B \cup C)^c = \{0,3,5,9\}$

$\boxed{A \cap (B \cup C)^c = \{3,5,9\}}$

7. Let U be the set of all A&M students. Let

$$A = \{x \in U | x \text{ owns an automobile}\}$$
$$D = \{x \in U | x \text{ lives in a dorm on campus}\}$$
$$F = \{x \in U | x \text{ is a freshman}\}$$

(a) Describe the set $(A \cap D) \cup F^c$ in words.

the set of all A&M students who both own an automobile and live in a dorm on campus, or who are not freshmen.

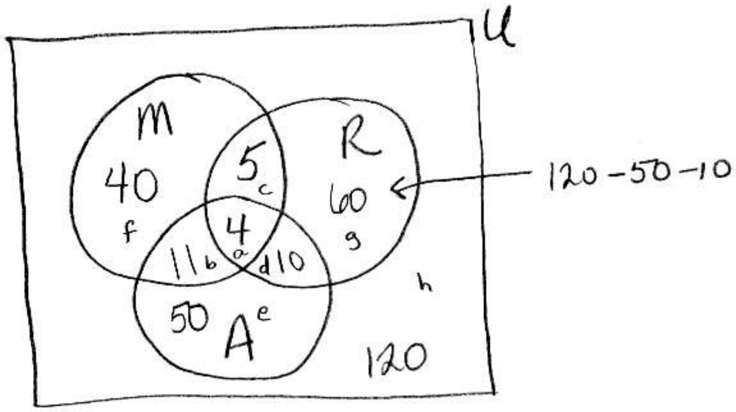
(b) Use set notation ($\cap, \cup, ^c$) to write the set of all A&M students who are freshmen living on campus in a dorm but do not own an automobile.

$F \cap D \cap A^C$

8. In a survey of 300 high school seniors:
   120 had not read *Macbeth* but had read *As You Like It* or *Romeo and Juliet.*
   6 61 had read *As You Like It* but not *Romeo and Juliet.*
   5 15 had read *Macbeth* and *As You Like It.*
   4 14 had read *As You Like It* and *Romeo and Juliet.*
   3 9 had read *Macbeth* and *Romeo and Juliet.*
   2 5 had read *Macbeth* and *Romeo and Juliet* but not *As You Like It.*
   1 40 had read only *Macbeth.*

   Let M = *Macbeth*, R = *Romeo and Juliet*, and A = *As You Like It.*

   (a) Fill in a Venn diagram illustrating the above information.



   (b) How many students read exactly one of these books?

   $40 + 60 + 50 = \boxed{150}$

   (c) How many students did not read *Romeo and Juliet*?

   $40 + 11 + 50 + 120 = \boxed{221}$

   (d) How many students read *Macbeth* or *As You Like It* and also read *Romeo and Juliet*?

   $5 + 4 + 10 = \boxed{19}$

   (e) Compute $n(M \cup (R^C \cap A)) =$ $11 + 50 + 40 + 4 + 5 = \boxed{110}$

   R^c - f, b, e, h
   A - a, b, d, e
   $R^c \cap A$ - b, e
   M - f, b, a, c
   $\cup$ $M \cup (R^c \cap A)$ - b, e, f, a, c

9. Find $n(A \cap B)$ if $n(A) = 8$, $n(B) = 9$, and $n(A \cup B) = 14$.

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$
$14 = 8 + 9 - n(A\cap B)$
$\boxed{3 = n(A\cap B)}$

10. A bag contains 3 pennies, a nickel, and two dimes. Two coins are selected at random from the bag and the monetary value of the coins (in cents) is recorded.

pp - 2
pn - 6
pd - 11
nd 15
dd 20

(a) What is the sample space of this experiment?

$S = \{2, 6, 11, 15, 20\}$

(b) Write the event E that the monetary value of the coins is less than 11 cents.

$E = \{2, 6\}$

(c) Write the event F that the nickel is drawn.

$F = \{6, 15\}$

(d) Are the events E and F mutually exclusive? Support your answer.

Since $E \cap F = \{6\} \neq \emptyset$, E and F are not mutually exclusive.

(e) Write the event G that the value of the coins is more than 25 cents.

$G = \{\ \} = \emptyset$

11. If p is a true statement and q is a false statement, find the truth value of each of the following statements:

(a) $\sim p \veebar \sim q$

F $\veebar$ T

T

True

(b) $p \vee \sim q$

T $\vee$ T

T

True

(c) $\sim (q \wedge \sim p)$

$\sim(F \wedge F)$
$= \sim(F)$
$= T$

True

12. Let $S = \{a, b, c, d, e, f\}$ be the sample space of an experiment with the following probability distribution:

Outcome	a	b	c	d	e	f
Probability	$\frac{3}{40}$	$\frac{4}{40}$	$\frac{7}{40}$	$\frac{14}{40}$	$\frac{9}{40}$	$\frac{3}{40}$

$\leftarrow \frac{24}{40} - \frac{7}{40} - \frac{14}{40}$

$1 - \frac{3}{40} - \frac{4}{40} - \frac{7}{40} - \frac{14}{40} - \frac{3}{40}$

Let $A = \{a, c, e\}$, $B = \{c, d, f\}$, and $C = \{b, d\}$ be events of the experiment and suppose $P(B) = \dfrac{24}{40}$.

(a) Fill in the missing probabilities in the probability distribution above.

(b) Is this a uniform sample space? Why or why not?

No. Outcomes are not equally likely.

(c) Find each of the following:

i. $P(A) = P(a) + P(c) + P(e) = \frac{3}{40} + \frac{7}{40} + \frac{9}{40} = \boxed{\frac{19}{40}}$

ii. $P(C) = P(b) + P(d)$
$= \frac{4}{40} + \frac{14}{40} = \boxed{\frac{18}{40}}$

iii. $P(B^c)$
$= 1 - P(B) = 1 - \frac{24}{40} = \frac{16}{40} = \boxed{\frac{2}{5}}$

iv. $P(A \cap B)$

$A \cap B = \{c\}$ $\boxed{P(A \cap C) = \frac{7}{40}}$

v. $P(A \cup B) = \frac{3}{40} + \frac{7}{40} + \frac{9}{40} + \frac{14}{40} + \frac{3}{40} = \frac{36}{40}$

$A \cup B = \{a, c, e, d, f\}$

(d) Are the events A and C mutually exclusive? Why or why not?

$A \cap C = \{c\} \neq \emptyset$ so not mutually exclusive.

13. One card is drawn at random from a standard deck of 52 playing cards. What is the probability that the card is

(a) a club? C-event that the card is a club

$$P(C) = \frac{n(C)}{n(S)} = \boxed{\frac{13}{52}}$$

(b) a face card?

F-event that the card is a face card.

$$P(F) = \frac{n(F)}{n(S)} = \boxed{\frac{12}{52}}$$

(c) a club or a face card?

$$P(C \cup F) = P(C) + P(F) - P(C \cap F)$$
$$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \boxed{\frac{22}{52}}$$

(d) neither a club nor a face card?

$$P(C^c \cap F^c) = P((C \cup F)^c) = 1 - P(C \cup F)$$
$$= 1 - \frac{22}{52} = \boxed{\frac{30}{52}}$$

14. Let E and F be two events of an experiment such that $P(E) = 0.35$, $P(F) = 0.4$, and $P(E \cup F) = 0.62$.

(a) Are E and F mutually exclusive events? Must find $P(E \cap F)$.

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$
$$0.62 = 0.35 + 0.4 - P(E \cap F)$$
$$0.13 = P(E \cap F)$$

since $P(E \cap F) > 0$, it is possible for these two events to occur at the same time. Therefore E and F are not mutually exclusive.

(b) Find $P(E^c \cup F^c)$.

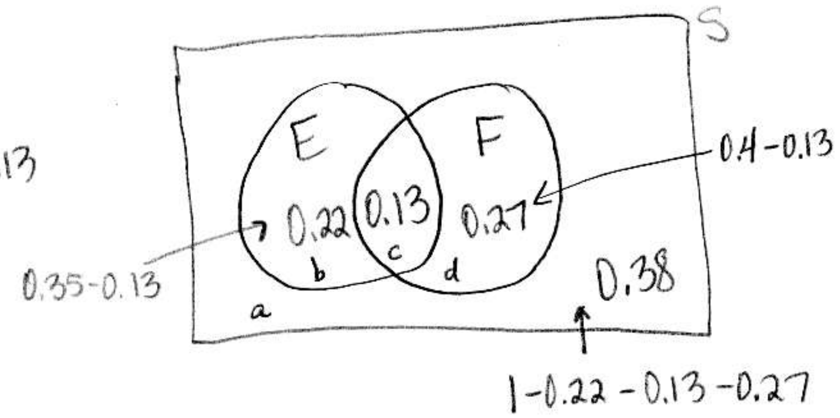
$$P(E^c \cup F^c) = P((E \cap F)^c)$$
$$= 1 - P(E \cap F) = 1 - 0.13 = \boxed{0.87}$$

(c) Find $P(E \cap F^c)$.

$$\boxed{0.22}$$

(d) Find $P(E \cup F^c) = 0.38 + 0.22 + 0.13$

$$= \boxed{0.73}$$



E - b,c

F^c - a,b

$E \cup F^c$ - a,b,c

15. A survey was conducted in which 1,000 students at Random University were asked how many hours they are currently taking. The results are given in the table below. Use the table to answer the following questions about a randomly selected student who participated in this survey.

F - freshmen
M - sophomores
J - Juniors
N - seniors

Classification	A 9 or less	B 10 to 13	C 14 to 17	D 18 or more	Total
Freshman	10	130	140	5	285
Sophomore	15	90	55	20	180
Junior	25	105	80	35	245
Senior	50	110	85	45	290
Total	100	435	360	105	1000

(a) What is the probability that the student is a freshman if he or she is registered for 18 or more hours?

given known

$$P(F|D) = \frac{5}{105}$$

← freshmen registered for 18 or more hours.

← students registered for 18 or more hours

(b) What is the probability that a senior is registered for 14-17 hours?

$$P(C|N) = \frac{85}{290}$$

← seniors registered for 14-17 hrs.

← seniors

(c) What is the probability that the student is a junior given that he or she is registered for 13 hours or less?

Juniors in reduced sample space

$$P(J|A \cup B) = \frac{25+105}{100+435} = \boxed{\frac{130}{535}}$$

← reduced sample space

16. A student has two exams in one day. The probability that he passes the first exam is 0.9, and the probability that he passes the second exam is 0.85. If the probability that the student passes at least one of the two exams is 0.97, are these two events independent?

A - passes 1st exam B - passes 2nd exam

Test

$P(A \cap B) \stackrel{?}{=} P(A)P(B)$

$0.78 \stackrel{?}{=} (0.9)(0.85)$

$0.78 \neq 0.765$

$\boxed{\text{not independent}}$

$P(A \cup B) = 0.97$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.97 = 0.9 + 0.85 - P(A \cap B)$

$0.78 = P(A \cap B)$

17. Let A, B, and C be three independent events of an experiment with $P(A) = 0.4$, $P(B) = 0.75$, and $P(C) = 0.3$. Calculate each of the following.

   (a) $P(A \cap B^C)$ $= P(A)\,P(B^C)$ (since independent)

   $= (0.4)(1 - 0.75)$

   $= \boxed{0.1}$

   (b) $P(A^C \cup C)$

   $P(A^C \cup C) = P(A^C) + P(C) - P(A^C \cap C)$

   $= (1 - 0.4) + 0.3 - (1 - 0.4)(0.3) = \boxed{0.72}$

   (c) $P(B|C)$

   $P(B|C) = P(B) = \boxed{0.75}$

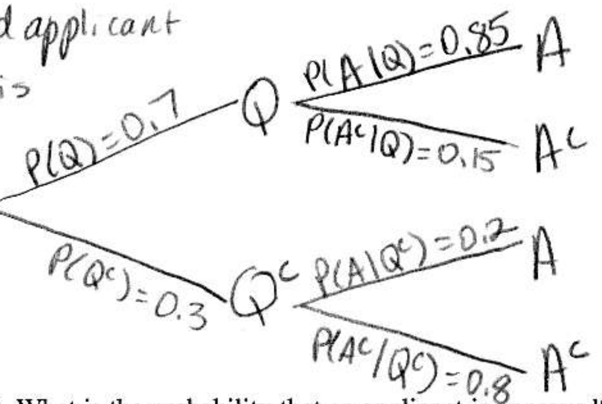
   (since Indep)

18. The personnel manager at a certain company claims that she approves qualified applicants for a certain job 85% of the time; she rejects an unqualified person 80% of the time. If 70% of all applicants for this job are qualified, find each of the following. (pg. 380 of *Finite Mathematics* by Lial, Greenwell, and Ritchey)

   (a) Draw a tree diagram (with probabilities and notation on all branches) representing the above information.

Q - qualified applicant

A - applicant is approved



   (b) What is the probability that an applicant is approved?

   $P(A) = (0.7)(0.85) + (0.3)(0.2) = \boxed{0.655}$

   (c) What is the probability that an applicant is approved for the job if he or she is unqualified?

   given

   $P(A|Q^c) = \boxed{0.2}$ (on tree)

   (d) What is the probability that an applicant is unqualified if he or she is approved for the job?

   given

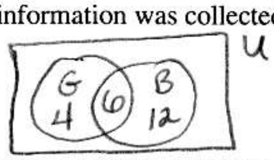
   $$P(Q^c|A) = \frac{P(Q^c \cap A)}{P(A)}$$

   $$= \frac{(0.3)(0.2)}{0.655} = \frac{12}{131} \approx 0.0916$$

   from (b)

19. A 7th grade class was selected and the following information was collected about the 30 students.

    4 students have only glasses.
    12 students have only braces.
    6 students have glasses and braces.



Determine whether the event that the student has glasses and the event that the student has braces are independent. Justify your answer.

G - student has glasses
B - " " braces

Test

$$P(G \cap B) \stackrel{?}{=} P(G)P(B)$$
$$\frac{6}{30} \stackrel{?}{=} \left(\frac{10}{30}\right)\left(\frac{18}{30}\right)$$
$$\frac{1}{5} \stackrel{?}{=} \frac{1}{5} \checkmark$$

$\boxed{\text{G and B are independent.}}$

20. The odds that it will not snow in College Station next winter are 17 to 2. What is the probability that it will snow in College Station next winter?

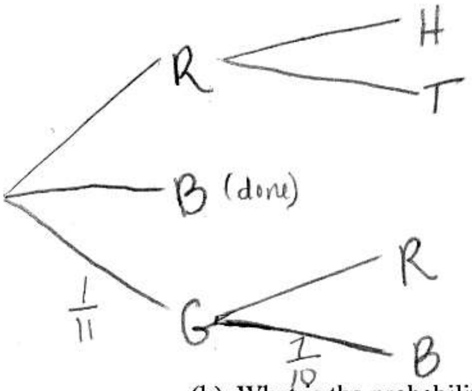
$$P(\text{will snow}) = \frac{2}{17+2} = \boxed{\frac{2}{19}}$$

odds 17 to 2 means 17 times it will not snow for every 2 times it does snow.

21. An experiment is as follows: One marble is drawn from a bag containing 3 red, 7 blue, and 1 green marble. The color of the marble selected is observed. If the marble is red, then a fair coin is flipped and the side landing up is observed. If the marble is green, then a second marble is drawn from the bag and the color is observed.

    (a) What is the sample space for this experiment?

R - red marble
B - blue marble
G - green marble



$$S = \{(R,H), (R,T), B, (G,R), (G,B)\}$$

    (b) What is the probability that the first marble is green and the second marble is blue?

$$P(\text{green 1st and blue 2nd}) = P(\text{green 1st})P(\text{blue 2nd} \mid \text{green 1st})$$
$$= \left(\frac{1}{11}\right)\left(\frac{7}{10}\right) = \boxed{\frac{7}{110}}$$

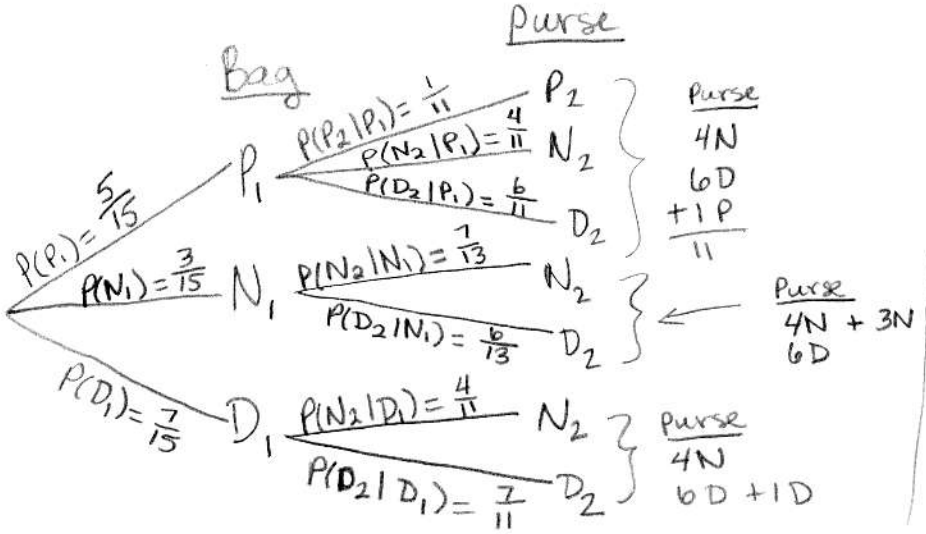
    (c) Write the event that a blue marble was selected during the experiment.

$$E = \{B, (G,B)\}$$

22. A bag contains 5 pennies, 3 nickels, and 7 dimes. A purse contains 4 nickels and 6 dimes. A coin is drawn from the bag and transferred to the purse, but if a nickel is selected from the bag, then all 3 of the nickels that were in the bag are transferred to the purse. A coin is then drawn from the purse. The type of coin drawn from each of the bag and purse is recorded. What is the probability that

P_1 - penny from bag
N_1 - nickel " "
D_1 - dime " "
P_2 - penny from purse
N_2 - nickel " "
D_2 - dime " "

(a) the transferred coin was a dime if a nickel was selected from the purse?



$$a)\ P(D_1|N_2) = \frac{P(D_1 \cap N_2)}{P(N_2)}$$
$$= \frac{(\frac{7}{15})(\frac{4}{11})}{(\frac{5}{15})(\frac{4}{11}) + (\frac{3}{15})(\frac{7}{13}) + (\frac{7}{15})(\frac{4}{11})}$$
$$= \frac{\frac{28}{165}}{\frac{57}{143}} = \boxed{\frac{364}{855}} \approx 0.4257$$

(b) both coins are pennies?

$$P(P_1 \cap P_2) = \left(\frac{5}{15}\right)\left(\frac{1}{11}\right) = \boxed{\frac{1}{33}}$$

(c) the coin drawn from the bag was a nickel or the coin drawn from the purse was a dime?

$$P(N_1 \cup D_2) = P(N_1) + P(D_2) - P(N_1 \cap D_2)$$
$$= \frac{3}{15} + \left[\left(\frac{5}{15}\right)\left(\frac{6}{11}\right) + \left(\frac{3}{15}\right)\left(\frac{6}{13}\right) + \left(\frac{7}{15}\right)\left(\frac{7}{11}\right)\right] - \left(\frac{3}{15}\right)\left(\frac{6}{13}\right) = \boxed{\frac{112}{165}}$$

(d) the coin drawn from the purse is a penny if the coin drawn from the bag was a nickel?
given

$P(P_2 | N_1) = 0$ (there are no pennies in the purse.)

23. Two cards are drawn at random from a standard deck of 52 playing cards. What are the odds that the second card drawn is a king given that the first card drawn was a queen?

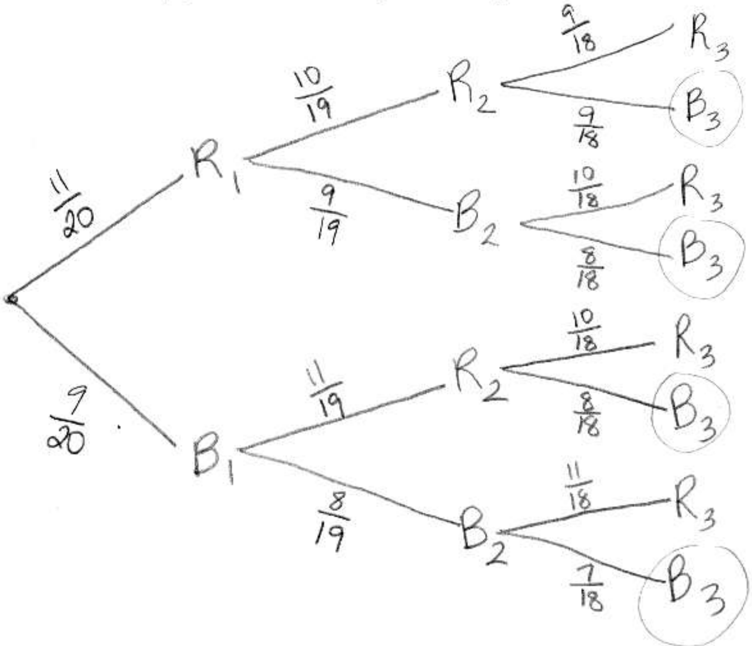
$$\text{probability} = \frac{4}{51}$$

$$\text{odds} = \frac{\frac{4}{51}}{1 - \frac{4}{51}} = \frac{\frac{4}{51}}{\frac{47}{51}} = \boxed{4 \text{ to } 47}$$

24. Three marbles are drawn one at a time without replacement from a bag containing 11 red and 9 blue marbles.

(a) Draw a tree diagram with probabilities on all branches for this experiment.

R_i = red marble on i^{th} d
B_i = blue marble on i^{th} d
$i = 1, 2, 3$



(b) What is the probability that the third marble selected is blue? $\frac{9}{20}$

$$P(B_3) = \left(\tfrac{11}{20}\right)\left(\tfrac{10}{19}\right)\left(\tfrac{9}{18}\right) + \left(\tfrac{11}{20}\right)\left(\tfrac{9}{19}\right)\left(\tfrac{8}{18}\right) + \left(\tfrac{9}{20}\right)\left(\tfrac{11}{19}\right)\left(\tfrac{8}{18}\right) + \left(\tfrac{9}{20}\right)\left(\tfrac{8}{19}\right)\left(\tfrac{7}{18}\right)$$

$$= \boxed{\tfrac{9}{20}}$$

(c) What is the probability that the third marble selected is blue if it is known that at least one of the first two marbles selected was red?

$$P(B_3 \mid \text{at least one red in } 1^{st} \text{ 2 draws}) = \frac{P(B_3 \cap \text{at least one red in } 1^{st} \text{ 2 draws})}{P(\text{at least one red in } 1^{st} \text{ 2 draws})}$$

$$= \frac{\left(\tfrac{11}{20}\right)\left(\tfrac{10}{19}\right)\left(\tfrac{9}{18}\right) \overset{(or)}{+} \left(\tfrac{11}{20}\right)\left(\tfrac{9}{19}\right)\left(\tfrac{8}{18}\right) \overset{(or)}{+} \left(\tfrac{9}{20}\right)\left(\tfrac{11}{19}\right)\left(\tfrac{8}{18}\right)}{\left(\tfrac{11}{20}\right)\left(\tfrac{10}{19}\right) + \left(\tfrac{11}{20}\right)\left(\tfrac{9}{19}\right) + \left(\tfrac{9}{20}\right)\left(\tfrac{11}{19}\right)}$$

$$= \frac{143/380}{77/95} = \boxed{\tfrac{13}{28}}$$

25. A building on campus has three vending machines: two Coke machines and a snack machine. Based on the model of the machines, the first Coke machine has a 12% chance of breaking down in a particular week, and the second Coke machine has a 4% chance of breaking down in a particular week. The snack machine has a 10% chance of breaking down in a particular week. Assuming independence, find the probability that exactly one machine breaks down.

C_1 - event that the 1st Coke machine breaks down.
C_2 - " " " 2nd " " "
N - " " " snack machine " "

$$P(\text{exactly 1 breaks down}) = P(C_1 \cap C_2^c \cap N^c) + P(C_1^c \cap C_2 \cap N^c) + P(C_1^c \cap C_2^c \cap N)$$

can multiply since events are independent $\longrightarrow$ $= (0.12)(1-0.04)(1-0.1) + (1-0.12)(0.04)(1-0.1) + (1-0.12)(1-0.04)(0.$

$$= \boxed{0.21984} = \frac{687}{3125}$$