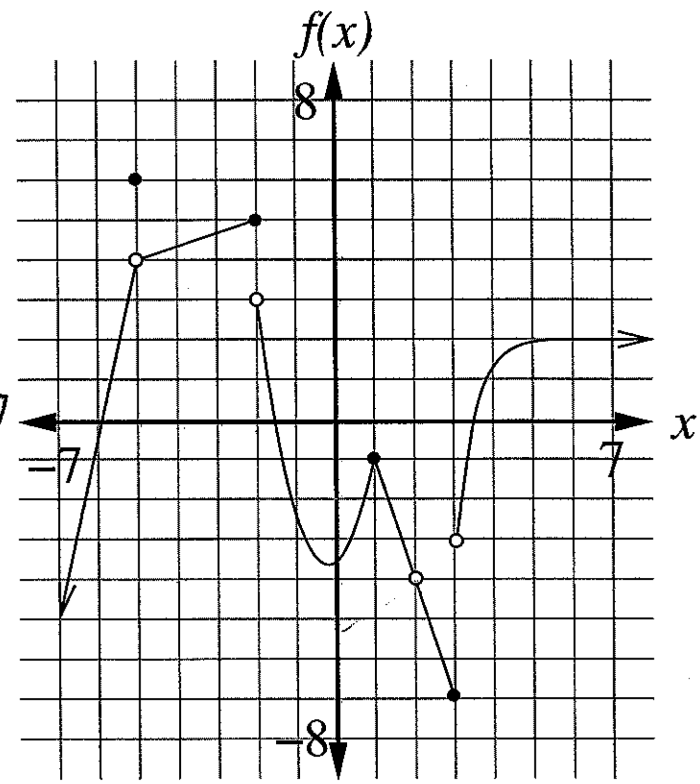
Math 142 - Exam 1 Review

NOTE: Exam 1 covers sections 1.2, 1.3, 2.1-2.5, Regression, and 3.1-3.3. This review is intended to highlight the material covered on Exam 1 but should not be used as your sole source of practice. Also refer to your instructor's lecture notes, previous week-in-reviews, suggested homework, supplemental homework, and the online homework as additional sources for review and exam preparation.

1. Use the graph of $f(x)$ below to answer each of the following.



(a) $\lim_{x\to 2} f(x) = -4$

(b) $\lim_{x\to -2^+} f(x) = 3$

(c) $\lim_{x\to 3} f(x)$ DNE since $\lim_{x\to 3^+} f(x) = -3 \neq \lim_{x\to 3^-} f(x) = -7$

(d) $f(-5) = 6$

(e) $\lim_{x\to \infty} f(x) = 2$

(f) $\lim_{x\to -\infty} f(x) = -\infty$

(g) Find all points of discontinuity of $f(x)$.

$x = -5, -2, 2, 3$

(h) Find all intervals over which $f(x)$ is continuous.

$(-\infty, -5) \cup (-5, -2) \cup (-2, 2) \cup (2, 3) \cup (3, \infty)$

(i) Find the domain and range of $f(x)$.

Domain: $(-\infty, 2) \cup (2, \infty)$

Range: $(-\infty, 4) \cup (4, 5] \cup \{6\}$

2. Find each of the following.

   (a) $\lim_{x \to -4} \frac{2x+8}{x^2-3x-28} = \lim_{x \to -4} \frac{2(x+4)}{(x+4)(x-7)} = \lim_{x \to -4} \frac{2}{(x-7)} = \frac{2}{-4-7} = \boxed{-\frac{2}{11}}$

   $\frac{0}{0}$ Indeterminate form

   Simplify!

   (b) $\lim_{x \to 10} f(x)$ where $f(x) = \begin{cases} 2x-5 & \text{if } x < 10 \\ k & \text{if } x = 10 \\ \frac{x^2-9x+5}{x-7} & \text{if } x > 10 \end{cases}$

   Check left and right limits!

   $\lim_{x \to 10^-} f(x) = 2(10)-5 = 15$

   $\lim_{x \to 10^+} f(x) = \frac{10^2-9(10)+5}{10-7} = \frac{15}{3} = 5$

   $\lim_{x \to 10} f(x)$ DNE since left and right limits are not equal.

   (c) Find all values of k for which $f(x)$ is continuous at $x = 10$. If this is not possible, explain why.

   For $f(x)$ to be continuous at $x=10$,

   ① $\lim_{x \to 10} f(x)$ must exist, ② $f(10)$ must exist, and ③ $\lim_{x \to 10} f(x) = f(10)$.

   Since $\lim_{x \to 10} f(x)$ DNE, there is no value of k for which $f(x)$ is continuous at $x=10$.

3. Determine the intervals where the following functions are continuous.

   (a) $f(x) = \log_5(x-7)$

   $x - 7 > 0$

   $x > 7$

   Continuous on $\boxed{(7, \infty)}$ (same as domain)

   (b) $g(x) = \frac{\sqrt[6]{x-4}}{x^2-8x+12} = \frac{\sqrt[6]{x-4}}{(x-6)(x-2)}$

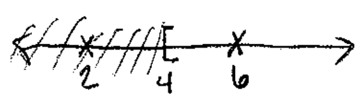
   $x - 4 \geq 0$

   $x \geq 4$

   AND

   $x \neq 6,\ x \neq 2$

   Continuous on $\boxed{[4,6) \cup (6, \infty)}$ (same as domain)



4. Find the domain of each of the following functions.

(a) $f(x) = \dfrac{\log_2(x-5)}{e^{2x}-3}$

$x - 5 > 0$
$x > 5$

AND

$e^{2x} - 3 \neq 0$
$e^{2x} \neq 3$
$2x \neq \ln 3$
$x \neq \dfrac{\ln 3}{2} \approx 0.5493$

$\boxed{\begin{array}{c}\text{Domain} \\ (5,\infty)\end{array}}$

(b) $g(x) = \begin{cases} \frac{1}{\ln(x-3)} & \text{if } 3 \leq x < 5 \\ \frac{1}{\sqrt[3]{4x-24}} & \text{if } x \geq 5 \end{cases}$

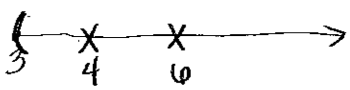
$x - 3 > 0$
$x > 3$

AND

$\ln 1 = 0$
so $x - 3 \neq 1$
$x \neq 4$

AND

$4x - 24 \neq 0$
$4x \neq 24$
$x \neq 6$



$\boxed{\begin{array}{l}\text{Domain} \\ (3,4) \cup (4,6) \cup (6,\infty)\end{array}}$

5. Evaluate each of the following.

(a) $\lim\limits_{x\to\infty} \dfrac{2x^2 - 4x^3 + 7}{9 + 4x + 7x^3} = \lim\limits_{x\to\infty} \dfrac{-4\cancel{x^3}}{7\cancel{x^3}} = \boxed{-\dfrac{4}{7}}$

(b) $\lim\limits_{x\to-\infty} \dfrac{5x - 2}{3x - 4x^2 + 3} = \lim\limits_{x\to-\infty} \dfrac{5x}{-4x^2} = \boxed{0}$ (highest power in denominator)

(c) $\lim\limits_{x\to-\infty} \dfrac{2x + 6x^2 + 1}{2 - 3x} = \lim\limits_{x\to-\infty} \dfrac{6x^2}{-3x} = \lim\limits_{x\to-\infty} -2x = \boxed{\infty}$

6. Find all asymptotes, x-coordinates of any holes, and intercepts of each of the following.

(a) $f(x) = \dfrac{4x^2 + 17x - 42}{x^2 + 4x - 12} = \dfrac{(4x-7)(x+6)}{(x+6)(x-2)}$

$x+6=0$ so $x=-6$

Hole at $x=-6$ (so there cannot be an x-intercept here)

vertical asymptote at $x=2$

$x-2=0$

$x=2$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \dfrac{4x^2}{x^2} = 4$ so **$y=4$ is a horizontal asymptote.**

y-int

$f(0) = \dfrac{-42}{-12} = \dfrac{7}{2}$ **$(0, \frac{7}{2})$ is the y-intercept**

x-int

$f(x)=0$ if and only if $4x^2+17x-42=0$

$(4x-7)(x+6)=0$

$x=-6$ was a hole, so no x-int. here

$4x-7=0$

$4x=7$

$x=\frac{7}{4}$

$(\frac{7}{4}, 0)$ is x-int

(b) $f(x) = \dfrac{8x - 16}{x^4 - 6x^3 + 8x^2}$

$f(x) = \dfrac{8(x-2)}{x^2(x^2-6x+8)} = \dfrac{8(x-2)}{x^2(x-2)(x-4)}$

Hole at $x=2$

$x^2(x-4)=0$

$x=0$, $x=4$ are vertical asymptotes

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \dfrac{8x}{x^4} = 0$

so **$y=0$ is a horizontal asymptote**

y-int

$f(0) = \dfrac{-16}{0}$ DNE

so **no y-intercept**

x-int

$f(x)=0$ if and only if $8x-16=0$

$8x=16$

$x=2$

This was the location of a hole, so

no x-intercepts

7. Acme Clothing produces game day wear for Aggie alumni. They can produce 31 polo style shirts for a total cost of $416 and 86 polo style shirts for a total cost of $636. They also know that when 100 shirts are demanded they can sell each shirt for $3, and when 30 shirts are demanded they can sell each shirt for $17.70. (courtesy Jenn Whitfield)

(a) How many shirts must Acme Clothing produce and sell to break even? (Assume the demand and cost functions are linear.)

$R(x) = px = (-0.21x+24)x = -0.21x^2+24x$

(100, 3)
(30, 17.70)
LinReg(ax+b) L_1, L_2

L_1	L_2
100	3
30	17.70

$p = -0.21x + 24$

$C(x) = cx + F = 4x + 292$
(31, 416)
(86, 636)
LinReg(ax+b) L_1, L_2

L_1	L_2
31	416
86	636

$R(x) = C(x)$

$-0.21x^2 + 24x = 4x + 292$ (Y_1 and Y_2)

Xmin = 0
xmax = 115
Zoom Fit
calc. Intersect

$X = 18.0032$
$X = 77.2349$

Approximately 18 or 77 shirts.

(Technically, it is not possible for Acme to exactly break even)

(b) If Acme's production of polo style shirts decreases by 15 shirts (Δx), what is the corresponding change in Acme's (Δy) total cost?

$C(x) = 4x + 292$

$m = 4 = \frac{\Delta y}{\Delta x}$ so $\frac{4}{1} = \frac{\Delta y}{-15}$

$\Delta y = -60$

Total cost will decrease by $60 if production is decreased by 15 shirts.

(c) How many shirts must be produced for the company to reach their maximum profit? (at vertex)

$P(x) = R(x) - C(x)$
$= -0.21x^2 + 24x - (4x + 292)$
$P(x) = -0.21x^2 + 20x - 292$

$x = \frac{-20}{2(-0.21)} = 47.6190$

$\approx$ 48 shirts

(d) At what price does the company need to sell their shirts to obtain maximum profit?

$p = -0.21x + 24$
$p = -0.21(48) + 24$
$p = \$13.92$

8. What sequence of graph transformations must be performed to obtain the graph of $g(x) = -5f(x+7) - 9$ from the graph of $f(x)$?

Shift the graph of f(x) 7 units to the left, then stretch vertically by a factor of 5, then reflect about the x-axis, and then shift down 9 units.

9. The following table shows the resale value of a particular model car as a function of mileage (how many miles the car has been driven).

Mileage (thousands of miles)	10	30	50	70	100	120
Resale Value (dollars)	13,700	10,200	7,900	6,500	5,000	4,300

Using either a linear, quadratic, or exponential model, find the best fitting regression model if used to predict the value of the car when the odometer reads

Note: neither prediction should be made with a linear model since the scatterplot shows definite curvature.

(a) 20,000 miles.

At $x=20$, the quadratic function seems to be a better fit: (INTERPOLATION)

$f(x) = 0.7588x^2 - 180.4245x + 15212.3492$ dollars, where x is the mileage of the car, in thousands of miles.

(b) 160,000 miles.

We have no reason to expect that the value of of the car will increase as the mileage increases past 120,000, so the exponential model would be more appropriate for this prediction.

$f(x) = 14108.7606(0.9897^x)$ dollars, where x is the mileage of the car in thousands of miles.

10. Bob invested an inheritance of $5,000 in an account paying 4.25% per year compounded semiannually, and 7 years later, he transferred the balance of this account into a new account paying 5.2% per year compounded continuously.

(a) Write a piecewise-defined function that gives the value of Bob's inheritance after t years.

$$A(t) = \begin{cases} 5000\left(1+\frac{0.0425}{2}\right)^{2t} & 0 \le t \le 7 \\ 6711.49e^{0.052(t-7)} & t > 7 \end{cases}$$

(6711.49 ↑ Balance of 1st account after 7 years)

(b) How much will Bob's inheritance be worth 10 years after the original investment?

$$A(10) = 6711.49e^{0.052(10-7)}$$

$$= \boxed{\$7844.57}$$

(c) How long will it take for Bob's investment to reach a value of $6,000?

Less than 7 years since $A(7) = 6711.49$

$$6000 = 5000\left(1+\frac{0.0425}{2}\right)^{2t}$$

$$\ln\left(\frac{6}{5}\right) = \ln(1.02125)^{2t}$$

$$\ln\left(\frac{6}{5}\right) = 2t\ln(1.02125)$$

$$t = \frac{\ln(6/5)}{2\ln(1.02125)}$$

$$\boxed{t = 4.3353 \text{ years}}$$

11. Find the domain, range, intercepts, and vertex of $p(x) = -5x^2 + 270x + 7$.

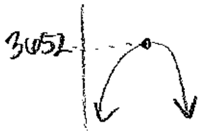
p(x) is a quadratic function, so **domain = $\mathbb{R}$**

Vertex

$x = \frac{-270}{2(-5)} = 27$

$p(27) = -5(27)^2 + 270(27) + 7$
$= 3,652$

(27, 3652) = vertex



Range: $(-\infty, 3652]$

y-int

$p(0) = 7$ so **(0,7) is the y-intercept**

x-int

$-5x^2 + 270x + 7 = 0$

$x = \frac{-270 \pm \sqrt{270^2 - 4(-5)(7)}}{2(-5)}$

$x = \frac{-270 \pm \sqrt{73040}}{-10}$

$x \approx -0.0259, 54.0259$

x-intercepts
(−0.0259, 0) and (54.0259, 0)

Look at #11 on pg 6 of the solutions to WIR #2 to see the method of completing the square.

12. Solve each of the following for x, y, or b as indicated. Give exact answers.

(a) $\log_2(\log(x^2 - 3x)) = 0$

$2^0 = \log(x^2 - 3x)$

so $\log_{10}(x^2 - 3x) = 1$

$10^1 = x^2 - 3x$
$0 = x^2 - 3x - 10$
$0 = (x-5)(x+2)$
$x = 5$ $x = -2$
(Both check alright)

(b) $\frac{9^{7x}}{81^3} = 27^{4x}$

$9^{7x} 81^{-3} = 27^{4x}$
$(3^2)^{7x}(3^4)^{-3} = (3^3)^{4x}$
$3^{14x} 3^{-12} = 3^{12x}$
$3^{14x-12} = 3^{12x}$

$14x - 12 = 12x$
$2x = 12$
$x = 6$

(c) $\log_{\frac{1}{4}} 16 = y$

$\frac{1}{4}^y = 16$

$4^{-y} = 16$

$-y = 2$ (since $4^2 = 16$)

$y = -2$

(d) $3\log x = \log(15x^2 - 10x) - \log 5$

$\log x^3 = \log\left(\frac{15x^2 - 10x}{5}\right)$

$x^3 = 3x^2 - 2x$
$x^3 - 3x^2 + 2x = 0$
$x(x^2 - 3x + 2) = 0$
$x(x-2)(x-1) = 0$

$x = 0$, **$x = 2$, $x = 1$**
($x = 0$: Not in Domain)

(e) $2 \cdot 7^x - 5 = 13$

$2 \cdot 7^x = 18$

$7^x = 9$

$\ln 7^x = \ln 9$

$x \ln 7 = \ln 9$

$x = \frac{\ln 9}{\ln 7}$

13. Describe the end behavior of each of the following.

(a) $p(x) = 3x - 7x^2 + 2x^3 - 5$

$\lim_{x \to \infty} p(x) = \infty$ and $\lim_{x \to -\infty} p(x) = -\infty$.

(As x increases without bound, p(x) increases without bound. As x decreases without bound, p(x) decreases without bound.)

(b) $g(x) = 5 - 7x + 6x^2 - 7x^4$

$\lim_{x \to \infty} g(x) = -\infty$ and $\lim_{x \to -\infty} g(x) = -\infty$.

14. If $\log x = 7$, $\log y = 4$, and $\log z = -2$, find $\log \frac{x^2}{yz^4}$.

$$\log \frac{x^2}{yz^4} = \log x^2 - \log yz^4$$
$$= 2\log x - (\log y + \log z^4)$$
$$= 2\log x - \log y - 4\log z$$
$$= 2(7) - 4 - 4(-2) = \boxed{18}$$

15. Find the limit as h approaches 0 of the difference quotient for $f(x) = 2 - 7x^2$ at $x = -1$.

Goal: $\lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h}$

① $f(-1+h) = 2 - 7(-1+h)^2$
$= 2 - 7(1 - 2h + h^2)$
$= 2 - 7 + 14h - 7h^2$
$= -7h^2 + 14h - 5$

② $f(-1) = 2 - 7(-1)^2 = -5$

③ $f(-1+h) - f(-1) = -7h^2 + 14h - 5 - (-5)$
$= -7h^2 + 14h$

④ $\frac{f(-1+h) - f(-1)}{h} = \frac{-7h^2 + 14h}{h} = \frac{h(-7h+14)}{h} = -7h + 14$

⑤ $\lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h}$

$= \lim_{h \to 0} (-7h + 14)$

$= \boxed{14}$