Note: Commands, requests, questions, and exclamations are not propositions.

Math 166 - Week in Review #~~3~~ #4 Spring 2007

Sections A.1 and A.2 - Propositions, Connectives, and Truth Tables

- A **proposition**, or *statement*, is a declarative sentence that can be classified as either true or false, but not both.
- **Prime propositions**, or *simple propositions*, are simple statements expressing a single complete thought.
- We use the lowercase letters p, q, r, etc. to denote prime propositions.
- Propositions that are combinations of two or more prime propositions are called **compound propositions**. The words used to combine propositions are called **logical connectives**.
- A **conjunction** is a statement of the form "p and q" and is represented symbolically by $p \wedge q$.

  The conjunction $p \wedge q$ is true if *both* p and q are true; it is false otherwise.
- A **disjunction** is a statement of the form "p or q" and is represented symbolically by $p \vee q$.

  The disjunction $p \vee q$ is **false** if *both* p and q are false; it is true in all other cases.
- An ***exclusive* disjunction** is a statement of the form "p or q" and is represented symbolically by $p \veebar q$.

  The disjunction $p \veebar q$ is **false** if *both* p and q are false AND it is **false** if both p and q are true; it is true only when exactly one of p and q is true.
- A negation is a proposition of the form "not p" and is represented symbolically by $\sim p$.

  The proposition $\sim p$ is true if p is false and vice versa.

Section 6.1 - Sets

- A *set* is a well-defined collection of objects.
- The objects in a set are called the *elements* of the set.
- Example of roster notation: $A = \{a, e, i, o, u\}$
- Example of set-builder notation: $B = \{x | x \text{ is a student at Texas A\&M}\}$
- Two sets are equal if and only if they have exactly the same elements.
- If every element of a set A is also an element of a set B, then we say that A is a *subset* of B and write $A \subseteq B$.
- If $A \subseteq B$ but $A \neq B$, then we say A is a proper subset of B and write $A \subset B$.
- The set that contains no elements is called the empty set and is denoted by $\emptyset$. (NOTE: $\{\} = \emptyset$, *but* $\{\emptyset\} \neq \emptyset$.)
- The *union* of two sets A and B, written $A \cup B$, is the set of all elements that belong either to A or to B or to both.
- The *intersection* of two sets A and B, written $A \cap B$, is the set of elements that A and B have in common.
- Two sets A and B are said to be **disjoint** if they have no elements in common, i.e., if $A \cap B = \emptyset$.
- If U is a universal set and A is a subset of U, then the set of all elements in U that are not in A is called the *complement* of A and is denoted A^c.
- De Morgan's Laws - Let A and B be sets. Then

  $(A \cup B)^c = A^c \cap B^c$

  $(A \cap B)^c = A^c \cup B^c$

Section 6.2 - The Number of Elements in a Finite Set

- The number of elements in a set A is denoted by $n(A)$.
- For any two sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
- For any three sets A, B, and C, $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.

1. Determine which of the following are propositions.

   (a) Do you know when the review starts? Not (a question)

   (b) What a surprise! Not. It's an exclamation.

   (c) She wore a black suit to the meeting. Is a proposition.

   (d) The number 4 is an odd number. Is a proposition.

   (e) $x - 5 = 4$ Not. Open ended sentence that cannot be classified as true or false.

   (f) Some of guests ate cake. Yes. Is a proposition

   (g) Please take off your hat before entering the MSC.
   Not. It's a command.

2. Write the negation of the following propositions.

(a) Bob will arrive before 8 p.m.

Bob will not arrive before 8pm.

At least 1 = One or more missed the meeting

(b) Some of the committee members missed the meeting.

negation: None of the committee members missed the meeting.

At least 1 is not listening

(c) Some of the kindergartners are not listening to the teacher.

None of the kindergartners are not listening to the teacher

or

☆ All of the kindergartners are listening to the teacher.

(d) All of the pencils have been sharpened.

Not all of the pencils have been sharpened.

or

At least one pencil has not been sharpened.

(e) None of the sodas are cold.

at least one soda is not cold.

or

Some of the sodas are cold.

3. Consider the following propositions:

p: Sally speaks Italian.
q: Sally speaks French.
r: Sally lives in Greece.

(a) Express the compound proposition, "Sally speaks Italian and French, and ~~but~~ she lives in Greece," symbolically.

$(p \wedge q) \wedge r$

(b) Express the compound proposition, "Sally lives in Greece, or she does not speak both Italian and French," symbolically.

$\vee$

$r \vee \sim (p \wedge q)$

(c) Write the statement $(p \veebar q) \wedge r$ in English.

Sally speaks either Italian or French (but not both), and she lives in Greece.

(d) Write the statement $\sim r \wedge \sim (p \vee q)$ in English.

Sally does not live in Greece and she does not speak Italian or French.

* Let n = the # of propositions. Then the truth table will have 2^n rows.

4. Construct a truth table for each of the following:

(a) $\sim(\sim p \vee \sim q)$

$2^2 = 4$ rows

p	q	$\sim p$	$\sim q$	$(\sim p \vee \sim q)$	$\sim(\sim p \vee \sim q)$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

(b) $(p \vee \sim q) \wedge q$

p	q	$\sim q$	$(p \vee \sim q)$	$(p \vee \sim q) \wedge q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	F
F	F	T	T	F

$2^3 = 8$ rows

(c) $\sim q \wedge \sim (p \vee r)$

and

p	q	r	$\sim q$	$(p \vee r)$	$\sim (p \vee r)$	$\sim q \wedge \sim (p \vee r)$
T	T	T	F	T	F	F
T	T	F	F	T	F	F
T	F	T	T	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	T	F	F	F	T	F
F	F	T	T	T	F	F
F	F	F	T	F	T	T

(d) $\sim(p \wedge q) \vee (q \wedge r)$

p	q	r	$p \wedge q$	$\sim(p \wedge q)$	$q \wedge r$	$\sim(p \wedge q) \vee (q \wedge r)$
T	T	T	T	F	T	T
T	T	F	T	F	F	F
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	F	T	T	T
F	T	F	F	T	F	T
F	F	T	F	T	F	T
F	F	F	F	T	F	T

5. Use set-builder notation to describe the collection of all history majors at Texas A&M University.

$\{x \mid x$ is a history major at Texas A&M$\}$

"such that"

6. Write the set $\{x|x$ is a letter in the word ABRACADABRA$\}$ in roster notation.

$\{a, b, r, c, d\}$

a list

7. Let U be the set of all A&M students. Define D, A, and C as follows:

$D = \{x \in U | x \text{ watches Disney movies}\}$

$A = \{x \in U | x \text{ watches action movies}\}$

$C = \{x \in U | x \text{ watches comedy movies}\}$

$\cup \to$ "or"

(a) Describe each of the following sets in words.

i. $A \cup C$ = The set of all A&M students who watch action or comedy movies (or both).

ii. $D \cap C \cap A^c$ The set of all A&M students who watch Disney movies and comedy movies but not action movies.

iii. $D \overset{\text{or}}{\cup} A \overset{\text{or}}{\cup} C$ The set of all A&M students who watch Disney or action or comedy movies (or some combination of the three).

iv. $C \cap (D \cup A)$ The set of all A&M students who watch comedy movies and who watch Disney or action movies.

(b) Write each of the following using set notation.

and → $\cap$

i. The set of all A&M students who watch comedy movies but not Disney movies.

$C \cap D^c$

ii. The set of all A&M students who watch only comedies of the three types of movies listed.

$C \cap D^c \cap A^c$

$C \cap (D \cup A)^c$

same by DeMorgan's Law

iii. The set of all A&M students who watch Disney movies or do not watch action movies.

$D \cup A^c$

DeMorgan's Law

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

8. Let $U=\{1,2,3,4,5,6,7,8,9,10\}$, $A=\{1,5,10\}$, $B=\{1,3,5,7,9\}$, and $C=\{2,4,6,10\}$. Find each of the following.

(a) $A\cup B = \{1,5,10,3,7,9\}$

(b) $B\cap C = \emptyset$

intersection : AND

Look for what B and C have in common

(c) $C^c = \{1,3,5,7,8,9\}$

(d) $A\cap(B\cup C)$

$B\cup C=\{1,3,5,7,9,2,4,6,10\}$

$\boxed{A\cap(B\cup C)=\{1,5,10\}}$

(e) $(A\cup C)^c\cup B$

$A\cup C=\{1,5,10,2,4,6\}$

$(A\cup C)^c=\{3,7,8,9\}$

$B=\{1,3,5,7,9\}$

union

$\boxed{(A\cup C)^c\cup B=\{3,7,8,9,1,5\}}$

FACT:

If A is a set with n elements, then A has 2^n subsets.

(f) How many subsets does C have?

C has 4 elements, so C has $2^4 = \boxed{16 \text{ subsets}}$

(g) How many proper subsets does C have?

$\boxed{16 - 1 = 15 \text{ proper subsets}}$

(h) Are A and C disjoint sets?

$A \cap C = \{10\}$ A and C are not disjoint because they have an element in common.

(i) Are B and C disjoint sets?

$B \cap C = \emptyset$, so B and C are disjoint.

$$S = \{a, b, c\}$$

The subsets of S:

$\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, $\{a, b, c\}$

not proper

To see if a set A is a proper subset of B:

1) Is every element in A also in B?
(If yes, then $A \subseteq B$.)

2) Does B have more?
(If yes, then $A \subset B$.) (proper subset)

9. Let $U = \{a,b,c,d,e,f,g,h,i\}$, $A = \{a,c,h,i\}$, $B = \{b,c,d\}$, $C = \{a,b,c,d,e,i\}$, and $D = \{d,b,c\}$.

(a) Find $n(A)$. $n(A) = 4$

(b) Find $n(B \cup C)$.

in common

$$n(B \cup C) = n(B) + n(C) - n(B \cap C)$$
$$= 3 + 6 - 3$$
$$= \boxed{6}$$

(c) Find $n(A \cap B)$.

$A \cap B = \{c\}$ $\boxed{n(A \cap B) = 1}$

Use the sets above to determine if the following are true of false.

(d) TRUE FALSE $A \subseteq C$ (There is no h in the set C.)

(e) TRUE FALSE $B \subset C$

(f) TRUE FALSE $D \subset B$ (B does not have more)

(g) TRUE FALSE $\emptyset \subseteq A$ The empty set is a subset of every set.

(h) TRUE FALSE $\{c\} \in A$ A would have to be $A = \{a, \{c\}, h, i\}$ for this to be true.

(i) TRUE FALSE $d \in C$

(j) TRUE FALSE $C \cup C^c = U$

(k) TRUE FALSE $A \cap A^c = 0$ zero $A \cap A^c = \emptyset$: This is true.

(l) TRUE FALSE $(B \cup B^c)^c = \emptyset$

$B^c \cap B = \emptyset$

DeMorgan's Law

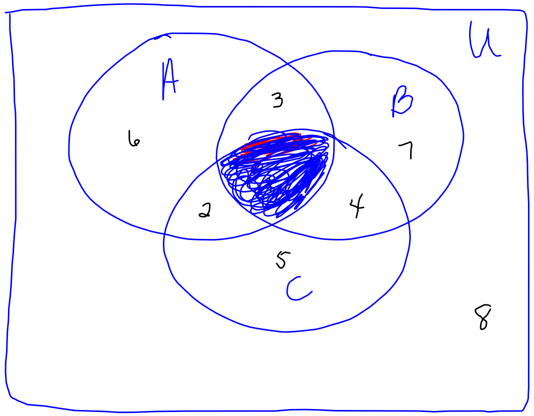
FACT:

$0 \neq \emptyset$

10. Draw a Venn diagram and shade each of the following.

(a) $A \cap B \cap C$

A and B and C



(b) $A \cup (B \cap C^c)$

B - 1, 3, 4, 7

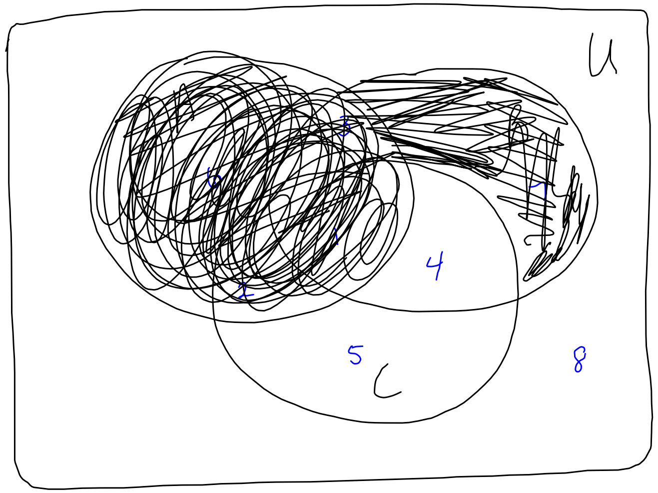
C^c - 3, 6, 7, 8

$\cap$

$B \cap C^c$ - 3, 7

A - 1, 2, 3, 6

$\cup$



$A \cup (B \cap C^c)$ - 1, 2, 3, 6, 7 ← shade these regions

(c) $A \cup (B \cap C)^c$

$B - 1,3,4,7$ $\Big\}\ \cap$
$C - 1,2,4,5$

$B \cap C - 1,4$

$(B \cap C)^c - 2,3,5,6,7,8$ $\Big\}$ union
$A - 1,2,3,6$

$A \cup (B \cap C)^c - 1,2,3,5,6,7,8$

(d) $(A \cap C)^c$

Section 6.2 Notes

1) $n(A)$ represents the number of elements in the set A.

2) $\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$

in common

Example:

$A = \{2, 4\}$

$B = \{3, 4, 5\}$

$A \cup B = \{2, 3, 4, 5\}$

$n(A) = 2$

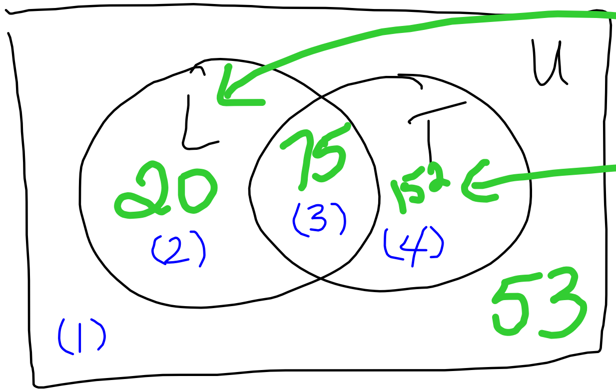
$n(B) = 3$

$n(A \cup B) = 2 + 3 - 1$

$= \boxed{4}$

11. A survey of 300 people found that 95 of those surveyed like licorice, 75 like taffy and licorice, and 53 like neither of these two candies.

(a) How many people surveyed like at least one of the two types of candy? Let L be the set of those surveyed who like licorice.
Let T - - - - - - - - - - - - taffy.



$95-75=20$

$300-20-75-53=152$

(a) Regions 2, 3, and 4 or $300-53=247$

$20+75+152=\boxed{247}$

(b) How many people surveyed like exactly one of these two types of candy?

Regions 2 and 4

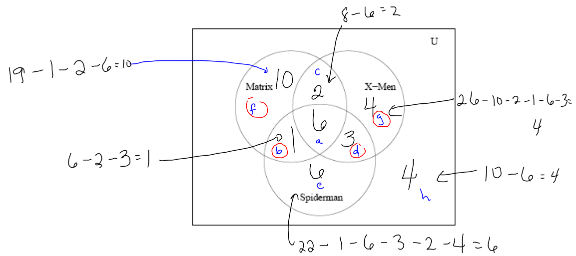
$20+152=\boxed{172}$

12. A survey of some college students was conducted to see which of the following three movies they had seen: *The Matrix*, *X-Men*, and *Spiderman*. It was found that

- 6 students had seen all three movies.
- 8 students had seen *The Matrix* and *X-Men*
- 3 students had seen *X-Men* and *Spiderman* but not *The Matrix*.
- 6 students had seen exactly 2 of the 3 movies. b, c, d
- 10 students had seen neither *X-Men* nor *The Matrix*. e and h
- 19 students had seen *The Matrix*.
- 26 students had seen *The Matrix* or (union) *X-Men*. f, c, g, b, a, d
- 22 students had seen *X-Men* or *Spiderman*. a, b, c, d, e, g

come back

(a) Fill in the Venn Diagram, illustrating the above information.



(b) How many students surveyed had seen at least one of the three movies?

$6 + 1 + 2 + 3 + 6 + 10 + 4 = \boxed{32}$

(c) How many students surveyed had seen only *Spiderman*?

$\boxed{6}$

(d) How many students surveyed had seen *The Matrix* or *X-Men* but not both?

$10 + 4 + 1 + 3 = \boxed{18}$

(e) How many students surveyed had seen *The Matrix* and *Spiderman*?

$1 + 6 = \boxed{7}$

13. Some students were asked whether they had one or more of the following types of animals as children: dog, cat, fish.

Let D be the set of those surveyed who had a dog.
Let C – — — – cat.
Let F — – — – fish.

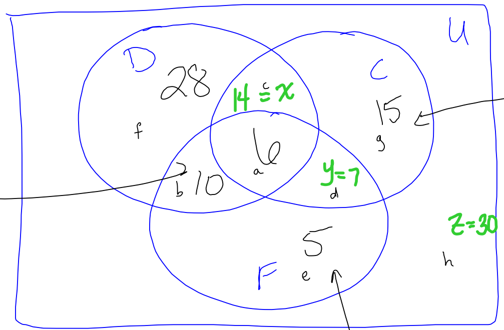
- ~~28~~ said they only had a dog.
- ~~6~~ said they had all three of these pets.
- ~~16~~ said they had a dog and a fish.
- ~~15~~ said they had a fish but did not have a cat.
- ~~48~~ said they only had one of these types of pets.
- 57 said they had a fish or a cat.
- 87 said they did not have a fish. ← $28 + 15 + x + z = 87$
- 57 said they did not have a dog.

need a system of eqns

$x + z + 43 = 87$

$x + z = 44$

(a) Fill in a Venn Diagram illustrating the above information.



$16 - 6 = 10$

$48 - 5 - 28 = 15$

$15 - 10 = 5$

57 had a fish or a cat:

$10 + 6 + 5 + 15 + x + y = 57$

$x + y + 36 = 57$

$x + y = 21$

System:

$$\begin{aligned} x + y \quad &= 21 \\ x \quad + z &= 44 \\ y + z &= 37 \end{aligned}$$

$$\begin{array}{ccc|c} x & y & z & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 21 \\ 1 & 0 & 1 & 44 \\ 0 & 1 & 1 & 37 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 14 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 30 \end{array}\right]$$

57 did not have a dog:

$15 + y + 5 + z = 57$

$y + z + 20 = 57$

$y + z = 37$

$x = 14$
$y = 7$
$z = 30$

(b) How many students were in the survey?

Add all #'s from all 8 regions!

$6 + 10 + 14 + 7 + 5 + 28 + 15 + 30 = \boxed{115}$