Math 141 - Week in Review #5

Section 4.1 - Simplex Method for Standard Maximization Problems

- A *standard maximization problem* is a linear programming problem that satisfies each of the following:

  1) The objective function is to be maximized.

  2) All variables are nonnegative.

  3) All constraints other than the nonnegativity (standard) constraints can be written in the form "variables"$\leq$ nonnegative number.

- Steps of the Simplex Method

  1. Set up the initial simplex tableau.
     (a) Create **slack variables**. Slack variables are used to convert inequalities into equalities.
     (b) Rewrite the objective function so that it is in the form $-c_1x_1 - c_2x_2 - \cdots - c_nx_n + P = 0$, i.e.
        – the coefficent of P is 1.
        – all variables and P are on the same side of the equal sign.
     (c) Place the constraints and the objective function in the *initial simplex tableau*–the augmented matrix that represents the new system of equalities formed by including slack variables in the constraints and with P as the last equation in the system.

  2. Determine whether or not the optimal solution has been reached.
     – The optimal solution has been reached if all entries in the last row to the left of the vertical line are non-negative.
     – If the optimal solution has been reached, skip to step 4.
     – If the optimal solution has not been reached, proceed to step 3.

  3. Perform Pivot Operations
     (a) Locate pivot element.
        – pivot column: Locate the **most negative** entry to the left of the vertical bar *in the last row*. The column containing this entry is the pivot column. (**NOTE:** If there are multiple columns with the same, most negative number, choose any one of these columns.)
        – pivot row: Divide each *positive* entry in the pivot column into its corresponding entry in the constants column. The pivot row is the row corresponding to the smallest ratio that results. (**NOTE:** If there are multiple rows with the same, smallest ratio, choose any one of these rows.)
        – pivot element: The element shared by the pivot row and pivot column.
     (b) Pivot about the pivot element. We will use the Simplex Program on our calculator to do the pivoting for us.
     (c) Return to step 2.

  4. Determine the solution: The value of the variable heading each unit column is given by the entry lying in the column of constants in the row containing a 1. The variables heading columns that are not unit columns are assigned the value 0.

Section 6.1 - Sets

- A *set* is a well-defined collection of objects.
- The objects in a set are called the *elements* of the set.
- Example of roster notation: $A = \{a, e, i, o, u\}$
- Example of set-builder notation: $B = \{x | x \text{ is a student at Texas A\&M}\}$
- Two sets are equal if and only if they have exactly the same elements.
- If every element of a set A is also an element of a set B, then we say that A is a *subset* of B and write $A \subseteq B$.
- If $A \subseteq B$ but $A \neq B$, then we say A is a proper subset of B and write $A \subset B$.
- The set that contains no elements is called the empty set and is denoted by $\emptyset$. (NOTE: $\{\} = \emptyset$, *but* $\{\emptyset\} \neq \emptyset$.)
- The *union* of two sets A and B, written $A \cup B$, is the set of all elements that belong either to A or to B or to both.
- The *intersection* of two sets A and B, written $A \cap B$, is the set of elements that A and B have in common.
- Two sets A and B are said to be **disjoint** if they have no elements in common, i.e., if $A \cap B = \emptyset$.
- If U is a universal set and A is a subset of U, then the set of all elements in U that are not in A is called the *complement* of A and is denoted A^c.
- De Morgan's Laws - Let A and B be sets. Then

  $(A \cup B)^c = A^c \cap B^c$

  $(A \cap B)^c = A^c \cup B^c$

Section 6.2 - The Number of Elements in a Finite Set

- The number of elements in a set A is denoted by $n(A)$.
- For any two sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
- For any three sets A, B, and C, $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.

Problems

1. The following simplex tableau is not in final form.

x	y	z	u	v	w	P	Constant	
0	5	1	-7	0	0	0	200	200/5 = 40
1	~~-4~~	0	5	0	7	0	300	
0	3	0	6	1	3	0	150	150/3 = 50
0	-2	0	3	0	-1	1	450	

Smallest ratio

(a) What is the value of each variable at this stage of the simplex method?

$x = 300$ $\quad u = 0$

$y = 0$ $\quad v = 150$ $\quad P = 450$

$z = 200$ $\quad w = 0$

(b) What is the location of the next pivot? You do not need to perform the pivot.

R 1 C 2

2. (a) Set up the initial simplex tableau for the following linear programming problem. Circle the location of the first pivot.

$$\begin{aligned} \text{Maximize} \quad & P = 3x + 5y + 2z \\ \text{subject to} \quad & x + 4y + 2z \le 2000 \\ & 2x + 4y + 5z \le 3200 \\ & x \ge 0, y \ge 0, z \ge 0 \end{aligned}$$

← add slack variables to these

$$x + 4y + 2z + u = 2000$$
$$2x + 4y + 5z + v = 3200$$
$$-3x - 5y - 2z + P = 0$$

$$\begin{array}{c} \\ \rightarrow \\ \\ \\ \end{array}\begin{array}{cccccc|c} x & y & z & u & v & P & \\ 1 & 4 & 2 & 1 & 0 & 0 & 2000 \\ 2 & 4 & 5 & 0 & 1 & 0 & 3200 \\ \hline -3 & -5 & -2 & 0 & 0 & 1 & 0 \\ & \uparrow & & & & & \end{array}$$

$2000/4 = 500$ ←

$3200/4 = 800$

R1 C2

(b) Perform the first pivot and write down the resulting matrix. Then find the value of each variable at this stage of solving the problem. Has the maximum value of P been reached?

After 1st Pivot

$$\begin{array}{cccccc|c} x & y & z & u & v & P & \\ 1/4 & 1 & 1/2 & 1/4 & 0 & 0 & 500 \\ 1 & 0 & 3 & -1 & 1 & 0 & 1200 \\ \hline -7/4 & 0 & 1/2 & 5/4 & 0 & 1 & 2500 \\ \uparrow & & & & & & \end{array}$$

$x = 0$ $\quad u = 0$

$y = 500$ $\quad v = 1200$

$z = 0$ $\quad P = 2500$

No, we have not found the maximum profit yet.

3. Use set-builder notation to describe the collection of all history majors at Texas A&M University.

give a rule for membership

$A = \{ x \mid x$ is a history major at A&M $\}$

"such that"

4. Write the set $\{x|x$ is a letter in the word ABRACADABRA$\}$ in roster notation.

$B = \{a, b, r, c, d\}$

5. Let U be the set of all A&M students. Define D, A, and C as follows:

← element of

$D = \{x \in U | x$ watches Disney movies$\}$

$A = \{x \in U | x$ watches action movies$\}$

$C = \{x \in U | x$ watches comedy movies$\}$

(a) Describe each of the following sets in words.

i. $A \cup C$ the set of all A&M students who watch action or comedy movies.

or

ii. $D \cap C \cap A^c$ ← complement "not in"

and and

the set of all A&M students who watch Disney and comedy and not action movies.

but

iii. $D \cup A \cup C$

or or

the set of all A&M students who watch at least one of Disney, action, or comedy movies.

iv. $C \cap (D \cup A)$

and or

the set of all A&M students who watch comedy and either Disney or action movies.

(b) Write each of the following using set notation. ← with unions, intersections, and complements.

i. The set of all A&M students who watch comedy movies but not Disney movies.

← same as "and"

$$\boxed{C \cap D^c}$$

ii. The set of all A&M students who watch only comedies of the three types of movies listed.

watch comedies and not action and not Disney

Both of these are correct.

$$\boxed{\underset{\text{(and)}}{C \cap A^c} \underset{\text{(and)}}{\cap D^c} = C \cap (D \cup A)^c}$$

iii. The set of all A&M students who watch Disney movies or do not watch action movies.

$$D \cup A^c$$

$$\boxed{D \cup A^c}$$

DeMorgan's Laws

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

6. Let $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{1,5,10\}$, $B = \{1,3,5,7,9\}$, and $C = \{2,4,6,10\}$. Find each of the following.

"or"

(a) $A \cup B = \{1,3,5,7,9,10\} = \{1,5,10,3,7,9\}$

Order in which you list elements does not matter!

"and"

(b) $B \cap C = \{\ \} = \emptyset$

in common

$\emptyset \neq 0$

very important!

(c) $C^c = \{1,3,5,7,9,8\}$

not in C

(d) $A \cap (B \cup C)$

$B \cup C = \{1,3,5,7,9,2,4,6,10\}$

$A = \{1,5,10\}$

Intersect $\rightarrow$ $\boxed{A \cap (B \cup C) = \{1,5,10\}}$

(e) $(A \cup C)^c \cup B$

Start with $A \cup C$, and then take the complement of it.

$(A \cup C)^c = (\{1,5,10,2,4,6\})^c = \{3,7,8,9\}$ $\leftarrow (A \cup C)^c$

$B = \{1,3,5,7,9\}$

UNION

$\boxed{(A \cup C)^c \cup B = \{3,7,8,9,1,5\}}$

(f) How many subsets does C have?

Fact: If a set has n elements, then the set has 2^n subsets.

Since C has 4 elements, C has $2^4 = 16$ subsets.

(g) How many proper subsets does C have?

$\boxed{15}$

(h) Are A and C disjoint sets?

check $A \cap C = \{10\}$

No, A and C are not disjoint because they have an element in common

(i) Are B and C disjoint sets?

$\boxed{B \cap C = \emptyset}$ so yes, B and C are disjoint.

New Example

List all the subsets of the set

$A = \{1, 2, 3\}$.

A has 3 elements, so
A has $2^3 = 8$ subsets.

* Note! The empty set is a subset of EVERY set.

Subsets of A

$\emptyset$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$,
$\{1,3\}$, $\{2,3\}$, $\{1,2,3\}$

↑ same as A, so not a proper subset.

8 subsets total, 7 of which are proper subsets of A.

Total # of subsets $= 2^n$ where $n =$ # of elements in the set.

Total # of proper subsets: $(2^n) - 1$.

NOTATION

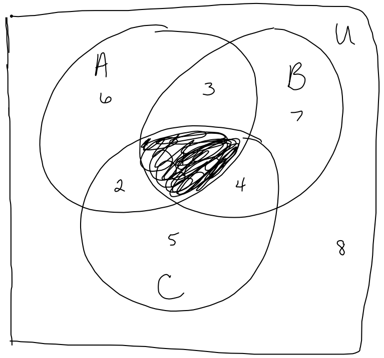
A is subset of B → $A \subseteq B$

A is a proper subset of B → $A \subset B$

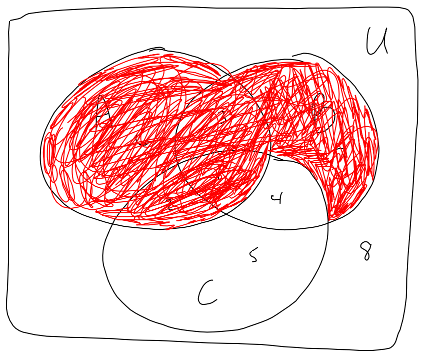
7. Draw a Venn diagram and shade each of the following.

(a) $\boxed{A \cap B \cap C}$

in A and in B and in C



(b) $A \cup (B \cap C^c)$



B – 7, 1, 3, 4
C^c – 7, 3, 6, 8

$\cap \rightarrow$ in common

$B \cap C^c$ – 7, 3

A – 6, 2, 1, 3

$\cup \rightarrow$ 7, 3, 6, 2, 1

$A \cup B^c \cup C^c$

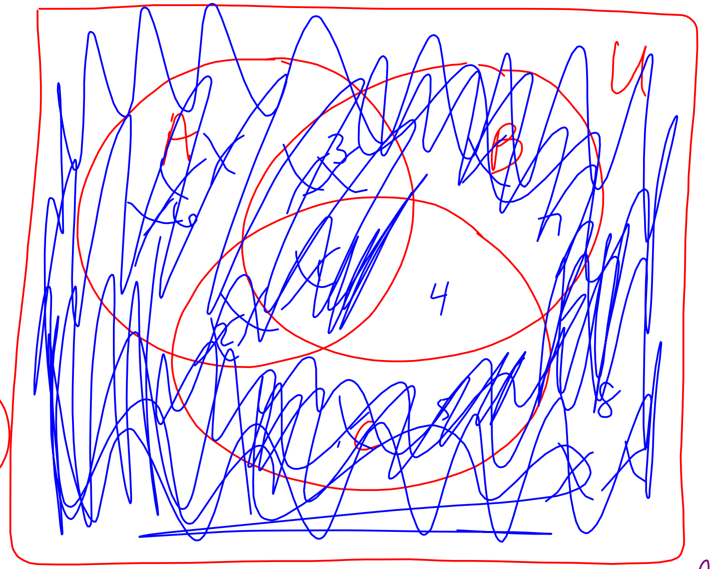
(c) $A \cup (B \cap C)^c$

$(B \cap C)^c - (1, 4)^c$

B and C -

$(B \cap C)^c$ — 2, 3, 5, 6, 7, 8

A — 1, 2, 3, 6



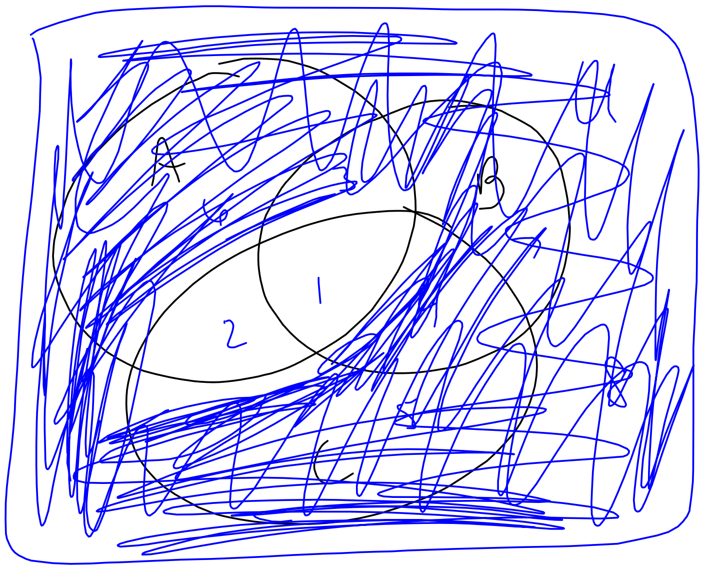
$A \cup (B \cap C)^c$ — 1, 2, 3, 5, 6, 7, 8 ← regions to be shaded

(d) $(A \cap C)^c$

$A \cap C$ - 1, 2

$(A \cap C)^c$ — 3, 4, 5, 6, 7, 8

regions to be shaded



8. Let $U = \{a,b,c,d,e,f,g,h,i\}$, $A = \{a,c,h,i\}$, $B = \{b,c,d\}$, $C = \{a,b,c,d,e,i\}$, and $D = \{d,b,c\}$.

(a) Find $n(A)$.

number of elements in $A = n(A) = 4$

(b) Find $n(B \cup C)$.

$$n(B \cup C) = n(B) + n(C) - n(B \cap C)$$
$$= 3 + 6 - 3 = \boxed{6}$$

(c) Find $n(A \cap B)$.

$n(A \cap B) = 1$

Use the sets above to determine if the following are true of false.

(d) TRUE FALSE $A \subseteq C$ because $h \in A$ but $h \notin C$

(e) TRUE FALSE $B \subset C$ Everything in B is also in C and C has more.

(f) TRUE FALSE $D \subset B$ It would have been true to write $D \subseteq B$

(g) TRUE FALSE $\emptyset \subseteq A$

(h) TRUE FALSE $\{c\} \in A$ → It would be true if we had written $c \in A$ or $\{c\} \subseteq A$.

(i) TRUE FALSE $d \in C$ ← d is an element of C.

(j) TRUE FALSE $C \cup C^c = U$

(k) TRUE FALSE $A \cap A^c = 0$ ← this would have been true if we had written $A \cap A^c = \emptyset$.

(l) TRUE FALSE $(B \cup B^c)^c = \emptyset$

$B^c \cap (B^c)^c$

$B^c \cap B = \emptyset$

"and"

9. A survey of 300 people found that 95 of those surveyed like licorice, 75 like taffy and licorice, and 53 like neither of these two candies.

Use a Venn Diagram.

(a) How many people surveyed like at least one (union) of the two types of candy?

Add #'s in regions 2, 3, and 4 $\quad 20 + 75 + 152 = \boxed{247}$

(b) How many people surveyed like exactly one of these two types of candy?

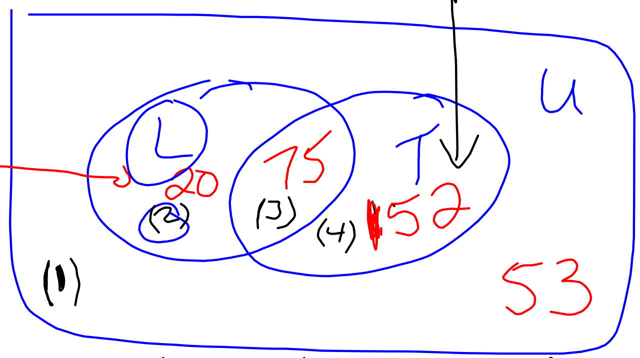
$20 + 152 = \boxed{172}$

$300 - 20 - 75 - 53$

Let L be the set of those surveyed who said they like licorice,

Let T - - - - - - - - - - - taffy.

$95 - 75$



(black #'s in parentheses represent the region number)

Regions 2 & 3 must add to 95

10. A survey of some college students was conducted to see which of the following three movies they had seen: *The Matrix*, *X-Men*, and *Spiderman*. It was found that

- 6 students had seen all three movies.
- 8 students had seen *The Matrix* and *X-Men* regions 1 + 3 add to 8
- 3 students had seen *X-Men* and *Spiderman* but not *The Matrix*.
- 6 students had seen exactly 2 of the 3 movies. regions 2, 3, 4 add to 6
- 10 students had seen neither *X-Men* nor *The Matrix*. regions 5 and 8
- ~~19~~ students had seen *The Matrix*. 1, 2, 3, 6
- 26 students had seen *The Matrix* or *X-Men*. 1, 2, 3, 4, 6, 7
- 22 students had seen *X-Men* or *Spiderman*. 1, 2, 3, 4, 5, 7

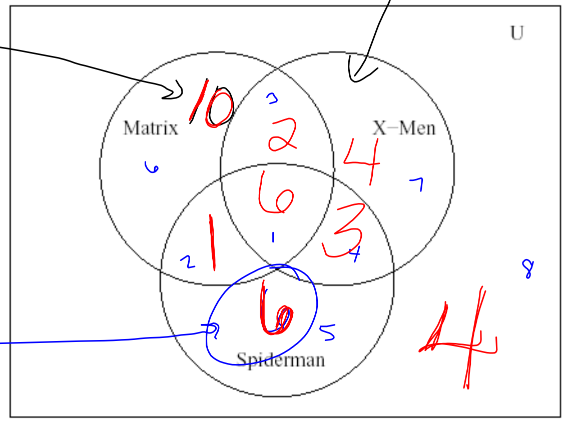
Blue #'s represent the region you are in.

(a) Fill in the Venn Diagram, illustrating the above information.

26 − 10 − 2 − 6 − 3 − 1

19 − 2 − 6 − 1

22 − 1 − 6 − 3 − 2 − 4



(b) How many students surveyed had seen at least one of the three movies?

10 + 2 + 4 + 6 + 1 + 3 + 6 = $\boxed{32}$ or union of all three − regions 1 thru 7.

(c) How many students surveyed had seen only *Spiderman*?

six students

(d) How many students surveyed had seen *The Matrix* or *X-Men* but not both?

regions 2, 6, 7, and 4 ← add 1 + 10 + 4 + 3 = $\boxed{18}$

(e) How many students surveyed had seen *The Matrix* and *Spiderman*?

1 + 6 = $\boxed{7}$

11. Some students were asked whether they had one or more of the following types of animals as children: dog, cat, fish.

Must use variables for these

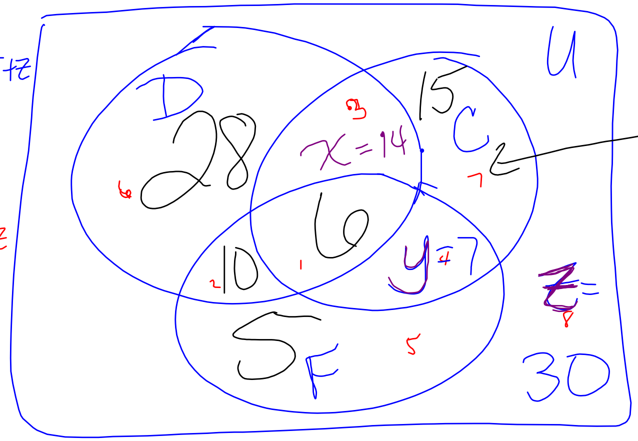
28 said they only had a dog.
6 said they had all three of these pets.
16 said they had a dog and a fish. regions 2 and
15 said they had a fish but did not have a cat.
48 said they only had one of these types of pets.
57 said they had a fish or a cat.
87 said they did not have a fish.
57 said they did not have a dog.

Let D be the set of those surveyed who had a dog as a child.
Let C — — — cat . . .
Let F — — fish . . .

(a) Fill in a Venn Diagram illustrating the above information.

$87 = 28 + x + 15 + z$
$x + z = 44$

$57 = 15 + y + 5 + z$
$y + z = 37$



$57 = 5 + 10 + 6 + x + y + 15$
$57 = x + y + 36$
$21 = x + y$
$x + y = 21$

Summary

$$\left.\begin{aligned} x + y &= 21 \\ x + z &= 44 \\ y + z &= 37 \end{aligned}\right\} \text{Solve.}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 21 \\ 1 & 0 & 1 & 44 \\ 0 & 1 & 1 & 37 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 14 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 30 \end{array}\right] \quad x=14 \quad y=7 \quad z=30$$

(b) How many students were in the survey?

Add all regions $\boxed{115}$ people.

12. With the time remaining, I will take requests from Joe Kahlig's counting handouts.