# Math 166 - Week in Review #5

Section 6.3 - Multiplication Principle

- If you wish to accomplish a big goal that requires intermediate steps and would like to know how many different ways there are to accomplish this big goal, simply list each of the individual steps required to meet this goal. Next to each step, write the number of ways that step can be done. To get the total number of ways of accomplishing the big goal, multiply all the numbers listed next to the individual steps. This is the multiplication principle.
- Formal Definition of the Generalized Multiplication Principle - Suppose that a task $T_1$ can be performed in $n_1$ ways, a task $T_2$ can be performed in $n_2$ ways, $\ldots$, and, finally, a task $T_k$ can be performed in $n_k$ ways. Then the number of ways of performing tasks $T_1, T_2, \ldots, T_k$ in succession is given by the product $n_1 n_2 \cdots n_k$.

Section 6.4 - Permutations and Combinations

- Permutation - Given a set of distinct objects, a *permutation* of the set is an arrangement of these objects in a definite order.
- For permutations, ORDER MATTERS.
- Factorial Notation: For any positive integer $n$, $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$. For example, $6! = 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1$.
- The factorial symbol ! can be found on the calculator by pressing $\boxed{\text{MATH}}$, arrowing over to PRB, and selecting option 4.
- Permutations of $n$ Distinct Objects - The number of permutations of $n$ distinct objects taken $r$ at a time is $P(n,r) = \dfrac{n!}{(n-r)!}$.
- In the notation $P(n,r)$, the $P$ stands for permutation, the $n$ is the number of distinct objects that you are starting with, and the $r$ is the number of those objects that you are arranging.
- The $nPr$ command can be found in the calculator by pressing $\boxed{\text{MATH}}$, arrowing over to PRB, and selecting option #2.
- Permutations of $n$ Objects, **Not All Distinct** - Given a set of $n$ objects in which the first type of object is repeated $n_1$ times, the second type of object is repeated $n_2$ times, $\ldots$ ,and, finally, the last type of object is repeated $n_r$ times so that $n_1 + n_2 + \cdots + n_r = n$, then the number of permutations of these $n$ objects taken $n$ at a time is given by $\dfrac{n!}{n_1! n_2! \cdots n_r!}$.
- Combination - A combination is a subset of objects chosen from a given set where the order in which the objects were chosen does not matter.
- For combinations, order DOES NOT MATTER.
- Combinations of $n$ Objects - The number of ways of choosing $r$ objects from $n$ distinct objects is given by $C(n,r) = \dfrac{n!}{r!(n-r)!}$.
- In the notation $C(n,r)$, the $C$ stands for combination, the $n$ is the number of objects that you are starting with, and the $r$ is the number of those objects that you are choosing to be in a subset.
- The $nPr$ command can be found in the calculator by pressing $\boxed{\text{MATH}}$, arrowing over to PRB, and selecting option #2.

1. Jeffrey is trying to find the perfect engagement ring for his girlfriend at a local jewelry store, and the jeweler has informed him that he has many decisions to make. He must first decide on the metal to be used–either yellow gold, white gold, or platinum. Then he must decide on the setting. The jewelry store has three types of settings: a solitaire setting, a setting with sidestones, and a multiple-stone setting. Next, he must choose the shape of the main diamond. The options are round, princess, emerald, asscher, marquise, oval, pear, and heart. After selecting the shape of diamond, he must choose from four different sizes for the diamond. How many possible engagement rings are there for Jeffrey to choose from?

$T_1$ – choose the metal $n_1 = 3$

$T_2$ – choose the setting $n_2 = 3$

$T_3$ – shape of diamond $n_3 = 8$

$T_4$ – size of diamond $n_4 = 4$

$3 \cdot 3 \cdot 8 \cdot 4 =$ $\boxed{\text{288 possible rings}}$

2. Bill and Sue and seven of their friends go to the movies. They all sit next to each other in the same row. How many ways can this be done if

(a) Sue and Bill must sit next to each other?

$T_1$ – pick 2 adjacent chairs for Bill and Sue $\quad n_1 = 8$

$T_2$ – arrange Bill + Sue in their 2 chairs $\quad n_2 = \underline{2} \cdot \underline{1} = 2! = P(2,2)$

$T_3$ – arrange the other 7 in the remaining chairs $\quad n_3 = 7! = P(7,7)$

Ans: $8 \cdot 2! \cdot 7! = \boxed{80,640}$

(b) Sue must not sit next to Bill?

Total # of arrangements with no restrictions – the arrangements where they do sit next to ea. othe.

$9! - 8 \cdot 2! \cdot 7! = \boxed{282,240}$

(c) Sue must sit in the middle seat?

$\underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{1} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = \boxed{8!}$

Sue

(d) Sue sits on one end of the row and Bill sits on the other end of the row?

$\underline{2} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} \cdot \underline{1} = \boxed{2! \cdot 7!}$

(e) Sue, Bill, or Jan sits in the middle seat?

$\underline{8}\ \underline{7}\ \underline{6}\ \underline{5}\ \underline{3}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = \boxed{3 \cdot 8!}$

(f) Sue, Bill, and Jan sit in the middle three seats?

$\underline{6}\ \underline{5}\ \underline{4}\ \underbrace{\underline{3}\ \underline{2}\ \underline{1}}\ \underline{3}\ \underline{2}\ \underline{1} = \boxed{3! \cdot 6!}$

3. Many U.S. license plates display a sequence of three letters followed by three digits.

(a) How many such license plates are possible?

no restrictions

$$\underbrace{26 \cdot 26 \cdot 26}_{\text{letters}} \cdot \underbrace{10 \cdot 10 \cdot 10}_{\text{digits}} = \boxed{17{,}576{,}000}$$

(b) How many of these have no repeated letters?

$$\underbrace{26 \cdot 25 \cdot 24}_{\text{letters no repeats}} \cdot \underbrace{10 \cdot 10 \cdot 10}_{\text{digits}} = \boxed{15{,}600{,}000}$$

(c) How many license plates with three letters followed by three digits have exactly two letters that are the same?

Total w/ no restrictions − # of license plates w/ no repeated letters − # of license plates w/ all 3 letters the same.

$$17{,}576{,}000 - 15{,}600{,}000 - \left(\underbrace{26 \cdot 1 \cdot 1}_{\text{letters}} \cdot \underbrace{10 \cdot 10 \cdot 10}_{\text{digits}}\right)$$

$$= \boxed{1{,}950{,}000}$$

Part (c) Alt. Version:

$$\underbrace{\_\ \_\ \_}_{\text{letters}}\ \underbrace{\_\ \_\ \_}_{\text{digits}}$$

$T_1$ – Choose the letter that is going to be repeated  $n_1 = 26 = C(26,1)$

$T_2$ – Choose the other letter  $n_2 = 25$

$T_3$ – arrange the letters  $n_3 = 3 = \frac{3!}{1!\,2!}$  (Example: ALL, LAL, LLA so 3 arrangements)

$T_4$ – fill in the 3 digits  $n_4 = 10 \cdot 10 \cdot 10 = 1000$

Answer: $26 \cdot 25 \cdot 3 \cdot 1000 = \boxed{1{,}950{,}000}$

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(d) In order to avoid confusion of letters with digits, some states do not use the letters I, O or Q on their license plates. How many of these license plates are possible (again, three letters followed by three digits)?

$$\underbrace{\underline{23}\cdot\underline{23}\cdot\underline{23}}_{\text{letters}}\cdot\underbrace{\underline{10}\cdot\underline{10}\cdot\underline{10}}_{\text{digits}}=\boxed{12{,}167{,}000}$$

(e) Assuming that the letter combinations VET, MDZ and DPZ are reserved for disabled veterans, medical practitioners, and disabled persons respectively, and also taking the restriction in part d into account, how many license plates are available for people who do not fall into one of those three categories?

$$12{,}167{,}000-\underbrace{\underline{3}}_{\substack{\text{VET or}\\ \text{MDZ or}\\ \text{DPZ}}}\cdot\underline{10}\cdot\underline{10}\cdot\underline{10}=\boxed{12{,}164{,}000}$$

4. Dripping wet after your shower, you have completely forgotten the combination of your lock. It is one of those "standard" combination locks, which uses a three number combination with each number in the range of 0 through 39. All you remember is that the second number is either 27 or 37, while the third number either is 5 or ends in a 5. In desperation, you decide to go through all possible combinations. Assuming that it takes about 10 seconds to try each combination, what is the longest possible time it can take to open your locker?

$\underline{40} \cdot \underline{2} \cdot \underline{4} = 320$ combo's to try

27 or 37

ends in 5

$320 \times 10\text{sec} = \boxed{3,200 \text{ seconds}}$

$= 53\frac{1}{3}$ minutes

5. How many 4-person committees are possible from a group of 9 people if:

(a) There are no restrictions?

$C(9,4) = 126$

(b) Both Jim and Mary must be on the committee?

$T_1$ - Choose both Jim + Mary to be on the committee $\quad n_1 = C(2,2) = 1$

$T_2$ - Choose 2 others $\quad n_2 = C(7,2)$

$\boxed{1 \cdot C(7,2) = 21}$

(c) Only Jim or only Mary is on the committee?

$T_1$ - Choose either Jim or Mary $\quad n_1 = C(2,1)$

$T_2$ - Choose 3 others $\quad n_2 = C(7,3)$ *

$\boxed{C(2,1)C(7,3)} = \boxed{70}$

6. A jewelry store chain with 8 stores in Georgia, 12 in Florida, and 10 in Alabama is planning to close 10 of these stores.

   (a) How many ways can this be done?

   $C(30,10)$

   (b) The company decided to close 2 stores in Georgia, 5 in Florida, and 3 in Alabama. How many ways can this be done?

   $T_1$ - choose 2 stores in Georgia $n_1 = C(8,2)$

   $T_2$ - choose 5 in FL $n_2 = C(12,5)$

   $T_3$ - choose 3 in AL $n_3 = C(10,3)$

   $\boxed{C(8,2)C(12,5)C(10,3)} = \boxed{2,661,120}$

7. Stan is having a mixed stroke of luck. He just got the phone number of his waitress, but he cannot read her handwriting. If he is certain that the first digit is a 5, the fourth digit is a 2 or a 7, and the last digit is a 6, what is the maximum number of phone numbers Stan must try? (Assume the phone number has only 7 digits.)

$$\underset{\text{is a 5}}{\underline{1}} \cdot \underline{10} \cdot \underline{10} \cdot \underset{\text{2 or 7}}{\underline{2}} \cdot \underline{10} \cdot \underline{10} \cdot \underset{\text{is a 6}}{\underline{1}} = \boxed{20{,}000}$$

8. You have a box that contains 8 red balls, 7 black balls, 2 green balls, and 6 purple balls. If you take a sample of six balls from the box, how many ways can you get

(a) exactly 2 purple balls and exactly 4 black ball?

$T_1$ – choose 2 purple $\quad n_1 = C(6,2)$

$T_2$ – choose 4 black $\quad n_2 = C(7,4)$

$$\boxed{C(6,2)C(7,4)} = \boxed{525}$$

(b) exactly 2 purple balls or exactly 4 black balls?

union

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

A – the set of all samples of 6 w/ exactly 2 purple balls

B = - - - - - - - - - - - - - - 4 black balls

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= \boxed{C(6,2)C(17,4) + C(7,4)C(16,2) - C(6,2)C(7,4)}$$

(c) exactly 2 red balls or exactly 1 green ball?

union

C – exactly 2 red

D – exactly 1 green

$$n(C \cup D) = n(C) + n(D) - n(C \cap D)$$

$$= \boxed{C(8,2)C(15,4) + C(2,1)C(21,5) - C(8,2)C(2,1)C(13,3)}$$

$= 62{,}902$

(d) at least 4 red balls?

4 or more

union

Exactly 4 red or Exactly 5 red or Exactly 6

$$\boxed{C(8,4)C(15,2) + C(8,5)C(15,1) + C(8,6)}$$

(e) at most 4 purple balls?

4 or less

Exactly 4 or 3 or 2 or 1 or 0

Quicker way:

(All samples of 6 w/no restrictions) − (# of samples of 6 w/ Exactly 5 purple) − (# of samples w/ exactly 6 purple)

$$\boxed{C(23,6) - C(6,5)C(17,1) - C(6,6)}$$

9. The state Motor Vehicle Department requires learners to pass a written test on the motor vehicle laws of the state. The exam consists of ten true/false questions, of which at least eight must be answered correctly to qualify for a permit. In how many different ways can a person who answers all the questions on the exam qualify for a permit?

Qualify: $\underbrace{\text{at least}}_{\text{8 or more}}$ 8 correct

Exactly 8 or Exactly 9 or Exactly 10

$C(10,8) + C(10,9) + C(10,10)$

$= \boxed{56}$

10. In a different state, the Motor Vehicle Department requires learners to pass a similar test with 10 multiple choice questions, of which at least 8 must be answered correctly to qualify for a permit. If each questions has 4 choices, in how many different ways can a person who answers all the questions on the exam qualify for a permit?

$\underbrace{\text{at least eight}}_{\text{8 or more}}$ correct

Exactly 8 or Exactly 9 or Exactly 10

$$\boxed{C(10,8)\cdot 1^8\cdot 3^2 + C(10,9)\cdot 1^9\cdot 3^1 + C(10,10)\cdot 1^{10}\cdot 3^0} = \boxed{436}$$

Exactly 8 correct

$\underline{1}\ \underline{1}\ \underline{3}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{3}\ \underline{1}\ \underline{1}$

$T_1$ - Choose 8 questions to answer correctly $\quad n_1 = C(10,8)$

$T_2$ - answer the chosen 8 questions correctly $\quad n_2 = 1^8$

$T_3$ - answer the remaining questions incorrectly. $\quad n_3 = 3^2$

$C(10,8)\cdot 1^8\cdot 3^2$

11. How many different arrangements can be made from the letters of MASSACHUSETTS?

$$\frac{n!}{n_1!\,n_2!\cdots n_r!} = \frac{13!}{1!\,2!\,4!\,1!\,1!\,1!\,1!\,2!} = \boxed{\frac{13!}{2!\,4!\,2!}}$$

$$= \boxed{64,864,800}$$

12. Jane has 3 yellow pillows, 6 purple pillows, 8 red pillows and 2 green pillows. In how many ways can Jane line up these pillows in a single row on her couch if pillows of the same color are identical?

$$\frac{19!}{3!\,6!\,8!\,2!}$$

13. Six students checked their backpacks at the entrance of the MSC Bookstore, but the backpacks are returned to the students at random. In how many ways can the backpacks be returned so that exactly 4 receive the correct backpack?

$T_1$ - choose 4 students to get the correct backpacks $\quad n_1 = C(6,4)$

$T_2$ - give the chosen 4 their correct backpacks $\quad n_2 = \underline{1} \cdot \underline{1} \cdot \underline{1} \cdot \underline{1} = 1$

$T_3$ - give the remaining 2 the wrong backpack $\quad n_3 = \underline{1} \cdot \underline{1} = 1$

$$C(6,4) \cdot 1 \cdot 1 = C(6,4) = \boxed{15}$$

14. Six boyfriend-girlfriend couples attend a party. They hire a photographer to take their picture. In how many ways can the group line up for the picture (in one row) if

← Permutation ↗

(a) there are no restrictions?

$$12! = P(12,12)$$

(b) they decide to take the picture with only 4 people in it?

Three different ways to work this problem:

$$\underline{12}\cdot\underline{11}\cdot\underline{10}\cdot\underline{9} = P(12,4) = \underbrace{C(12,4)}_{T_1}\cdot\underbrace{4!}_{T_2} = \boxed{11{,}880}$$

$T_1$ - Choose 4 people to be in the picture.

$T_2$ - Line up the 4 chosen people: $\underline{4}\cdot\underline{3}\cdot\underline{2}\cdot\underline{1} = 4!$

(c) they decide to take the picture with only 2 *couples* in them? (Assume that there are no restrictions on the order the people stand in line for the picture.)

$T_1$ - Choose 2 couples to be in the picture $\quad n_1 = C(6,2)$

$T_2$ - arrange the 4 people for the picture $\quad n_2 = P(4,4) = \underline{4}\cdot\underline{3}\cdot\underline{2}\cdot\underline{1} = 4!$

$$\boxed{C(6,2)\cdot 4!} = 360$$