Math 142 - Week in Review #5

- 1. Suppose an object moves along the y-axis so that its location is $y = 3x^2 + 5x$ at time x, where y is measured in meters and x is measured in seconds.
 - (a) Find the average rate of change of y with respect to x from x = 4 to x = 9. Interpret your answer.
 - (b) Using the limit definition, find the instantaneous rate of change of y at x = 4. Interpret your answer.
 - (c) Confirm your answer to (b) using basic differentiation properties.
- 2. Let $f(x) = \sqrt{x-5} + 3$.
 - (a) Use the limit definition of the derivative to find f'(x).
 - (b) Find the slope of the tangent line to f(x) at x = 21.
 - (c) Find the instantaneous rate of change of f(x) at x = 6.
 - (d) Find the equation of the tangent line at x = 14.
- 3. The graph of a function f is shown below. Find all values of x for which f(x) is not differentiable.



- 4. Find the derivative of each of the following.
 - (a) f(x) = 7x + 5
 - (b) $g(x) = 10x^5 7x^4 + 3x^2 4$
 - (c) $h(x) = 7\sqrt{x} \frac{8}{x^{9.1}} + 10\sqrt[3]{x^5} \frac{11}{\sqrt[9]{x^4}} + 2x^{-1.32} 6\pi^3$
- 5. Find the value(s) of x where the tangent line to $f(x) = 2x^5 30x^3 + e^4$ is horizontal.
- 6. The total sales of a company t months from now are given by $S(t) = 0.004t^4 + 0.3t^3 + 2.7t^2 + 5t 2$ thousand dollars.
 - (a) Find a model for the rate of change of sales with respect to time.
 - (b) Find and interpret S(4) and S'(4).
 - (c) Use your answers in (b) to estimate the company's total sales 5 months from now and 6 months from now.
- 7. Consider the following graph of the function f(x).



- (a) Between which two consecutive labeled points is the average rate of change positive? Negative?
- (b) Between which two consecutive labeled points is the average rate of change largest? Smallest?
- (c) At which labeled point(s) is the instantaneous rate of change positive? Negative?
- (d) At which labeled point(s) is the instantaneous rate of change zero?
- (e) At which labeled point(s) is the instantaneous rate of change largest? Smallest?

8. Let $f(x) = \frac{1}{x}$.

- (a) Use the limit definition of the derivative to find f'(x).
- (b) Confirm your answer using basic differentiation properties.
- (c) Are there any values of x for which f'(x) does not exist?
- 9. If h(x) = 7x + 3f(x) 2g(x) + 8, f'(3) = 4, and g'(3) = -5, find h'(3).
- 10. Acme, Inc.'s market research department has determined the price-demand function for its graphing calculators to be p = 201 0.03x dollars per calculator, where x is the number of graphing calculators demanded. Acme has a fixed cost of \$81,180 and a variable cost of \$36.90 per calculator.
 - (a) Find a model for revenue, and state its domain.
 - (b) Find a model for marginal revenue.
 - (c) Find R(3000) and R'(3000) and interpret your answers.
 - (d) What is the approximate revenue generated by the sale of the 3,001st calculator?
 - (e) Find the exact revenue generated by the sale of the 3,001st calculator.
 - (f) Find the cost and marginal cost of producing 2,500 calculators and use your answers to estimate the total cost for producing 2,501 calculators and producing 2,515 calculators.
 - (g) What is the average revenue when 6,000 calculators are sold?
 - (h) Find the average profit function.
 - (i) Find $\overline{P}'(3000)$ and interpret your answer.
 - (j) Find a model for marginal average revenue.
- 11. Annette plans to invest some money in an account paying 7.25% per year compounded continuously. How long will it take for her money to triple?
- 12. Bob has invested \$5,400 into a savings account paying 5.9% per year compounded continuously.
 - (a) How much interest will Bob earn during the next 5 years?
 - (b) How long will it take for Bob's account to reach \$8,000?
- 13. A company's revenue can be modeled by $R(x) = 300x 0.15x^2$ million dollars, where x is the number of items sold.
 - (a) Find the average revenue when 900 items are sold.
 - (b) Find the marginal revenue when 900 items are sold.
 - (c) Find the marginal average revenue when 900 items are sold.
 - (d) Find the revenue when 900 items are sold.