## Math 142 - Week in Review \#5

1. Suppose an object moves along the $y$-axis so that its location is $y=3 x^{2}+5 x$ at time $x$, where $y$ is measured in meters and $x$ is measured in seconds.
(a) Find the average rate of change of $y$ with respect to $x$ from $x=4$ to $x=9$. Interpret your answer.
(b) Using the limit definition, find the instantaneous rate of change of $y$ at $x=4$. Interpret your answer.
(c) Confirm your answer to (b) using basic differentiation properties.
2. Let $f(x)=\sqrt{x-5}+3$.
(a) Use the limit definition of the derivative to find $f^{\prime}(x)$.
(b) Find the slope of the tangent line to $f(x)$ at $x=21$.
(c) Find the instantaneous rate of change of $f(x)$ at $x=6$.
(d) Find the equation of the tangent line at $x=14$.
3. The graph of a function $f$ is shown below. Find all values of $x$ for which $f(x)$ is not differentiable.

4. Find the derivative of each of the following.
(a) $f(x)=7 x+5$
(b) $g(x)=10 x^{5}-7 x^{4}+3 x^{2}-4$
(c) $h(x)=7 \sqrt{x}-\frac{8}{x^{9.1}}+10 \sqrt[3]{x^{5}}-\frac{11}{\sqrt[9]{x^{4}}}+2 x^{-1.32}-6 \pi^{3}$
5. Find the value(s) of $x$ where the tangent line to $f(x)=2 x^{5}-30 x^{3}+e^{4}$ is horizontal.
6. The total sales of a company $t$ months from now are given by $S(t)=0.004 t^{4}+0.3 t^{3}+2.7 t^{2}+5 t-2$ thousand dollars.
(a) Find a model for the rate of change of sales with respect to time.
(b) Find and interpret $S(4)$ and $S^{\prime}(4)$.
(c) Use your answers in (b) to estimate the company's total sales 5 months from now and 6 months from now.
7. Consider the following graph of the function $f(x)$.

(a) Between which two consecutive labeled points is the average rate of change positive? Negative?
(b) Between which two consecutive labeled points is the average rate of change largest? Smallest?
(c) At which labeled point(s) is the instantaneous rate of change positive? Negative?
(d) At which labeled point(s) is the instantaneous rate of change zero?
(e) At which labeled point(s) is the instantaneous rate of change largest? Smallest?
8. Let $f(x)=\frac{1}{x}$.
(a) Use the limit definition of the derivative to find $f^{\prime}(x)$.
(b) Confirm your answer using basic differentiation properties.
(c) Are there any values of $x$ for which $f^{\prime}(x)$ does not exist?
9. If $h(x)=7 x+3 f(x)-2 g(x)+8, f^{\prime}(3)=4$, and $g^{\prime}(3)=-5$, find $h^{\prime}(3)$.
10. Acme, Inc.'s market research department has determined the price-demand function for its graphing calculators to be $p=201-0.03 x$ dollars per calculator, where $x$ is the number of graphing calculators demanded. Acme has a fixed cost of $\$ 81,180$ and a variable cost of $\$ 36.90$ per calculator.
(a) Find a model for revenue, and state its domain.
(b) Find a model for marginal revenue.
(c) Find $R(3000)$ and $R^{\prime}(3000)$ and interpret your answers.
(d) What is the approximate revenue generated by the sale of the 3,001 st calculator?
(e) Find the exact revenue generated by the sale of the 3,001 st calculator.
(f) Find the cost and marginal cost of producing 2,500 calculators and use your answers to estimate the total cost for producing 2,501 calculators and producing 2,515 calculators.
(g) What is the average revenue when 6,000 calculators are sold?
(h) Find the average profit function.
(i) Find $\bar{P}^{\prime}(3000)$ and interpret your answer.
(j) Find a model for marginal average revenue.
11. Annette plans to invest some money in an account paying $7.25 \%$ per year compounded continuously. How long will it take for her money to triple?
12. Bob has invested $\$ 5,400$ into a savings account paying $5.9 \%$ per year compounded continuously.
(a) How much interest will Bob earn during the next 5 years?
(b) How long will it take for Bob's account to reach $\$ 8,000$ ?
13. A company's revenue can be modeled by $R(x)=300 x-0.15 x^{2}$ million dollars, where $x$ is the number of items sold.
(a) Find the average revenue when 900 items are sold.
(b) Find the marginal revenue when 900 items are sold.
(c) Find the marginal average revenue when 900 items are sold.
(d) Find the revenue when 900 items are sold.
