- For combinations, order DOES NOT MATTER.
- Combinations of n Objects - The number of ways of choosing r objects from n distinct objects is given by $C(n,r) = \dfrac{n!}{r!(n-r)!}$.
- In the notation $C(n,r)$, the C stands for combination, the n is the number of objects that you are starting with, and the r is the number of those objects that you are choosing to be in a subset.
- The nPr command can be found in the calculator by pressing MATH, arrowing over to PRB, and selecting option #2.

---

1. Bob loaned \$3,500 to his friend Ed. When Ed paid the money back to Bob, Bob charged him simple interest at a rate of 5.5% so that the amount Bob received from Ed was \$3,654. How many days did Ed have Bob's money?

in years

$I = Prt$

$154 = 3500(.055)t$

$.8 = t$
years

$.8\text{ yr} * \dfrac{365\text{ days}}{1\text{ yr}} = \boxed{292\text{ days}}$

2. Ann invested \$400 for 6 years in an account paying simple interest at a rate of 3.8%. What is the accumulated amount?

$A = P + I$

$I = 400(0.038)(6)$

$I = \$91.20$

$A = 400 + 91.20 = \boxed{\$491.20}$

3. Sophie wants to have \$1,000,000 in her retirement account when she retires at age 65. If she is currently 22 years old and has an account that pays 7.5% interest per year compounded monthly, what monthly payments should she make to reach this goal?

$N = 12 * 43$

$I\% = 7.5$

$PV = 0$

$PMT = ?$

$FV = 1000000$

$P/Y = C/Y = 12$

$\boxed{\$261.48}$

4. The Jones's plan to purchase a $170,000 home. They are able to make a 30% down payment and finance the balance with a 30-year mortgage charging 4.85%/year compounded monthly.

   (a) What monthly payments should the Jones's make to amortize the loan for 30 years?

   Down PMT = .3(170000) = 51,000
   Loan Amt = 170,000 − 51,000

   N = 30*12   PMT = ? $\boxed{\$627.95}$
   I% = 4.85   FV = 0
   119,000   P/Y = C/Y = 12

   (b) How much of the first payment actually goes toward paying off the principle?

   1st find interest owed after 1 month:
   119,000 * 0.0485/12 = $480.96
   AMT toward principle = 627.95 − 480.96 = $\boxed{\$146.99}$

   (c) How much will the Jones's still owe after 20 years of payments?

   = 20*12   PMT = −627.95
   I% = 4.85   FV = ? $\boxed{\$59,616.85}$
   PV = 119000   P/Y = C/Y = 12

   (d) How much equity will the Jones's have after 20 years?

   Equity = Value of house − what you still owe.
   = 170,000 − 59616.85
   = $\boxed{\$110,383.15}$

   (e) How much interest will they pay?

   Interest pd = total amt pd for house − value of house
   = 51,000 + 627.95 * 30*12 − 170000
   = 277,062 − 170 000 = $\boxed{\$107,062}$

5. How much money should Dan deposit now into an account paying 9.5%/year compounded semiannually so that in 8 years the balance is $1,000? How much interest will he earn?

   N = 2*8   PMT = 0
   I% = 9.5   FV = 1000
   PV = ?   P/Y = C/Y = 2

   $\boxed{\$475.92}$

6. Consider the propositions

p: Bob will have a hamburger for lunch.
q: Bob will have pizza for lunch.
r: Fred will have a hamburger for lunch.

(a) Write the proposition $r \wedge (p \veebar q)$ in words.

Fred will have a hamburger for lunch and Bob will have either a hamburger or pizza (but not both) for lunch.

(b) Write the proposition $(p \vee \sim q) \wedge r$ in words.

Bob will have a hamburger or he will not have pizza for lunch, and Fred will have a hamburger for lunch

(c) Write the proposition "Bob and Fred will both have a hamburger for lunch, or Bob will have pizza for lunch," symbolically.

$(p \wedge r) \vee q$

(d) Write the proposition "Bob will not have a hamburger or pizza for lunch, but Fred will have a hamburger for lunch," symbolically.

$\sim(p \vee q) \wedge r$

Check: $(p \wedge \sim q) \vee (p \wedge \sim q)$

7. Write a truth table for each of the following.

(a) $\sim(\sim p \vee q) \vee (p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$\sim(\sim p \vee q)$	$(p \wedge \sim q)$	$\sim(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	F	F	T	F	F	F
T	F	F	T	F	T	T	T
F	T	T	F	T	F	F	F
F	F	T	T	T	F	F	F

(b) $\sim r \wedge (q \vee \sim p)$

p	q	r	$\sim p$	$\sim r$	$q \vee \sim p$	$\sim r \wedge (q \vee \sim p)$
T	T	T	F	F	T	F
T	T	F	F	T	T	T
T	F	T	F	F	F	F
T	F	F	F	T	F	F
F	T	T	T	F	T	F
F	T	F	T	T	T	T
F	F	T	T	F	T	F
F	F	F	T	T	T	T

The majority of the following Chapter 6 problems are courtesy of Joe Kahlig.

8. True or False. $U = \{0,1,2,3,4,5,6,7,8,9\}$ and $A = \{0,1,2,3,4,5\}$

T F	$\emptyset \in A$		T F	$n(A) = 5$		T F	$n(\{3,4\}) = 2$
T F	$\emptyset \subseteq A$		T F	$\{1,3,5\} \in A$		T F	$n(\emptyset) = 1$
T F	$\{1,2,3\} \subseteq A$		T F	$2 \in A$		T F	$3 \in A^C$
T F	$2 \subseteq A$		T F	$\{\emptyset\} = \emptyset$		T F	$0 = \emptyset$

9. A = $\{a,b,c\}$

(a) List all subsets of A.

$\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$, $\{a,b,c\}$

(b) List all of the proper subsets of A.

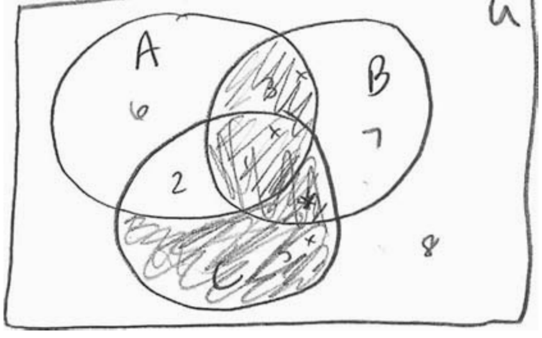
$\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$

(c) Give an example of two subsets of A that are disjoint. If this is not possible, then explain why.

$\{a\}$ and $\{b\}$ are disjoint
(they have no elements in common.)

10. Shade the part of the Venn diagram that is represented by

(a) $(A^C \cup B) \cap (C \cup A)$



A - 1,2,3,6

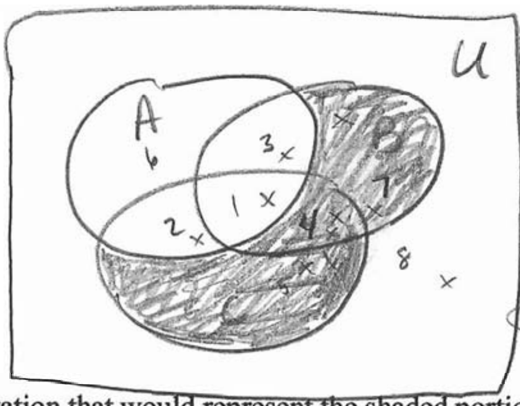
A^c - 4,5,7,8 } union $A^c \cup B$ - 4,5,7,8,1,3
B - 1,3,4,7

Intersect

C - 1,2,4,5 } union - $C \cup A$ - 1,2,4,5,3,6
A - 1,2,3,6

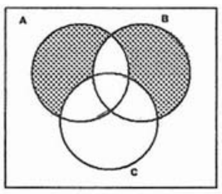
$(A^c \cup B) \cap (C \cup A)$ - 1,3,4,5

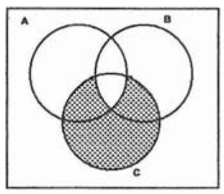
(b) $(B\cup C)\cap A^C$



B - 1,3,4,7 } union
C - 1,2,4,5
$B\cup C$ - 1,2,4,5,7
A - 1,2,3,6
A^c - 4,5,7,8 ← Intersect
$(B\cup C)\cap A^c$ - 4,5,7

11. Write down the set notation that would represent the shaded portion of the Venn diagram.





$(A\cap B^c\cap C^c)\cup(B\cap A^c\cap C^c)$

$C\cap(A\cap B\cap C)^c$

or

$C\cap(A\cap B)^c$

or

$(C\cap A\cap B^c)\cup(C\cap B\cap A^c)\cup(C\cap A^c\cap B^c)$

12. $U=\{0,1,2,3,4,5,6,7,8,9\}$, $A=\{1,3,5,7,9\}$, $B=\{1,2,4,7,8\}$, and $C=\{2,4,6,8\}$. Compute the following.

a) $(A\cap B)\cup C$

$A\cap B=\{1,7\}$ } union
$=\{2,4,6,8\}$
$(A\cap B)\cup C=\boxed{\{1,7,2,4,6,8\}}$

b) $A^C\cap B$

$A^c=\{0,2,4,6,8\}$ } Intersect
$B=\{1,2,4,7,8\}$
$A^c\cap B=\boxed{\{2,4,8\}}$ (b)

c) $A\cap(B\cup C)^C$

$B\cup C=\{1,2,4,7,8,6\}$
$(B\cup C)^c=\{0,3,5,9\}$ ← Intersect w/A
$A\cap(B\cup C)^c=\boxed{\{3,5,9\}}$ (c)

13. Any problem like numbers 39-50 from section 6.1.

13H. Let U be the set of all A&M students. Let

$$A=\{x\in U|x \text{ owns an automobile}\}$$
$$D=\{x\in U|x \text{ lives in a dorm on campus}\}$$
$$F=\{x\in U|x \text{ is a freshman}\}$$

a) Describe the set $(A\cap D)\cup F^c$ in words.

The set of all A&M students who both own an automobile and live in a dorm on campus or who are not freshmen.

b) Use set notation ($\cap,\cup,{}^c$) to write the set of all A&M students who are freshmen living on campus in a dorm but do not own an automobile.

$F\cap D\cap A^c$

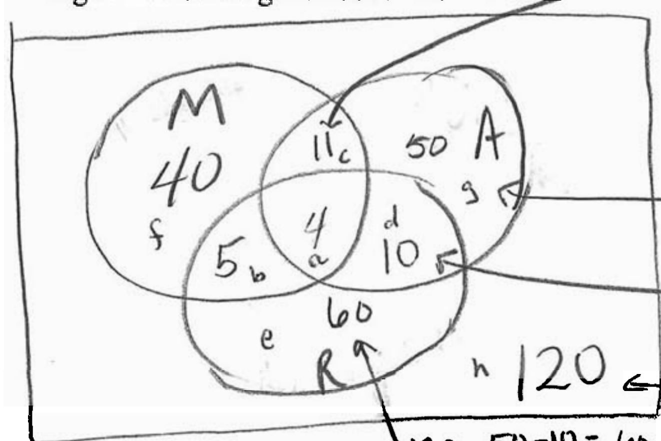
14. In a survey of 300 high school seniors:
(7) 120 had not read *Macbeth* but had read *As You Like It* or *Romeo and Juliet*.
(6) 61 had read *As You Like It* but not *Romeo and Juliet*.
(5) 15 had read *Macbeth* and *As You Like It*.
(4) 14 had read *As You Like It* and *Romeo and Juliet*.
(3) 9 had read *Macbeth* and *Romeo and Juliet*.
(2) 5 had read *Macbeth* and *Romeo and Juliet* but not *As You Like It*.
(1) 40 had read only *Macbeth*.

Let M = *Macbeth*, R = *Romeo and Juliet*, and A = *As You Like It*.

Let M be the set of seniors who read Macbeth
Let A - As You Like It.
Let R - Romeo + Juliet

(a) Fill in a Venn diagram illustrating the above information.

15 − 4 = 11



61 − 11 = 51

14 − 4 = 10

300 − 40 − 11 − 50 − 5 − 4 − 10 − 60 = 120

120 − 50 − 10 = 60

(b) How many students read exactly one of these books?

$40 + 50 + 60 = \boxed{150}$

(c) How many students did not read *Romeo and Juliet*?

$40 + 11 + 50 + 120 = \boxed{221}$

(d) How many students read either *Macbeth* or *As You Like It* and read *Romeo and Juliet*?

$5 + 4 + 10 = \boxed{19}$

(e) Compute $n(M \cup (R^C \cap A)) =$

R - a, b, d, e

A - a, d, g, c

R^c - c, f, g, h

Intersect

$R^c \cap A$ - g, c
M - a, b, c, f
} union

$M \cup (R^c \cap A)$ - a, b, c, f, g

$4 + 5 + 11 + 40 + 50 = \boxed{110}$

part (e) answer

(f) Compute $n(A^C \cap (R \cup M)) =$

R - a, b, d, e
M - a, b, c, f
} union

$R \cup M$ = a, b, d, e, c, f
A^c - f, b, e, h
} Intersect

$A^c \cap (R \cup M)$ - b, e, f

$5 + 60 + 40 = \boxed{105}$

15. Find $n(A \cap B)$ if $n(A) = 8$, $n(B) = 9$, and $n(A \cup B) = 14$.

$n(A \cup B) = n(A) + n(B) - n(A$

$14 = 8 + 9 - n(A \cap B)$

$-3 = -n(A \cap B)$

$\boxed{n(A \cap B) = 3}$

16. Bill and Sue and four of their friends go to the movies. They all sit next to each other in the same row. How many ways can this be done if

(a) Sue and Bill must sit next to each other?

T_1 - pick 2 chairs together for Sue + Bill $n_1 = 5$

T_2 - Arrange Bill + Sue in their chairs $n_2 = 2!$

T_3 - arrange 4 others $n_3 = 4!$

$5 \cdot 2! \cdot 4! = \boxed{240}$

(b) Sue must not sit next to Bill?

All possible arrangements − arrangements where they do sit together

$6! - 240 = \boxed{480}$

(c) Sue sits on one end of the row and Bill sits on the other end of the row?

$\underset{\uparrow}{\underline{2}} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} \cdot \underset{\uparrow}{\underline{1}} = 2! \, 4! = \boxed{48}$

Bill or Sue

17. Many U.S. license plates display a sequence of three letters followed by three digits.

(a) How many such license plates are possible?

$\underbrace{\underline{26} \cdot \underline{26} \cdot \underline{26}}_{\text{letters}} \cdot \underbrace{\underline{10} \cdot \underline{10} \cdot \underline{10}}_{\text{digits}} = \boxed{17{,}576{,}000}$

(b) In order to avoid confusion of letters with digits, some states do not use the letters I, O or Q on their license plates. How many of these license plates are possible?

$\underline{23} \cdot \underline{23} \cdot \underline{23} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = \boxed{12{,}167{,}000}$

(c) Assuming that the letter combinations VET, MDZ and DPZ are reserved for disabled veterans, medical practitioners, and disabled persons respectively, and also taking the restriction in part b into account, how many license plates are possible? (meaning those that are not reserved)

How many have VET, MDZ, or DPZ to begin with?

$\underline{3} \; \underline{10} \; \underline{10} \; \underline{10} = 3{,}000$ ← what we don't want

Total from (b) − what we don't want

$12{,}167{,}000 - 3000 = \boxed{12{,}164{,}000}$

18. Dripping wet after your shower, you have completely forgotten the combination of your lock. It is one of those "standard" combination locks, which uses a three number combination with each number in the range of 0 through 39. All you remember is that the second number is either 27 or 37, while the third number ends in a 5. In desperation, you decide to go through all possible combinations. Assuming that it takes about 10 seconds to try each combination, what is the longest possible time it can take to open your locker?

$40 \cdot 2 \cdot 4 = 320$ combo's to try

27 or 37

ends in 5: 05, 15, 25, 35

$320 * 10$ sec.

$\boxed{3200 \text{ seconds}}$ ($53\frac{1}{3}$ hrs)

19. Compute $C(20,5) =$ $\frac{20!}{5!(20-5)!} = \boxed{15504}$ 20nCr5

$P(20,5) =$ $\frac{20!}{15!} = \boxed{1860480}$ 20nPr5

20. How many 4-person committees are possible from a group of 9 people if:

(a) There are no restrictions?

$\boxed{C(9,4)}$

(b) Both Jim and Mary must be on the committee?

T_1 - Choose Jim + Mary $n_1 = C(2,2)$
T_2 - choose 2 others $n_2 = C(7,2)$

$\boxed{C(2,2)C(7,2)}$

(c) Only Jim or only Mary is on the committee?

$\boxed{C(7,3) + C(7,3)} - 0$

can't have only Jim and only Mary at same time

OTHER WAY:
T_1 - Choose Jim or Mary $n_1 = C(2,1)$
T_2 - Choose 3 others $n_2 = C(7,3)$

$\boxed{C(2,1)C(7,3)}$

21. A jewelry store chain with 8 stores in Georgia, 12 in Florida, and 10 in Alabama is planning to close 10 of these stores.

$8+12+10 = 30$

(a) How many ways can this be done?

$C(30,10)$

(b) The company decided to close 2 stores in Georgia, 5 in Florida, and 3 in Alabama. How many ways can this be done?

$C(8,2)C(12,5)C(10,3)$

22. The U.B.S. Television company is considering bids submitted by seven different firms for three different contracts. In how many ways can the contracts be awarded among these firms if no firm is to receive more than two contracts?

This means no firm will get all three contracts, so take the total # of ways of distributing the contracts w/ no restrictions minus the # of ways of giving all 3 to 1 firm.

$$\underset{\text{contract }1}{7} \cdot \underset{2}{7} \cdot \underset{3}{7} - 7 = \boxed{336}$$

(give all 3 to firm 1 or firm 2 or ... or firm 7)
7 ways to do this

23. You have a box that contains 3 red balls, 4 black balls, 2 green balls, and 5 purple balls. If you take a sample of three balls from the box, how many ways can you get

3R
4B
2G
5P

(a) 2 black balls and one green ball?

$\boxed{C(4,2)C(2,1)} = 12$

(b) exactly 2 red balls or exactly one purple ball?

union

A - set of all samples with exactly 2 red.
B - set of all samples w/ exactly 1 purple

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = \boxed{C(3,2)C(11,1) + C(5,1)C(9,2) - C(3,2)C(5,1)} = 198$

(c) at least two purple balls?

2 or more

union → $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Exactly 2 purple or Exactly 3 purple

$C(5,2)C(9,1) + \underbrace{C(5,3)C(9,0)}_{\text{equals }1} - 0$

can't have exactly 2 purple AND exactly 3 purple at the same time

$\boxed{C(5,2)C(9,1) + C(5,3)}$

24. The state Motor Vehicular Department requires learners to pass a written test on the motor vehicle laws of the state. The exam consists of ten true/false questions, of which at least eight must be answered correctly to qualify for a permit. In how many different ways can a person who answers all the questions on the exam qualify for a permit?

Exactly 8 or exactly 9 or Exactly 10 correct

$C(10,8) + C(10,9) + C(10,10)$

24H. In a different state, the Motor Vehicle Deparment requires learners to pass a similar test with 10 multiple choice questions, of which at least 8 must be answered correctly to qualify for a permit. If each question has 4 choices, in how many different ways can a person who answers all the questions on the exam qualify for a permit?

Exactly 8

T_1 - choose 8 questions to get correct $n_1 = C(10,8)$

T_2 - answer those 8 correctly $n_2 = 1^8$

T_3 - answer other 2 incorrectly $n_3 = 3^2$

1 3 1 1 1 3 1 1 1 1

3 wrong answers

Exactly 8 or Exactly 9 or Exactly 10

$\boxed{C(10,8)1^8 3^2 + C(10,9)1^9 3^1 + C(10,10)\cdot 1^{10} 3^0}$

$= 405 + 30 + 1$

$= \boxed{436}$

25. How many different arrangements can be made from the letters of MASSACHUSETTS?

$$\frac{13!}{1!\,2!\,4!\,1!\,1!\,1!\,1!\,1!\,2!} = \frac{13!}{2!\,4!\,2!} = \boxed{64{,}864{,}800}$$

25H. Jane has 3 yellow pillows, 6 purple pillows, 8 red pillows and 2 green pillows. In how many ways can Jane line up these pillows in a single row on her couch if pillows of the same color are identical?

$$\frac{19!}{3!\,6!\,8!\,2!} = \boxed{349{,}188{,}840}$$

26. An experiment consists of tossing a 4 sided die and flipping a coin.

(a) Describe an appropriate sample space for this experiment.

$S = \{(1,H), (2,H), (3,H), (4,H), (1,T), (2,T), (3,T), (4,T)\}$

(b) Give two events of this sample space that are mutually exclusive.

means no outcomes in common (like disjoint sets)

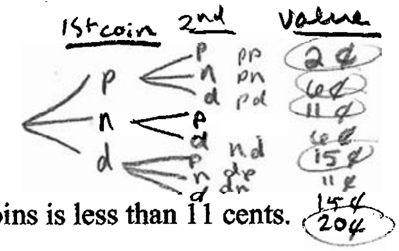
$E = \{(1,H), (2,H)\}$

$F = \{(3,H), (4,H)\}$

$E \cap F = \emptyset$, so E and F are mutually exclusive.

(There are many correct answers to this problem.)

27. A bag contains 3 pennies (p), a nickel (n), and two dimes (d). Two coins are selected at random from the bag and the monetary value of the coins (in cents) is recorded.



(a) What is the sample space of this experiment?

$S = \{2, 6, 11, 15, 20\}$

(b) Write the event E that the monetary value of the coins is less than 11 cents.

$E = \{2, 6\}$

(c) Write the event F that the nickel is drawn.

$F = \{6, 15\}$

(d) Are the events E and F mutually exclusive? Support your answer.

NO $E \cap F = \{6\} \neq \phi$ Mutually exclusive events have no elements in common.

(e) Write the event G that the value of the coins is more than 25 cents.

$G = \phi$ (impossible event)

28. An experiment consists of randomly selecting an integer multiple of 3 that is between 3 and 21 (inclusive).

(a) What is the sample space of this experiment?

$S = \{3, 6, 9, 12, 15, 18, 21\}$

(b) Write the event E that the number selected is even.

$E = \{6, 12, 18\}$

(c) Write the event F that the number selected is a multiple of 4.

$F = \{12\}$

(d) Write the event G that the number selected is odd and less than 15.

$G = \{3, 9\}$

(e) Which pairs of the events E, F, and G are mutually exclusive?

$F \cap G = \phi$ so F and G are mutually exclusive.
$E \cap G = \phi$ so E and G are mutually e

(f) If the number selected was 12, which of the events E, F, and G have occurred?

E and F have occurred.