

Math 166 - Exam 2 Review

NOTE: For reviews of the other sections on Exam 2, refer to the first page of WIR #4 and #5.

Section 7.1 - Experiments, Sample Spaces, and Events

- An **experiment** is an activity with observable results (called **outcomes**).
- **Sample Space** - the set of all possible outcomes of an experiment
- **Event** - a subset of a sample space of an experiment
- An event E is said to **occur** in a trial of an experiment whenever E contains the observed outcome.
- Unions, intersections, and complements of events are found in the same ways as they are for sets.
- Two events E and F are **mutually exclusive** if $E \cap F = \emptyset$ (i.e., if it is impossible for both E and F to occur at the same time).

Section 7.2 - Definition of Probability

- The probability of an event is a number between 0 and 1 inclusive that indicates the likelihood of that event occurring. The closer the probability is to 1, the more likely the event is to occur.
- **Probability Distribution** - a table that lists all of the simple events of an experiment and their corresponding probabilities.

NOTE: The sum of all probabilities in a probability distribution is always 1.

- **Uniform Sample Space** - a sample space in which all outcomes are equally likely.
- If $E = \{s_1, s_2, \dots, s_k\}$ is an event of an experiment with sample space S , then $P(E) = P(s_1) + P(s_2) + \dots + P(s_k)$.

Section 7.3 - Rules of Probability

Let S be a sample space of an experiment and suppose E and F are events of the experiment. Then

1. $0 \leq P(E) \leq 1$ for any E .
2. $P(S) = 1$
3. If E and F are mutually exclusive, then $P(E \cup F) = P(E) + P(F)$.
4. If E and F are any two events of an experiment, then $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.
5. $P(E^c) = 1 - P(E)$ (Rule of Complements)

NOTE: When calculating probabilities, Venn Diagrams can sometimes be useful. De Morgan's Laws may also come in handy from time to time: $(E \cap F)^c = E^c \cup F^c$ and $(E \cup F)^c = E^c \cap F^c$.

Section 7.4 - Use of Counting Techniques in Probability

- **Computing the Probability of an Event in a Uniform Sample Space** - Let S be a uniform sample space and let E be any event. Then

$$P(E) = \frac{\text{number of favorable outcomes in } E}{\text{number of possible outcomes in } S} = \frac{n(E)}{n(S)}$$

1. Consider the propositions

p : Bob will have a hamburger for lunch.

q : Bob will have pizza for lunch.

r : Fred will have a hamburger for lunch.

(a) Write the proposition $r \wedge (p \vee q)$ in words.

Fred will have a hamburger for lunch, and Bob will have either a hamburger or pizza (but not both) for lunch.

(b) Write the proposition $(q \vee \sim p) \wedge r$ in words.

Bob will have pizza or not a hamburger for lunch, but Fred will have a hamburger for lunch.

(c) Write the proposition "Bob and Fred will both have a hamburger for lunch, or Bob will have pizza for lunch," symbolically.

$$(p \wedge r) \vee q$$

(d) Write the proposition "Bob will not have a hamburger or pizza for lunch, but Fred will have a hamburger for lunch," symbolically.

$$\sim(p \vee q) \wedge r$$

2. Write a truth table for each of the following.

(a) $\sim (\sim p \vee q) \vee (p \wedge \sim q)$

See scanned answers

$$(b) \sim r \wedge (q \vee \sim p)$$

see scanned answers

Problems 3, 4, 5, 6, 7, and 10 are courtesy of Joe Kahlig.

3. True or False. $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{0, 1, 2, 3, 4, 5\}$

T F $\emptyset \in A$	T F $n(A) = 5$	T F $n(\{3, 4\}) = 2$
T F $\emptyset \subseteq A$	T F $\{1, 3, 5\} \in A$	T F $n(\emptyset) = 1$
T F $\{1, 2, 3\} \subseteq A$	T F $2 \in A$	T F $3 \in A^C$
T F $2 \subseteq A$	T F $\{\emptyset\} = \emptyset$	T F $0 = \emptyset$

4. $A = \{a, b, c\}$

(a) List all subsets of A.

$\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$

(b) List all of the proper subsets of A.

1st seven

(c) Give an example of two subsets of A that are disjoint. If this is not possible, then explain why.

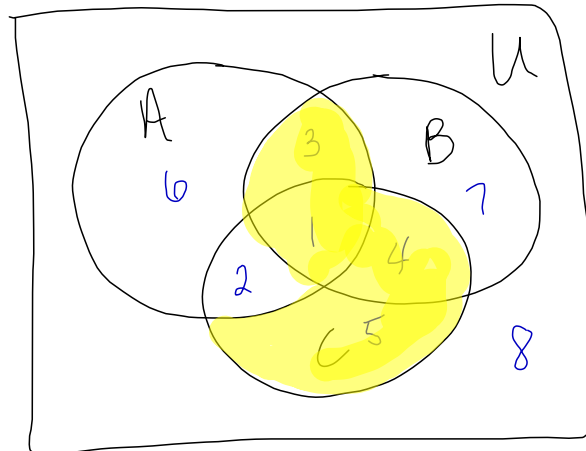
$$\{a\} \cap \{b, c\} = \emptyset$$

These are disjoint

(There are many correct answers.)

5. Shade the part of the Venn diagram that is represented by

(a) $(A^c \cup B) \cap (C \cup A)$

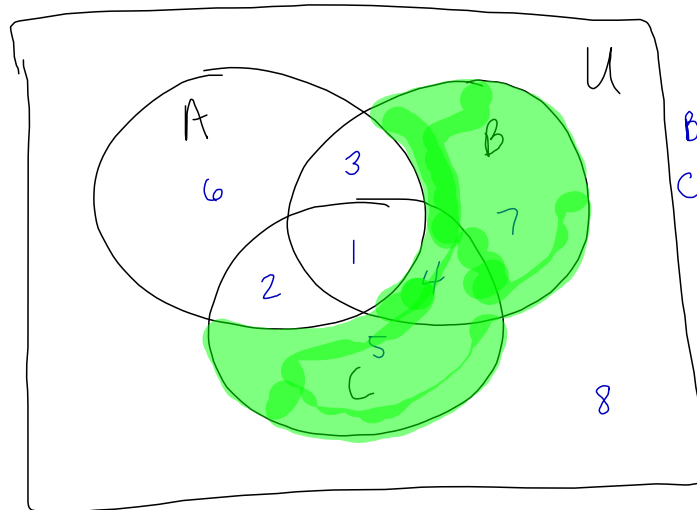


$A^c = \{4, 5, 7, 8\}$
 $B = \{1, 3, 4, 7\}$
 $A^c \cup B = \{4, 5, 7, 8, 1, 3\}$

 $C = \{1, 2, 4, 5\}$
 $A = \{1, 2, 3, 6\}$
 $C \cup A = \{1, 2, 4, 5, 3, 6\}$

(b) $(B \cup C) \cap A^c$

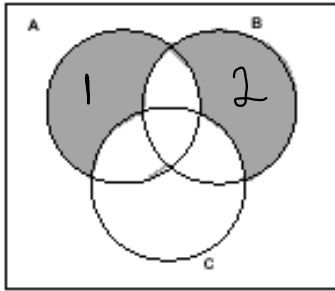
$\{1, 3, 4, 5\}$



$B = \{1, 3, 4, 7\}$
 $C = \{1, 2, 4, 5\}$
 $B \cup C = \{1, 3, 4, 7, 2, 5\}$
 $A^c = \{4, 5, 7, 8\}$
 $(B \cup C) \cap A^c = \{4, 5, 7\}$

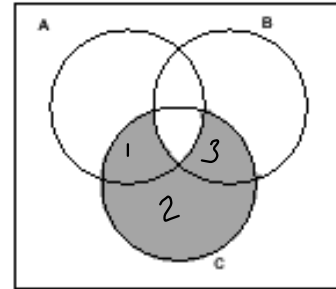
6. Write down the set notation that would represent the shaded portion of the Venn diagram.

a)



a) $(A \cap B^c \cap C^c) \cup (B \cap A^c \cap C^c)$

b)



b) $C \cap (A \cap B \cap C)^c$

Also correct for (b):

$$(C \cap A \cap B^c) \cup (C \cap A^c \cap B^c) \cup (C \cap B \cap A^c)$$

7. $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{1, 2, 4, 7, 8\}$, and $C = \{2, 4, 6, 8\}$.

Compute the following.

a) $(A \cap B) \cup C$

$$A \cap B = \{1, 7\}$$

$$(A \cap B) \cup C = \{1, 7, 2, 4, 6, 8\}$$

b) $A^c \cap B$

$$A^c = \{0, 2, 4, 6, 8\}$$

$$A^c \cap B = \{2, 4, 8\}$$

c) $A \cap (B \cup C)^c$

$$B \cup C = \{1, 2, 4, 7, 8, 6\}$$

$$(B \cup C)^c = \{0, 3, 5, 9\}$$

$$A \cap (B \cup C)^c = \{3, 5, 9\}$$

8. Any problem like numbers 39-50 from section 6.1.

9. Let U be the set of all A&M students. Let

$$A = \{x \in U \mid x \text{ owns an automobile}\}$$

$$D = \{x \in U \mid x \text{ lives in a dorm on campus}\}$$

$$F = \{x \in U \mid x \text{ is a freshman}\}$$

(a) Describe the set $(A \cap D) \cup F^c$ in words.

only write once in description

The set of all A&M students who own an automobile and live in a dorm on campus, or who are not freshmen.

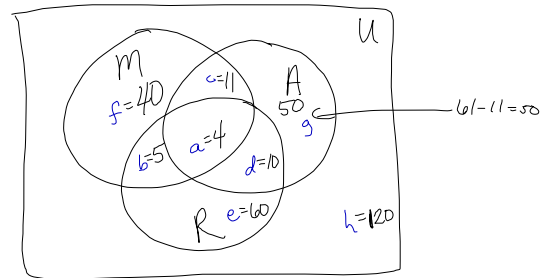
(b) Use set notation ($\cap, \cup, ^c$) to write the set of all A&M students who are freshmen living on campus in a dorm but do not own an automobile.

$$F \cap D \cap A^c$$

10. In a survey of 300 high school seniors:
- $e+d+g=120$ had not read *Macbeth* but had read *As You Like It* or *Romeo and Juliet*.
 - $c+g=61$ had read *As You Like It* but not *Romeo and Juliet*.
 - $a+c=15$ had read *Macbeth* and *As You Like It*.
 - $a+d=14$ had read *As You Like It* and *Romeo and Juliet*.
 - $a+b=9$ had read *Macbeth* and *Romeo and Juliet*.
 - $b=5$ had read *Macbeth* and *Romeo and Juliet* but not *As You Like It*.
 - $f=40$ had read only *Macbeth*.

Let $M = \text{Macbeth}$, $R = \text{Romeo and Juliet}$, and $A = \text{As You Like It}$.

- (a) Fill in a Venn diagram illustrating the above information.



- (b) How many students read exactly one of these books?

$$40 + 50 + 60 = \boxed{150}$$

- (c) How many students did not read *Romeo and Juliet*?

$$f, c, g, h \\ 40 + 11 + 50 + 120 = \boxed{221}$$

- (d) How many students read *Macbeth* (or *As You Like It*) and also read *Romeo and Juliet*?

$$n((M \cup A) \cap R)$$

$M - \{f, c, b, a\}$
 $A - \{c, g, a, d\}$
 $M \cup A - \{f, c, b, g, d\}$
 $R - \{a, b, d, e\}$
 $(M \cup A) \cap R - \{a, b, d\}$

$$\underline{4 + 5 + 10 = \boxed{19}}$$

- (e) Compute $n(M \cup (R^c \cap A)) = 110$

See scanned copy for how to get these answers.

- (f) Compute $n(A^c \cap (R \cup M)) = 105$

11. Find $n(A \cap B)$ if $n(A) = 8$, $n(B) = 9$, and $n(A \cup B) = 14$.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$14 = 8 + 9 - n(A \cap B)$$

$$-3 = -n(A \cap B)$$

$$3 = n(A \cap B)$$

12. An experiment consists of tossing a 4-sided die and flipping a coin.

(a) Describe an appropriate sample space for this experiment.

$$S = \{ (1,H), (1,T), (2,H), (2,T), (3,H), (3,T), (4,H), (4,T) \}$$

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑
each of these is a different outcome.

(8 outcomes total)

(b) Give two events of this sample space that are mutually exclusive.

subsets

$$\{ (1,H) \} \cap \{ (4,H) \} = \emptyset$$

(disjoint)

so they are mutually exclusive.

$$\{ (1,H), (4,T) \}$$

$$\{ (1,T), (4,H) \}$$

← These two events are also mutually exclusive.

13. A bag contains 3 pennies, a nickel, and two dimes. Two coins are selected at random from the bag and the monetary value of the coins (in cents) is recorded.

(a) What is the sample space of this experiment?

$$S = \{2, 6, 11, 15, 20\}$$

2 pennies	→	2¢
1 penny, 1 nickel		6¢
1 penny, 1 dime		11¢
1 nickel, 1 dime		15¢
2 dimes		20¢

(b) Write the event E that the monetary value of the coins is less than 11 cents.
yes subset.

$$E = \{2, 6\}$$

(c) Write the event F that the nickel is drawn.

$$F = \{6, 15\}$$

(d) Are the events E and F mutually exclusive? Support your answer.

No. They have the outcome 6 in common.

(e) Write the event G that the value of the coins is more than 25 cents.

$$G = \emptyset$$

14. Let $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ be the sample space of an experiment with the following probability distribution:

Outcome	s_1	s_2	s_3	s_4	s_5	s_6
Probability	$\frac{3}{40}$	$\frac{4}{40}$	$\frac{7}{40}$	$\frac{14}{40}$	$\frac{9}{40}$	$\frac{3}{40}$

$$1 - \frac{3}{40} - \frac{4}{40} - \frac{7}{40} - \frac{14}{40} - \frac{9}{40} - \frac{3}{40}$$

Let $A = \{s_1, s_3, s_5\}$, $B = \{s_3, s_4, s_6\}$, and $C = \{s_2, s_4\}$ be events of the experiment and suppose $P(B) = \frac{24}{40}$.

- (a) Fill in the missing probabilities in the probability distribution above.

$$P(B) = P(A_3) + P(A_4) + P(A_6)$$

$$\frac{24}{40} = \frac{7}{40} + \frac{14}{40} + P(A_6)$$

$$\frac{3}{40} = P(A_6)$$

- (b) Is this a uniform sample space? Why or why not?

No. The outcomes are not equally likely.

- (c) Find each of the following:

i. $P(A) = \frac{3}{40} + \frac{7}{40} + \frac{9}{40} = \frac{19}{40}$

ii. $P(C) = \frac{4}{40} + \frac{14}{40} = \frac{18}{40}$

iii. $P(B^c) = 1 - P(B) = 1 - \frac{24}{40} = \frac{16}{40}$

- iv. $P(A \cap B)$

$$A \cap B = \{s_3\}$$

$$P(A \cap B) = \frac{7}{40}$$

v. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $= \frac{19}{40} + \frac{18}{40} - \frac{7}{40}$
 $= \frac{30}{40}$

- (d) Are the events A and C mutually exclusive? Why or why not?

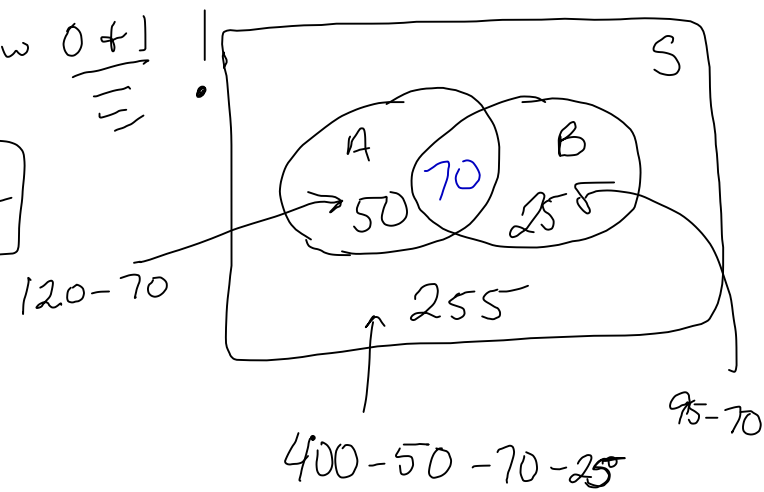
Yes since $A \cap C = \emptyset$.

A - Magazine A B - Magazine B

15. Acme, Inc. advertised its products in two magazines: Magazine A and Magazine B. A survey of 400 customers revealed that 120 learned of its products from Magazine A, 95 learned of its products from Magazine B, and 70 learned of its products from both magazines. What is the probability that a person selected at random from this group saw Acme, Inc.'s advertisement in

(a) exactly one of these magazines?

$$\frac{50 + 25}{400} = \frac{75}{400} = \boxed{\frac{3}{16}}$$



(b) at least one of these magazines?

or more

$$\frac{50 + 70 + 25}{400} = \frac{145}{400} = \boxed{\frac{29}{80}}$$

(c) in Magazine A but not Magazine B?

$$\frac{50}{400} = \boxed{\frac{1}{8}}$$

Information about a Standard Deck of 52 Cards

- There are 4 suits: hearts, diamonds, clubs, and spades.
- Hearts and diamonds are red; clubs and spades are black.
- There are 13 cards in each suit: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King.
- Jacks, Queens, and Kings are called face cards, so there are 12 face cards in a standard deck of 52 cards.

16. One card is drawn at random from a standard deck of 52 playing cards. What is the probability that the card is

(a) a club? $\frac{13}{52}$

(b) a face card? $\frac{12}{52}$

(c) a club or a face card?

C - event that the card is a club
F - - - - - face card.

$$P(C \cup F) = P(C) + P(F) - P(C \cap F)$$
$$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \boxed{\frac{22}{52}}$$

(d) neither a club nor a face card?

$$P(C^c \cap F^c) = P((C \cup F)^c)$$
$$= 1 - P(C \cup F)$$
$$= 1 - \frac{22}{52}$$
$$= \boxed{\frac{30}{52}}$$

17. Acme, Inc. ships lightbulbs in lots of 50. Before each lot is shipped, a sample of 8 lightbulbs is selected from the lot for testing. If any of the bulbs is defective, the entire lot is rejected. What is the probability that a lot containing 3 defective lightbulbs will still get shipped?

E - event that the lot gets shipped.
(i.e., event that no defectives were in the sample of 8)

$$P(E) = \frac{n(E)}{n(S)} = \frac{C(47,8)}{C(50,8)}$$

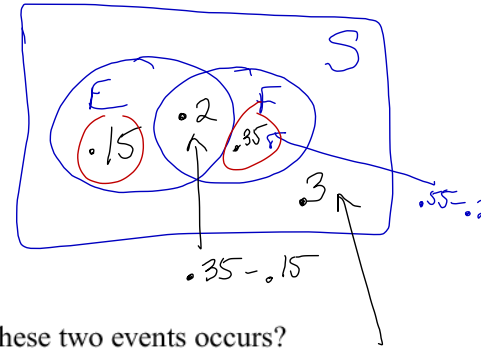
18. Is the following statement correct? "The probability that Kurt spends less than \$15 on a new DVD is 0.4. Therefore the probability that Kurt spends more than \$15 on a new DVD is 0.6."

No, It should say, "The probab. that Kurt spends less than \$15 on a new DVD is .4. Therefore the probab that Kurt spends \$15 or more on a new DVD is .6."

19. Let E and F be two events of an experiment with $P(E) = 0.35$, $P(F) = 0.55$, and $P(E \cap F^c) = 0.15$.

(a) Find $P(E \cap F)$.

$$\boxed{.2}$$



(b) What is the probability that exactly one of these two events occurs?

$$.15 + .35 = \boxed{.5}$$

$$1 - .15 - .2 - .35$$

(c) Are E and F mutually exclusive?

Since $P(E \cap F) = .2$ (instead of 0) these events can occur at the same time. Therefore they are not mutually exclusive.

(d) Find the probability that at least one of the two events occurs.

15 more

$$P(E \cup F) = .15 + .2 + .35 = \boxed{.7}$$

20. 16 people are selected at random. What is the probability that at least 2 of the people in this group were born in the same week? (There are 52 weeks in a year. Assume that all weeks are equally likely.)

E - event that at least 2 of the 16 were born in the same week.

E^c - event that all 16 people were born in different weeks.

$$P(E) = 1 - P(E^c)$$

$$= 1 - \frac{n(E^c)}{n(S)} = \boxed{1 - \frac{P(52, 16)}{52^{16}}} \approx \boxed{.9241}$$

$$n(S):$$

$$\frac{52 \cdot 52 \cdot 52 \cdot \dots \cdot 52}{(16 \text{ blanks})}$$

$$= 52^{16}$$

$$n(E^c)$$

$$\frac{52 \cdot 51 \cdot 50 \cdot \dots \cdot 37}{(16 \text{ blanks})}$$

$$= P(52, 16)$$

21. How many different 5-card poker hands are there when playing with a standard deck of 52 cards?

$$C(52, 5)$$

22. How many different 5-card poker hands are there that have

(a) exactly 2 aces?

$$T_1 - \text{Choose 2 aces} \quad n_1 = C(4, 2)$$

$$T_2 - \text{choose 3 non-aces} \quad n_2 = C(48, 3)$$

$$C(4, 2)C(48, 3)$$

(b) two pairs? (A pair is two cards with the same number or face on them.)

$$T_1 - \text{Choose 2 \#s or faces to have pairs of} \quad n_1 = C(13, 2)$$

$$T_2 - \text{Choose 2 of the 1st type of card} \quad n_2 = C(4, 2)$$

$$T_3 - \text{Choose 2 of the other type of card} \quad n_3 = C(4, 2)$$

$$T_4 - \text{Choose 1 more card of some other type} \quad n_4 = C(44, 1)$$

Answer:

$$C(13, 2)C(4, 2)C(4, 2)C(44, 1)$$

23. What is the probability that a 5-card poker hand

(a) is a full house? (A full house is a hand that has 1 pair and 1 three-of-a-kind.)

E - event that the hand is a full house.

$$P(E) = \frac{n(E)}{n(S)} = \frac{13 \cdot C(4,2) \cdot 12 \cdot C(4,3)}{C(52,5)}$$

n(E):

- T₁ - decide what face or # to have a pair of $n_1 = 13$
- T₂ - choose 2 cards of that type $n_2 = C(4,2)$
- T₃ - decide the # or face to have 3 of a kind of $n_3 = 12$
- T₄ - choose 3 of that type $n_4 = C(4,3)$

$13 \cdot C(4,2) \cdot 12 \cdot C(4,3)$

(b) is a flush? (A flush is hand in which all cards are of the same suit.)

F - event that the hand is a flush.

$$P(F) = \frac{n(F)}{n(S)} = \frac{4 \cdot C(13,5)}{C(52,5)}$$

- T₁ - choose the suit $n_1 = 4$
- T₂ - choose 5 of that suit $n_2 = C(13,5)$

25. Rosa has 3 different red dresses, 6 different black dresses, 7 different blue dresses, and 2 different green dresses.

18 total

(a) In how many ways can Rosa arrange these dresses in her closet if there are no restrictions?

$$P(18, 18) = 18!$$

(b) In how many ways can Rosa arrange these dresses in her closet if dresses of the same color must be kept together?

T_1 - decide the order of the colors

$$n_1 = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1} = 4! = P(4, 4)$$

T_2 - Arrange the red dresses in their spot
etc.

$$n_2 = 3!$$

etc.

$$4! \cdot 3! \cdot 6! \cdot 7! \cdot 2!$$

(c) Rosa has decided to pack 4 dresses for a trip. How many ways can she choose 4 dresses if

i. there are no restrictions?

$$C(18, 4)$$

ii. at most 1 of the dresses is blue?

n 1 or less

Exactly blue or no blue

$$\boxed{C(7,1)C(11,3) + C(11,4)}$$

iii. exactly 1 is blue and exactly 2 are red?

$$\boxed{C(7,1)C(3,2)C(8,1)}$$

(d) If Rosa randomly selects 6 dresses from her closet, what is the probability that

i. exactly 4 are black ^{or} exactly 2 are green?

union

B - event that exactly 4 are black.

G - - - - - 2 are green.

$$P(B \cup G) = P(B) + P(G) - P(B \cap G)$$

$$= \frac{n(B)}{n(S)} + \frac{n(G)}{n(S)} - \frac{n(B \cap G)}{n(S)}$$

$$= \frac{C(6,4)C(12,2)}{C(18,6)} + \frac{C(2,2)C(16,4)}{C(18,6)} - \frac{C(6,4)C(2,2)}{C(18,6)}$$

ii. at least 1 is blue?

E - at least 1 is blue

E^c is the event that none are blue.

$$P(E) = 1 - P(E^c)$$

$$= 1 - \frac{n(E^c)}{n(S)}$$

$$= \boxed{1 - \frac{C(11,6)}{C(18,6)}}$$

26. Referring to the previous problem, if dresses of the same color were identical, how many distinguishable arrangements of the dresses would be possible?

$$\frac{n!}{n_1! n_2! \dots n_r!} = \frac{18!}{(3! 6! 7! 2!)}$$

