Math 142 - Exam 2 Review

NOTE: Exam 2 covers sections 3.4, 3.5, 3.7, 4.1-4.4, 4.7, 5.1, and 5.2. This review is intended to highlight the material covered on Exam 2 but should not be used as your sole source of practice. Also refer to your instructor's lecture notes, previous week-in-reviews, suggested homework, supplemental homework, and the online homework as additional sources for review and exam preparation.

1. Acme, Inc. has determined the price-demand equation for its model airplane to be $25p+x=1000$.

   (a) Find the elasticity of demand, $E(p)$.

   $$x = f(p) = 1000-25p \qquad E(p) = \frac{-pf'(p)}{f(p)} = \frac{-p(-25)}{1000-25p}$$

   $$f'(p) = -25 \qquad E(p) = \frac{25p}{25(40-p)} = \boxed{\frac{p}{40-p}}$$

   (b) Find and interpret $E(13)$.

   $$E(13) = \frac{13}{40-13} = \boxed{0.4815} < 1$$

   Demand is inelastic when p= $13. A small change in price will produce a smaller change in demand.

   (c) If the current price of $13 per model airplane were increased by 5%, what would be the approximate change in demand?

   If price goes up, demand goes down

   percentage change in demand $\approx E(13)\cdot 5\% = 0.4815 * 5\% = 2.4074\%$

   $\boxed{\text{Demand will decrease by } 2.4074\%}$

   (d) Find and interpret $E(30)$.

   $$E(30) = \frac{30}{40-30} = \boxed{3} > 1$$

   Demand is elastic when p = $30. A small change in price will produce a larger change in demand.

   (e) If the current price is $30 per model airplane, should Acme increase or decrease this price to produce an increase in revenue?

   (Demand is elastic, so price and revenue act oppositely)

   $\boxed{\text{Since demand is elastic, Acme should decrease the price to produce an increase in revenue.}}$

   (f) Use elasticity of demand to find the price that maximizes revenue.

   Revenue is maximized when demand is unit elastic, i.e, $E(p)=1$.

   $$E(p)=1 \quad \text{so} \quad \frac{p}{40-p} = 1 \qquad \text{(cross-multiply)}$$

   $$p = 40-p$$

   $$2p=40$$

   $$\boxed{p = \$20}$$

domain = $\mathbb{R}$, so sign chart is $-\infty \longleftrightarrow \infty$

2. Let $f(x) = e^x(x+2)^2$. Using calculus techniques, find the intervals on which $f(x)$ is increasing, the intervals on which it is decreasing, and the locations of local extrema.

$f'(x) = e^x(x+2)^2 + e^x(2(x+2)'(1))$

$= e^x(x+2)^2 + 2e^x(x+2)$

$= e^x(x+2)[(x+2)+2]$

$= e^x(x+2)(x+4) = 0$

$e^x = 0$ No soln. $\quad x=-2 \quad x=-4$

Test #	f '(x)
-5	f'(-5) = 0.0202
-3	f'(-3) = -0.0498
0	f'(0) = 8

- f(x) is increasing on $(-\infty, -4)$ and $(-2, \infty)$.
- f(x) is decreasing on $(-4, -2)$.
- f(x) has a local maximum at $x = -4$ and a local minimum at $x = -2$.

3. Let $f(x) = 4x - x\ln x$. Using calculus techniques, find the intervals on which $f(x)$ is concave upward, the intervals on which it is concave downward, and the locations of any inflection points.

* First note that the domain of f(x) is $(0, \infty)$ because of the natural log. This means all sign charts should be made for $(0,\infty)$.

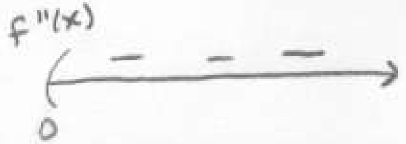
$f'(x) = 4 - [(1)\ln x + x(\frac{1}{x})]$

$f'(x) = 4 - \ln x - 1$

$f'(x) = -\ln x + 3$

$f''(x) = -\frac{1}{x} = 0$ ← not possible

$f''(x)$ DNE for $x = 0$



Test #	f ''(x)
1	$f''(1) = -\frac{1}{1} = -1$

$\boxed{\text{f(x) is concave downward on } (0,\infty) \text{ and has no inflection pts.}}$

4. Suppose that the number of students enrolled in a certain university can be modeled by $E(t) = 4t^3 - 87t^2 + 189t + 18{,}982$ students, where t is the number of years since September 1990, $1 \le t \le 17$.

   (a) Find the relative rate of change of enrollment in September 1995 $t=5$

$E(5) = 18252$ (in calc)

$E'(t) = 12t^2 - 174t + 189$

$E'(5) = -381$

Relative r.o.c $= \frac{f'(5)}{f(5)} = \frac{-381}{18252} = \boxed{-0.0209}$

   (b) Find the percentage rate of change of enrollment in September 2003 $t=13$

$E(13) = 15{,}524$

$E'(13) = -45$

% r.o.c. $= \frac{f'(13)}{f(13)} 100\% = \frac{-45}{15524} \cdot 100\% = \boxed{-0.2899\%}$

5. Let $f(x) = (x^2 - 7)(x^2 - 1)$.

(a) Analyze $f(x)$ by finding its domain, x and y-intercepts, and locations of any asymptotes.

Domain $= \mathbb{R}$

X-intercepts

$f(x) = 0$

$(x^2-7)(x^2-1)=0$

$x^2-7=0$ $\quad$ $x^2=1$

$x^2=7$ $\quad$ $x=\pm 1$

$x=\pm\sqrt{7}$

y-intercept

$f(0) = (0^2-7)(0^2-1) = 7$

$(0,7)$

Asymptotes: none since f(x) is a polynomial.

(b) Find the intervals on which $f(x)$ is increasing, the intervals on which it is decreasing, and the coordinates of all local extrema.

$f'(x) = 2x(x^2-1) + (x^2-7)(2x)$

$= 2x[(x^2-1)+(x^2-7)]$

$= 2x(2x^2-8)$

$= 4x(x^2-4) = 0$

$4x=0$ $\quad$ $x^2-4=0$

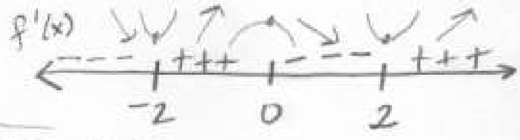
$x=0$ $\quad$ $x=\pm 2$

y-coords. of extrema

$f(0)=7$

$f(-2)=-9$

$f(2)=-9$



Test #	f'(x)
-3	f'(-3) = -60
-1	f'(-1) = 12
1	f'(1) = -12
3	f'(3) = 60

- f(x) is increasing on (-2,0) and (2,∞).
- f(x) is decreasing on (-∞,-2) and (0,2).
- f(x) has a local maximum at (0,7) and local minima at (-2,-9) and (2,-9).

(c) Find the intervals on which $f(x)$ is concave upward, the intervals on which it is concave downward, and the coordinates of any inflection points.

$f'(x) = 4x^3 - 16x$

$f''(x) = 12x^2 - 16 = 0$

$12x^2 = 16$

$x^2 = \frac{4}{3}$

$x = \pm\sqrt{\frac{4}{3}}$

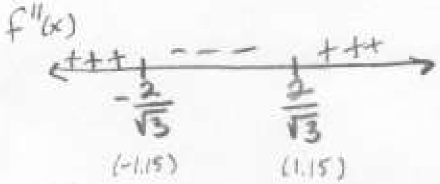
$= \pm\frac{2}{\sqrt{3}}$

y-coords of infl. pts

$f(-\frac{2}{\sqrt{3}}) = -\frac{17}{9}$

$f(\frac{2}{\sqrt{3}}) = \frac{-17}{9}$

≈ -1.89

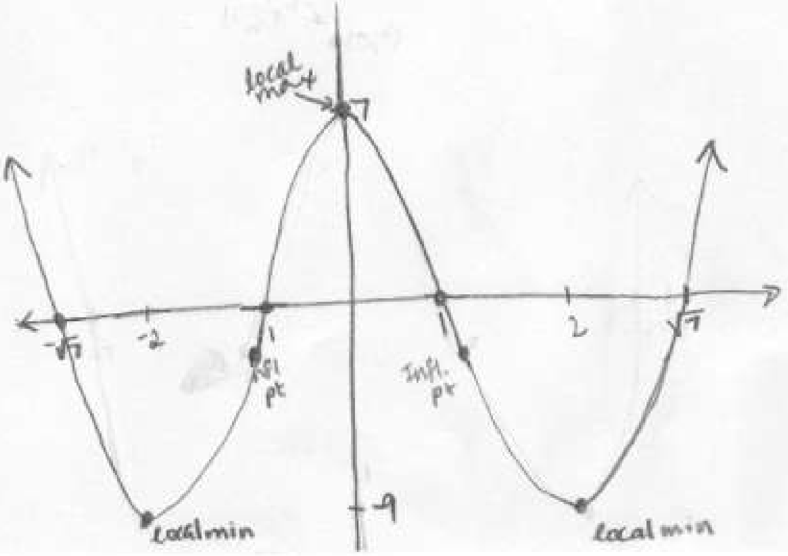


Test#	f''(x)
-2	f''(-2) = 32
0	f''(0) = -16
2	f''(2) = 32

- f(x) is concave up on $(-\infty, -\frac{2}{\sqrt{3}})$ and $(\frac{2}{\sqrt{3}}, \infty)$.
- f(x) is concave down on $(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$.
- f(x) has inflection pts at $(-\frac{2}{\sqrt{3}}, -\frac{17}{9})$ and $(\frac{2}{\sqrt{3}}, -\frac{17}{9})$

$\approx$ (-1.15, -1.89) $\quad$ (1.15, -1.89)

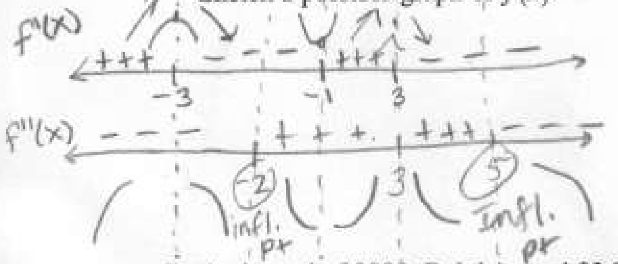
(d) Sketch a graph of $f(x)$.

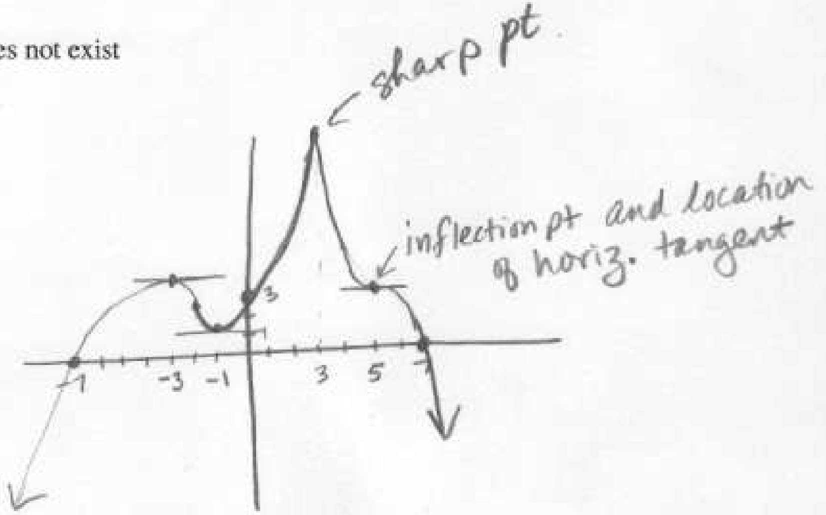


6. Suppose that f is a continuous function that satisfies the following.

- $f(-7) = 0$, $f(7) = 0$, and $f(0) = 3$
- $f'(-3) = f'(-1) = f'(5) = 0$ and $f'(3)$ does not exist
- $f'(x) > 0$ on $(-\infty, -3)$ and $(-1, 3)$
- $f'(x) < 0$ on $(-3, -1)$ and $(3, \infty)$
- $f''(x) > 0$ on $(-2, 3)$ and $(3, 5)$
- $f''(x) < 0$ on $(-\infty, -2)$ and $(5, \infty)$

Sketch a possible graph of $f(x)$.





7. At the end of 2002, Bob invested \$2,000 in an account paying 5.4% per year compounded continuously.

$a = 3$ $b = 6$

(a) Find the average rate of change of the account's value from the end of 2005 to the end of 2008.

$$f(x) = A = Pe^{rt} = \underbrace{2000e^{0.054t}}_{\text{Store in } Y_1}$$

$$\text{Avg. r.o.c} = \frac{f(b) - f(a)}{b - a} = \frac{Y_1(6) - Y_1(3)}{6 - 3} = \frac{\$413.5741}{3 \text{ yrs}} = \boxed{\$137.86 \text{ per year}}$$

(b) How quickly is the account's value growing at the end of 2007?

Instantaneous r.o.c.

$$f'(x) = 2000e^{0.054t}(0.054)$$

$$f'(5) = 2000e^{0.054(5)}(0.054) = \boxed{\text{increasing by } \$141.48 \text{ per year}}$$

8. Find the first [redacted] derivative of $g(x) = \frac{4x}{e^x + 3}$.

$$g'(x) = \frac{(e^x + 3)(4) - 4x(e^x)}{(e^x + 3)^2}$$

$$\boxed{g'(x) = \frac{4e^x + 12 - 4xe^x}{(e^x + 3)^2}}$$

9. Use the limit definition to find the instantaneous rate of change of $f(x) = 3 - 4x^2$ at $x = 5$.

① $f(5+h) = 3-4(5+h)^2$

$= 3-4(25+10h+h^2)$

$= 3-100-40h-4h^2$

$= -97-40h-4h^2$

Check

$f'(x) = -8x$

$f'(5) = -8(5) = -40$ ✓

② $f(5) = 3-4(5)^2 = -97$

③ $f(5+h) - f(5) = -97-40h-4h^2-(-97)$

$= -40h-4h^2$

④ $$\lim_{h\to 0} \frac{f(5+h)-f(5)}{h} = \lim_{h\to 0} \frac{-40h-4h^2}{h}$$

$$= \lim_{h\to 0}(-40-4h) = \boxed{-40}$$

10. Find the derivative of each of the following.

(a) $h(x) = 7 \cdot 3^{\ln 7x^2}$

$$h'(x) = 7(\ln 3)\cdot 3^{\ln 7x^2} \cdot \frac{d}{dx}(\ln 7x^2)$$

$$= (7\ln 3)\cdot 3^{\ln 7x^2} \cdot \left(\frac{1}{7x^2}\right)(14x)$$

$$= \boxed{\left(\frac{14\ln 3}{x}\right)\left(3^{\ln 7x^2}\right)}$$

(b) $j(x) = \log_8 \frac{3x}{x-9}$

$j(x) = \log_8 3x - \log_8(x-9)$

$j'(x) = \left(\frac{1}{\ln 8}\right)\left(\frac{1}{3x}\right)(3) - \left(\frac{1}{\ln 8}\right)\left(\frac{1}{x-9}\right)(1)$ ← unsimplified

$= \frac{1}{\ln 8}\left[\frac{1}{x} - \frac{1}{x-9}\right]$ (can be combined into one fraction)

$= \frac{1}{\ln 8}\left[\frac{x-9}{x(x-9)} - \frac{x}{x(x-9)}\right] = \boxed{\left(\frac{1}{\ln 8}\right)\left[\frac{-9}{x(x-9)}\right]} = j'(x)$

simplified

11. Acme Pool Supplies has determined the price-demand function for a 50 lb bucket of its chlorine tabs to be $35x + 100p = 17,500$, where x is the number of buckets that can be sold at a unit price of p dollars.

(a) Approximate the revenue earned from the sale of the 31st bucket of chlorine tabs.

use $R'(30)$

$R(x) = px$

$35x + 100p = 17500$

$100p = 17500 - 35x$

$p = 175 - 0.35x$

$R(x) = 175x - 0.35x^2$

$R'(x) = 175 - 0.7x$

$R'(30) = 175 - 0.7(30) = \$154$ per bucket

31st bucket will bring about \$154 in revenue.

(b) Find the revenue and marginal revenue from the sale of 55 buckets of chlorine tabs.

$R(55) = 175(55) - 0.35(55)^2 = \$8566.25$

$R'(55) = 175 - 0.7(55) = \$136.50$ per bucket

(c) Use your answers in (b) to approximate the revenue earned from the sale of 58 buckets of chlorine tabs.

$$R(58) \approx R(55) + 3R'(55)$$
$$= 8566.25 + 3(136.50)$$
$$= \boxed{\$8975.75}$$

(d) Acme Pool Supplies has a fixed cost of \$7,400 and a production cost of \$34 per bucket. Find a model for average cost.

$C(x) = 34x + 7400$

$\overline{C}(x) = \frac{C(x)\ \$}{x\ \text{bucket}} = \frac{34x + 7400}{x} = 34 + \frac{7400}{x}$ dollars per bucket

$\overline{C}(x) = 34 + \frac{7400}{x}$ dollars per bucket, where x is the number of 50-lb buckets of chlorine tabs produced

(e) Find the marginal average profit of producing and selling 42 buckets of chlorine tabs.

$P(x) = R(x) - C(x) = 175x - 0.35x^2 - (34x + 7400) = -0.35x^2 + 141x - 7400$ dollars

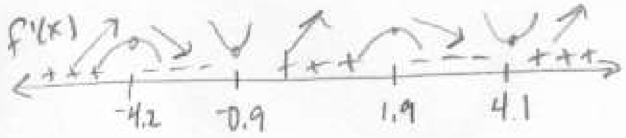
$\overline{P}(x) = \frac{P(x)}{x} = \frac{-0.35x^2 + 141x - 7400}{x} = -0.35x + 141 - \frac{7400}{x}$

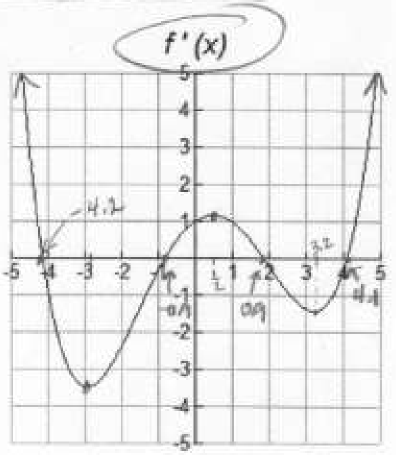
so $\overline{P}(x) = -0.35x + 141 - 7400x^{-1}$ dollars/bucket

$\overline{P}'(x) = -0.35 + 7400x^{-2}$ dollars/bucket per bucket

$\overline{P}'(42) = -0.35 + 7400(42)^{-2} = \boxed{\$3.85/\text{bucket per bucket}}$

12. The following graph represents the first derivative of a function f. Approximate your answers to one decimal place.





(a) Where is $f(x)$ increasing? Decreasing?

f(x) is increasing where $f'(x) > 0$: $(-\infty, -4.2)$ and $(-0.9, 1.9)$ and $(4.1, \infty)$

f(x) is decreasing where $f'(x) < 0$: $(-4.2, -0.9)$ and $(1.9, 4.1)$

(b) Where does $f(x)$ have local extrema?

$x = -4.2$	$x = -0.9$	$x = 1.9$	$x = 4.1$
local max	local min	local max	local min

(c) Where is $f(x)$ concave up? Concave down?

f(x) is concave up where slope ($f'(x)$) is increasing: $(-3, 0.5)$ and $(3.2, \infty)$

f(x) is concave down where slope($f'(x)$) is decreasing: $(-\infty, -3)$ and $(0.5, 3.2)$

(d) Where does $f(x)$ have inflection points?

$x = -3$ $\quad$ $x = 0.5$ $\quad$ $x = 3.2$

(f(x) has inflection pts at the same x's where $f'(x)$ has local extrema)

13. Find the derivative of $f(x) = (4x-3)^7 \ln(2x-7)$.

$$f'(x) = \left[7(4x-3)^6(4)\right]\ln(2x-7) + (4x-3)^7\left[\left(\frac{1}{2x-7}\right)(2)\right]$$

14. Find the derivative of $g(x) = \left(\frac{5x}{4^x - 6}\right)^3$.

$$g'(x) = 3\left(\frac{5x}{4^x-6}\right)^2 \cdot \frac{d}{dx}\left(\frac{5x}{4^x-6}\right)$$

$$\boxed{g'(x) = 3\left(\frac{5x}{4^x-6}\right)^2\left[\frac{(4^x-6)(5) - 5x(4^x \ln 4)}{(4^x-6)^2}\right]}$$

15. A rational function f, which has domain $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$, has first derivative $f'(x) = \frac{16x}{(x^2-4)^2}$.

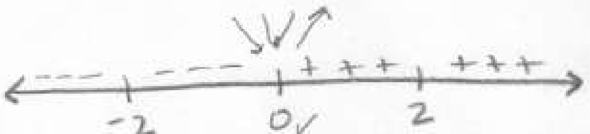
(a) Find the intervals where f is increasing and decreasing. Identify any local extrema.

$f'(x) = 0$ only if $16x = 0$

$x = 0$

$f'(x)$ DNE for $x = \pm 2$

Note: $x=0$, -2, and 2 are all partition #'s, but since $x=0$ is the only one in the domain of $f(x)$, $x=0$ is the only critical #



- $f(x)$ is increasing on $(0,2)$ and $(2,\infty)$.
- $f(x)$ is decreasing on $(-\infty,-2)$ and $(-2,0)$
- $f(x)$ has a local minimum at $x = 0$.

Test #	$f'(x)$
-3	$f'(-3) = -1.92$
-1	$f'(-1) = -1.78$
1	$f'(1) = 1.78$
3	$f'(3) = 1.92$

(b) Find $f''(x)$ and simplify.

$$f''(x) = \frac{(x^2-4)^2(16) - 16x\left[2(x^2-4)^1(2x)\right]}{(x^2-4)^4}$$

$$= \frac{16(x^2-4)\left[(x^2-4) - (2x)(2x)\right]}{(x^2-4)^{\cancel{4}\,3}}$$

$$= \frac{16\left[x^2-4-4x^2\right]}{(x^2-4)^3} = \boxed{\frac{16(-3x^2-4)}{(x^2-4)^3} = f''(x)}$$

(c) Find the intervals where f is concave upward and concave downward. Identify any ~~local extrema~~. inflection pts.

① $\frac{16(-3x^2-4)}{(x^2-4)^3} = 0$ only if $16(-3x^2-4) = 0$

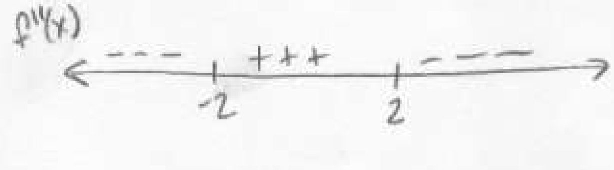
$-3x^2-4 = 0$

$-3x^2 = 4$

$x^2 = -\frac{4}{3}$

No soln (no $\sqrt{\text{neg}}$ #)

② $f''(x)$ DNE for $x = \pm 2$



Test #	$f''(x)$
-3	$f''(-3) = -3.968$
0	$f''(0) = 1$
3	$f''(3) = -3.968$

- $f(x)$ is concave upward on $(-2,2)$ and concave downward on $(-\infty,-2)$ and $(2,\infty)$.
- $f(x)$ HAS NO INFLECTION POINTS since $x=-2$ and $x=2$ are not in the domain of $f(x)$.