Math 141 - Week in Review #~~6~~ 7

Section 7.1 - Experiments, Sample Spaces, and Events

- An **experiment** is an activity with observable results (called **outcomes**).
- Sample Space - the set of all possible outcomes of an experiment
- Event - a subset of a sample space of an experiment
- An event E is said to **occur** in a trial of an experiment whenever E contains the observed outcome.
- Unions, intersections, and complements of events are found in the same ways as they are for sets.
- Two events E and F are **mutually exclusive** if $E \cap F = \emptyset$ (i.e., if it is impossible for both E and F to occur at the same time).

Section 7.2 - Definition of Probability

- The probability of an event is a number between 0 and 1 inclusive that indicates the likelihood of that event occurring. The closer the probability is to 1, the more likely the event is to occur.

- Probability Distribution - a table that lists all of the simple events of an experiment and their corresponding probabilities.

  NOTE: The sum of all probabilities in a probability distribution is always 1.

- Uniform Sample Space - a sample space in which all outcomes are equally likely.

- If $E = \{s_1, s_2, \ldots, s_k\}$ is an event of an experiment with sample space S, then $P(E) = P(s_1) + P(s_2) + \cdots + P(s_k)$.

Section 7.3 - Rules of Probability

Let S be a sample space of an experiment and suppose E and F are events of the experiment. Then

1. $0 \le P(E) \le 1$ for any E.
2. $P(S) = 1$
3. If E and F are mutually exclusive, then $P(E \cup F) = P(E) + P(F)$.
4. If E and F are any two events of an experiment, then $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.
   or
5. $P(E^c) = 1 - P(E)$ (Rule of Complements)

NOTE: When calculating probabilities, Venn Diagrams can sometimes be useful. De Morgan's Laws may also come in handy from time to time: $(E \cap F)^c = E^c \cup F^c$ and $(E \cup F)^c = E^c \cap F^c$.

Section 7.4 - Use of Counting Techniques in Probability

- Computing the Probability of an Event in a Uniform Sample Space - Let S be a uniform sample space and let E be any event. Then

$$P(E) = \frac{number\ of\ favorable\ outcomes\ in\ E}{number\ of\ possible\ outcomes\ in\ S} = \frac{n(E)}{n(S)}$$

1. One card is selected at random from a standard deck of 52 playing cards, and the suit of the card is recorded. What is the sample space of this experiment?

$S = \{\text{hearts, diamonds, spades, clubs}\}$

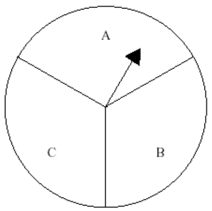
2. One card is selected at random from a standard deck of 52 playing cards, and the color is recorded. What is the sample space of this experiment?

$S = \{\text{black, red}\}$

3. The numbers 0, 1, 2, and 3 are written onto 4 individual slips of paper and placed in a bag. Two numbers are selected at random from the bag and their product is recorded. What is the sample space of this experiment?

$S = \{0, 2, 3, 6\}$

4. A fair coin is tossed and the hand on the spinner below is spun, and the side landing up on the coin and region in which the arrow stops are recorded. (Exclude the possibility that the spinner lands on a line.) What is the sample space of this experiment?



$S = \{(H,A), (H,B), (T,A), (H,C), (T,B), (T,C)\}$

The # of subsets (events) is 2^n where $n = n(S)$

2^6 subsets $= 64$

5. Two fair four-sided dice are cast and the numbers that land up on the first and second dice are recorded.

(a) What is the sample space of this experiment?

$S = \{(1,1), (1,2), (1,3), (1,4),$
$(2,1), (2,2), (2,3), (2,4),$
$(3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$

(b) Is this a uniform sample space? Why or why not?

Yes. because all outcomes are equally likely.

(c) Write the event that the sum of the dice is 6.

subset

$E = \{(2,4), (3,3), (4,2)\}$

(d) What is the probability that the first die is a 2 and the second die is even?

$F = \{(2,2), (2,4)\}$

$P(F) = \frac{2}{16} = \boxed{\frac{1}{8}}$

(e) What is the probability that the sum of the numbers shown on the dice is less than 4 or at least one die shows a 1?

union

Let A be the event that the sum is less than 4.
Let B - - - - - at least 1 die shows the # 1.

$A = \{(1,1), (1,2), (2,1)\}$

$B = \{(1,1), (1,2), (1,3), (1,4), (2,1), (3,1), (4,1)\}$

$A \cup B = \{(1,1), (1,2), (2,1), (1,3), (1,4), (3,1), (4,1)\}$

$\boxed{P(A \cup B) = \frac{7}{16}}$

6. Two fair four-sided dice are cast and the sum of the numbers landing up is recorded.

(a) What is the sample space of this experiment?

$S = \{2, 3, 4, 5, 6, 7, 8\}$

(b) Find the probability distribution for this experiment.

make a table of probabilities!

~~Outcome~~ Simple Event	$\{2\}$	$\{3\}$	$\{4\}$	$\{5\}$	$\{6\}$	$\{7\}$	$\{8\}$
Probability	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

Probab that the sum is 2

must all add to 1

(c) Is this a uniform sample space? Why or why not?

No! The outcomes are not all equally likely.

(d) Write the event that the sum of the dice is 6.

Subset of the given sample space.

$E = \{6\}$

(different from 5c)

Title: Mar 27 - 4:20 PM (5 of 17)

7. Let $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ be the sample space of an experiment with the following probability distribution:

must add to 1.

$P(s_5) = 1 - \frac{3}{40} - \frac{4}{40} - \frac{7}{40} - \frac{14}{40} - \frac{3}{40} = 9/40$

Outcome	s_1	s_2	s_3	s_4	s_5	s_6
Probability	$\frac{3}{40}$	$\frac{4}{40}$	$\frac{7}{40}$	$\frac{14}{40}$	$\frac{9}{40}$	$\frac{3}{40}$

Let $A = \{s_1, s_3, s_5\}$, $B = \{s_3, s_4, s_6\}$, and $C = \{s_2, s_4\}$ be events of the experiment and suppose $P(B) = \frac{24}{40}$.

(a) Fill in the missing probabilities in the probability distribution above.

$P(B) = P(s_3) + P(s_4) + P(s_6)$

$24/40 = 7/40 + 14/40 + P(s_6)$

Solve for $P(s_6)$

$P(s_6) = \frac{3}{40}$

(b) Is this a uniform sample space? Why or why not?

No because outcomes are not equally likely.

(c) Find each of the following:

i. $P(A) = P(s_1) + P(s_3) + P(s_5) = \frac{3}{40} + \frac{7}{40} + \frac{9}{40} = \boxed{\frac{19}{40}}$

ii. $P(C) = P(s_2) + P(s_4) = \frac{4}{40} + \frac{14}{40} = \boxed{\frac{18}{40}}$

iii. $P(B^c) = 1 - P(B) = 1 - \frac{24}{40} = \frac{16}{40} = \boxed{\frac{2}{5}}$

iv. $P(A \cap B) =$ Find $A \cap B = \{s_3\}$ $\quad P(A \cap B) = P(s_3) = \boxed{\frac{7}{40}}$

v. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{19}{40} + \frac{24}{40} - \frac{7}{40} = \boxed{\frac{36}{40}}$

(d) Are the events A and C mutually exclusive? Why or why not?

Find $A \cap C = \{\ \} = \phi$

They are mutually exclusive b/c they have no outcomes in common.

8. A bag contains 3 pennies, a nickel, and two dimes. Two coins are selected at random from the bag and the monetary value of the coins (in cents) is recorded.

(n, 1 penny) (2 pennies)

pp
pn
nd
pd
dd

(a) What is the sample space of this experiment?

$S = \{6, 2, 11, 20, 15\}$

(b) Write the event E that the monetary value of the coins is less than 11 cents.

$E = \{2, 6\}$

(c) Write the event F that the nickel is drawn.

$F = \{6, 15\}$

(d) Are the events E and F mutually exclusive? Support your answer.

No because they have the outcome 6 in common.

(e) Write the event G that the value of the coins is more than 25 cents.

$G = \emptyset$ (impossible)

9. An experiment consists of randomly selecting an integer multiple of 3 that is between 3 and 21 (inclusive).

(a) What is the sample space of this experiment?

$S = \{3, 6, 9, 12, 15, 18, 21\}$

(b) Write the event E that the number selected is even.

$E = \{6, 12, 18\}$

(c) Write the event F that the number selected is a multiple of 4.

$F = \{12\}$

(d) Write the event G that the number selected is odd and less than 15.

$G = \{3, 9\}$

(e) Which pairs of the events E, F, and G are mutually exclusive?

F and G / E and G are mutually exclusive.

(f) If the number selected was 12, which of the events E, F, and G have occurred?

E and F

10. Let E and F be two events that are mutually exclusive and suppose $P(E) = 0.35$ and $P(F) = 0.4$. Find

← means $E \cap F = \emptyset$

(a) $P(E \cap F) = P(\emptyset) = \underline{0}$

(b) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

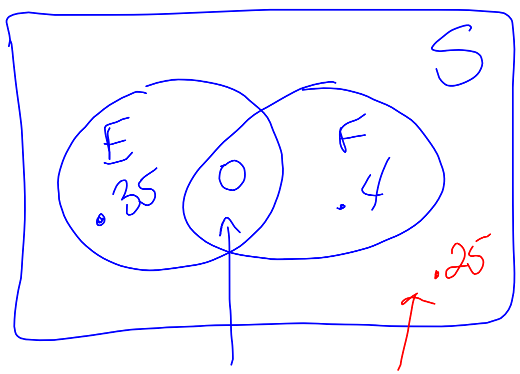
$= .35 + .4 - 0$

$= \boxed{.75}$

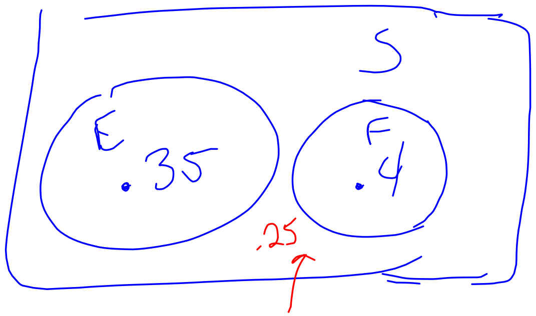
(c) $P(E^c \cup F^c) = P((E \cap F)^c) = 1 - P(E \cap F) = 1 - 0 = \boxed{1}$

(d) $P(E \cap F^c) = \boxed{.35}$

Use a Venn Diagram — all regions must sum to 1 (probabilities)



or



$1 - .35 - .4$

11. Acme, Inc. advertised its products in two magazines: Magazine A and Magazine B. A survey of 400 customers revealed that 120 learned of its products from Magazine A, 95 learned of its products from Magazine B, and 70 learned of its products from both magazines. What is the probability that a person selected at random from this group saw Acme, Inc.'s advertisement in

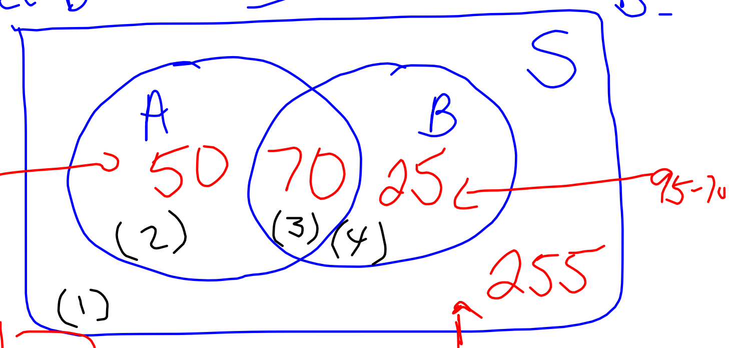
Let A be the event the person saw the ads in Magazine A.
Let B - - - - - - - - B.

E:

(a) exactly one of these magazines?

$$P(E) = \frac{50+25}{400} = \frac{75}{400} = \boxed{\frac{3}{16}}$$

120 − 70



400 − 50 − 70 − 25

(b) at least one of these magazines?

$$P(A \cup B) = \frac{50+70+25}{400} = \frac{145}{400} = \boxed{\frac{29}{80}}$$

(c) in Magazine A but not Magazine B?

and

$$P(A \cap B^c) = \frac{50}{400} = \boxed{\frac{1}{8}}$$

Information about a Standard Deck of 52 Cards

- There are 4 suits: hearts, diamonds, clubs, and spades.
- Hearts and diamonds are red; clubs and spades are black.
- There are 13 cards in each suit: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King.
- Jacks, Queens, and Kings are called face cards, so there are 12 face cards in a standard deck of 52 cards.

12. One card is drawn at random from a standard deck of 52 playing cards. What is the probability that the card is

(a) a club?

$$P(\text{club}) = \frac{13}{52} = \boxed{\frac{1}{4}}$$

(b) a face card?

$$P(\text{face card}) = \frac{12}{52} = \frac{3}{13}$$

(c) a club or a face card?

union

Let C be the event we get a club.
Let F - - - - - - a face card.

$$P(C \cup F) = P(C) + P(F) - P(C \cap F)$$
$$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \boxed{\frac{11}{26}}$$

(d) neither a club nor a face card?

$$P(C^c \cap F^c) = P((C \cup F)^c)$$
$$= 1 - P(C \cup F)$$
$$= 1 - \frac{11}{26}$$
$$= \boxed{\frac{15}{26}}$$

13. A box contains 2 red marbles, 8 yellow marbles, 6 red gumballs, 5 yellow gumballs, and 3 blue jawbreakers. If a sample of 5 objects is randomly chosen from the box (without replacement), what is the probability that

(a) exactly 3 yellow marbles are chosen?

Let E be the event that exactly 3 yellow marbles are chosen.

$n(S)$: $C(24,5)$

$$P(E) = \frac{n(E)}{n(S)} = \boxed{\frac{C(8,3)C(16,2)}{C(24,5)}}$$

$n(E) = C(8,3)C(16,2)$

T_1 - choose 3 yellow marbles $n_1 = C(8,3)$

T_2 - choose 2 other objects $n_2 = C(16,2)$

(b) exactly 3 red gumballs and exactly 2 yellow objects are chosen?

Let F be the event we get exactly 3 red gumballs and 2 yellow objects.

$$P(F) = \frac{n(F)}{n(S)} = \boxed{\frac{C(6,3)C(13,2)}{C(24,5)}}$$

$n(F)$:

T_1 - choose 3 red gumballs. $n_1 = C(6,3)$

$\times$

T_2 - choose 2 yellow objects $n_2 = C(13,2)$

(c) exactly 4 yellow marbles or exactly 1 blue jawbreaker is chosen?

union

Let E be the event we get exactly 4 yellow marbles.

Let F - - - - exactly 1 blue jawbreaker.

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$= \boxed{\frac{C(8,4)C(16,1)}{C(24,5)} + \frac{C(3,1)C(21,4)}{C(24,5)} - \frac{C(8,4)C(3,1)}{C(24,5)}}$$

$P(E) = \frac{n(E)}{n(S)} = \frac{}{C(24,5)}$

$n(E): C(8,4)C(16,1)$

$P(F) = \frac{n(F)}{n(S)}$

$n(F) = C(3,1)C(21,4)$

$P(E \cap F) = \frac{n(E \cap F)}{n(S)}$

$n(E \cap F) = C(8,4)C(3,1)$

(d) at least 1 yellow marble is chosen?

1 or more

Exactly 1 or 2 or 3 or 4 or 5 yellow marbles.

Let G be the event we get at least 1 yellow marble. Use the complement!

$P(G) = 1 - P(G^c)$, where G^c is the event that the sample has no yellow marbles.

$$P(G) = 1 - P(G^c)$$

$$= 1 - \frac{n(G^c)}{n(S)}$$

$$= \boxed{1 - \frac{C(16,5)}{C(24,5)}}$$

$n(G^c)$ = the # of ways of choosing 5 objects with no yellow marbles,

$n(G^c) = C(16,5)$

14. Acme, Inc. ships lightbulbs in lots of 50. Before each lot is shipped, a sample of 8 lightbulbs is selected from the lot for testing. If any of the bulbs is defective, the entire lot is rejected. What is the probability that a lot containing 3 defective lightbulbs will still get shipped?

Let E be the event that the lot still gets shipped. In other words, E is the event that there are no defectives in the sample of 8 objects.

choose 0 defectives

$$P(E) = \frac{n(E)}{n(S)} = \boxed{\frac{C(47,8)}{C(50,8)}}$$

$n(S) = C(50,8)$

15. License plates in a certain state are made with 6 symbols using letters and digits. If a person is given a new license plate, what is the probability that the license plate

(a) has 3 letters followed by 3 digits? Let E be the event the license plate has 3 letters followed by 3 digits.

$$P(E) = \frac{n(E)}{n(S)} = \frac{26^3 \cdot 10^3}{(36)^6} \approx \boxed{0.0081}$$

$n(S) = \underline{36} \cdot \underline{36} \cdot \underline{36} \cdot \underline{36} \cdot \underline{36} \cdot \underline{36}$

$n(E) = \underbrace{\underline{26} \cdot \underline{26} \cdot \underline{26}}_{\text{letters}} \cdot \underbrace{\underline{10} \cdot \underline{10} \cdot \underline{10}}_{\text{digits}}$

(b) starts with the letter A or B and ends in an odd digit?

Let F be the event that it

$$P(F) = \frac{n(F)}{n(S)} = \frac{10 \cdot (36)^4}{(36)^6} = \boxed{\frac{5}{648}}$$

$n(F) = \underline{2} \cdot \underline{36} \cdot \underline{36} \cdot \underline{36} \cdot \underline{36} \, \underline{5}$

(c) has no letter or digit repeated?

Let G be the event that no letter or digit is repeated.

$$P(G) = \frac{n(G)}{n(S)} = \frac{36 \cdot 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31}{(36)^6} = \approx \boxed{.6443}$$

$n(G) = \underline{36} \cdot \underline{35} \cdot \underline{34} \cdot \underline{33} \, \underline{32} \cdot \underline{31}$

$= P(36, 6)$

16. Harry, Sally, and 5 of their friends go to the movies and randomly sit in seven adjacent chairs. What is the probability that

(a) Harry and Sally sit on opposite ends from each other?

Let E be the event that Harry + Sally sit on opposite ends.

$$P(E) = \frac{n(E)}{n(S)} = \frac{2!\,5!}{7!} = \boxed{\frac{1}{21}}$$

$n(S) = P(7,7) = 7!$

$n(E) = \underline{2}\cdot\underline{5}\cdot\underline{4}\cdot\underline{3}\cdot\underline{2}\cdot\underline{1}\cdot\underline{1}$

$= 2!\,5!$

(b) Sally sits in the middle chair?

Let F be the event that Sally sits in the middle chair.

$$P(F) = \frac{n(F)}{n(S)} = \frac{6!}{7!} = \boxed{\frac{1}{7}}$$

$n(F) = \underline{6}\cdot\underline{5}\cdot\underline{4}\cdot\underline{1}\cdot\underline{3}\cdot\underline{2}\cdot\underline{1}$

$= 6!$

(c) Harry and Sally sit together?

Let G be the event that Harry and Sally sit together.

$$P(G) = \frac{n(G)}{n(S)} = \frac{6\cdot 2!\cdot 5!}{7!} = \boxed{\frac{2}{7}}$$

$n(G) = 5\cdot 4\cdot 2\cdot 1\cdot 3\cdot 2\cdot 1$

T_1 - Choose 2 chairs next to each other for Harry + Sally. $n_1 = 6$

T_2 - line up Harry + Sally in their 2 chairs $n_2 = 2!$

T_3 - lining the others in the remaining chairs $n_3 = 5!$

(d) Harry and Sally do not sit together?

$$P(G^c) = 1 - P(G)$$

$$= 1 - \frac{2}{7}$$

$$= \boxed{\frac{5}{7}}$$

17. Three married couples go to the movies. If these 6 people randomly sit in 6 adjacent chairs, what is the probability that each person sits next to his or her spouse (i.e., married couples sit together)?

Let E be the event that married couples sit together.

$$P(E) = \frac{n(E)}{n(S)} = \frac{3!\,2!\,2!\,2!}{6!} = \frac{3!(2!)^3}{6!} = \boxed{\frac{8}{15}}$$

$n(S) = P(6,6) = 6!$

$$n(E) = \overbrace{\underbrace{2 \cdot 1}_{3} \cdot \underbrace{2 \cdot 1}_{2} \cdot \underbrace{2 \cdot 1}_{1}}$$

T_1 - arrange the 3 couples $n_1 = 3!$

T_2 - arrange the spouses ~~people~~ in the 1st couple $n_2 = 2 \cdot 1 = 2!$

T_3 - arrange spouses in 2nd couple $n_3 = 2!$

T_4 - arrange spouse in 3rd couple $n_4 = 2!$

$n(E) = 3!(2!)^3$