**Math 166 - Week in Review #7**

Section 7.2 - Definition of Probability

- The probability of an event is a number between 0 and 1 inclusive that indicates the likelihood of that event occurring. The closer the probability is to 1, the more likely the event is to occur.
- Probability Distribution - a table that lists all of the simple events of an experiment and their corresponding probabilities.

  NOTE: The sum of all probabilities in a probability distribution is always 1.
- Uniform Sample Space - a sample space in which all outcomes are equally likely.
- If $E = \{s_1, s_2, \dots, s_k\}$ is an event of an experiment with sample space $S$, then $P(E) = P(s_1) + P(s_2) + \cdots + P(s_k)$.

Section 7.3 - Rules of Probability

Let $S$ be a sample space of an experiment and suppose $E$ and $F$ are events of the experiment. Then

1. $0 \leq P(E) \leq 1$ for any $E$.
2. $P(S) = 1$
3. If $E$ and $F$ are mutually exclusive, then $P(E \cup F) = P(E) + P(F)$.
4. If $E$ and $F$ are any two events of an experiment, then $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.
5. $P(E^c) = 1 - P(E)$ (Rule of Complements)

NOTE: When calculating probabilities, Venn Diagrams can sometimes be useful. De Morgan's Laws may also come in handy from time to time: $(E \cap F)^c = E^c \cup F^c$ and $(E \cup F)^c = E^c \cap F^c$.

Section 7.4 - Use of Counting Techniques in Probability

- Computing the Probability of an Event in a Uniform Sample Space - Let $S$ be a uniform sample space and let $E$ be any event. Then

$$P(E) = \frac{number\ of\ favorable\ outcomes\ in\ E}{number\ of\ possible\ outcomes\ in\ S} = \frac{n(E)}{n(S)}$$

1. Two fair four-sided dice are cast and the numbers that land up on the first and second dice are recorded.

(a) What is the sample space of this experiment?

$$S = \{(1,1),(1,2),(1,3),(1,4),$$
$$(2,1),(2,2),(2,3),(2,4),$$
$$(3,1),(3,2),(3,3),(3,4),$$
$$(4,1),(4,2),(4,3),(4,4)\}$$

(b) Is this a uniform sample space? Why or why not?

Yes. All outcomes are equally likely.

(c) Write the ~~event~~ subset that the sum of the dice is 6.

$$E = \{(4,2),(3,3),(2,4)\}$$

(d) What is the probability that the first die is a 2 and the second die is even?

F - event that the 1st die is 2 & the 2nd die is even.

$$F = \{(2,2),(2,4)\} \qquad P(F) = \frac{2}{16} = \boxed{\frac{1}{8}}$$

(e) What is the probability that the sum of the numbers shown on the dice is less than 4 or at least one die shows a 1?

$\cup$

A - event that the sum is less than 4.

B - - — - - at least 1 die shows a 1.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{3}{16} + \frac{7}{16} - \frac{3}{16} = \boxed{\frac{7}{16}}$$

$$A = \{(1,1),(2,1),(1,2)\}$$
$$B = \{(1,1),(1,2),(1,3),(1,4),(2,1),(3,1),(4,1)\}$$

$$A \cap B = \{(1,1),(2,1),(1,2)\}$$

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2. Two fair four-sided dice are cast and the sum of the numbers landing up is recorded.

(a) What is the sample space of this experiment?

$S = \{2, 3, 4, 5, 6, 7, 8\}$

(b) Find the probability distribution for this experiment.

make a table

| Outcome | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| Probability | $\frac{1}{16}$ | $\frac{2}{16}$ | $\frac{3}{16}$ | $\frac{4}{16}$ | $\frac{3}{16}$ | $\frac{2}{16}$ | $\frac{1}{16}$ |

(c) Is this a uniform sample space? Why or why not?

No. The outcomes are <u>not</u> equally likely.

(d) Write the ~~event~~ that the sum of the dice is 6.

subset of the sample space.

$E = \{6\}$

3. Let $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ be the sample space of an experiment with the following probability distribution:

$P(s_5)$ : $1 -$ sum of all the other probab's

| **Outcome** | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $s_5$ | $s_6$ |
|---|---|---|---|---|---|---|
| **Probability** | $\frac{3}{40}$ | $\frac{4}{40}$ | $\frac{7}{40}$ | $\frac{14}{40}$ | $\frac{9}{40}$ | $\frac{3}{40}$ |

Let $A = \{s_1, s_3, s_5\}$, $B = \{s_3, s_4, s_6\}$, and $C = \{s_2, s_4\}$ be events of the experiment and suppose $P(B) = \frac{24}{40}$.

(a) Fill in the missing probabilities in the probability distribution above.

$$P(B) = P(s_3) + P(s_4) + P(s_6)$$

$$\frac{24}{40} = \frac{7}{40} + \frac{14}{40} + P(s_6)$$

$$P(s_6) = 3/40$$

(b) Is this a uniform sample space? Why or why not?

No. The outcomes are not equally likely.

(c) Find each of the following:

i. $P(A) = P(s_1) + P(s_3) + P(s_5)$

$$= \frac{3}{40} + \frac{7}{40} + \frac{9}{40} = \boxed{\frac{19}{40}}$$

ii. $P(C) = P(s_2) + P(s_4)$

$$= \frac{4}{40} + \frac{14}{40} = \frac{18}{40} = \boxed{\frac{9}{20}}$$

iii. $P(B^c) = 1 - P(B)$

$$= 1 - \frac{24}{40} = \frac{16}{40} = \boxed{\frac{2}{5}}$$

iv. $P(A \cap B)$

$$A \cap B = \{s_3\}$$

$$P(A \cap B) = P(s_3) = \boxed{\frac{7}{40}}$$

v. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{19}{40} + \frac{24}{40} - \frac{7}{40} = \frac{36}{40} = \boxed{\frac{9}{10}}$$

(d) Are the events $A$ and $C$ mutually exclusive? Why or why not?

$A \cap C = \emptyset$ Since $A$ and $C$ have no outcomes in common, they are mutually exclusive.

4. Let $E$ and $F$ be two events that are mutually exclusive and suppose $P(E) = 0.35$ and $P(F) = 0.4$. Find

(a) $P(E \overset{\text{and}}{\cap} F) = P(\phi) = \boxed{0}$

(b) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= .35 + .4 - 0$

$= \boxed{.75}$

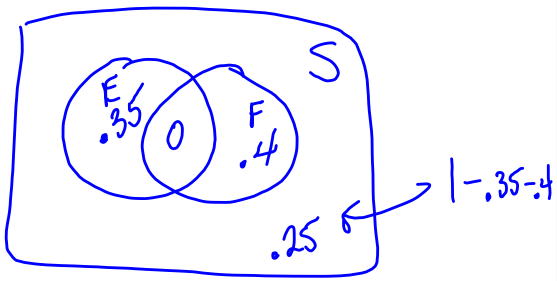
(c) $P(E^c \cup F^c) = P\left((E \cap F)^c\right)$

$= 1 - P(E \cap F)$

$= 1 - 0 = \boxed{1}$

(d) $P(E \cap F^c) = \boxed{.35}$ (see venn diagram)

Note about using Venn Diagrams for probab: Probab's from all regions in the Venn Diagram must add to 1.



5. Acme, Inc. advertised its products in two magazines: Magazine A and Magazine B. A survey of 400 customers revealed that 120 learned of its products from Magazine A, 95 learned of its products from Magazine B, and 70 learned of its products from both magazines. What is the probability that a person selected at random from this group saw Acme, Inc.'s advertisement in

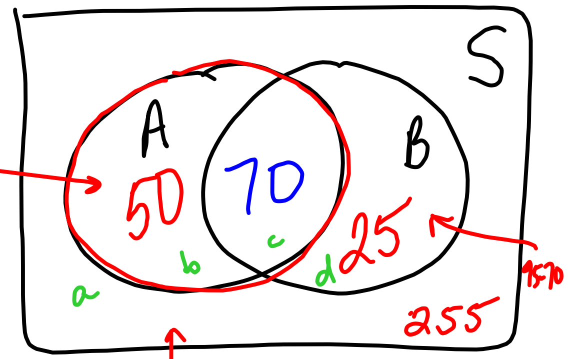
A- event that the person saw the ad in mag. A.
B- — — — — mag. B.

(a) exactly one of these magazines?

$$\frac{50+25}{400} = \frac{75}{400} = \boxed{\frac{3}{16}}$$

see next page for notation

120-70




(b) at least one of these magazines?

1 or more

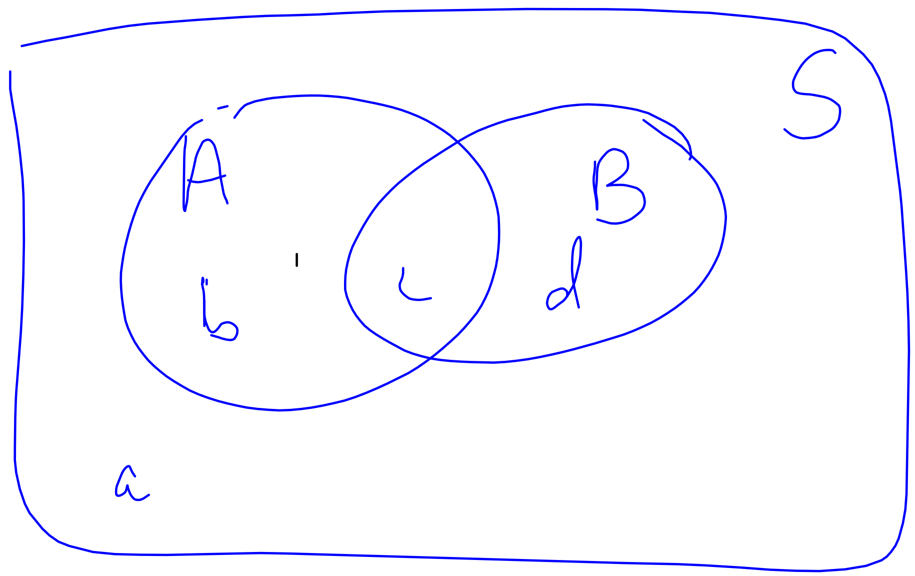
$$P(A \cup B) = \frac{50+70+25}{400} = \frac{145}{400} = \boxed{\frac{29}{80}}$$

400-50-70-25

(c) in Magazine A but not Magazine B?

and

$$P(A \cap B^c) = \frac{50}{400} = \boxed{\frac{1}{8}}$$




Regions b and d

$$P\left(\left(A \cap B^c\right) \cup \left(B \cap A^c\right)\right)$$

(notation for probability in #5a.)

Information about a Standard Deck of 52 Cards

- There are 4 suits: hearts, diamonds, clubs, and spades.
- Hearts and diamonds are red; clubs and spades are black.
- There are 13 cards in each suit: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King.
- Jacks, Queens, and Kings are called face cards, so there are 12 face cards in a standard deck of 52 cards.

6. One card is drawn at random from a standard deck of 52 playing cards. What is the probability that the card is

(a) a club? E - event that the card is a club.

$$P(E) = \frac{n(E)}{n(S)} = \frac{13}{52} = \boxed{\frac{1}{4}}$$

(b) a face card? F - event that the card is a face card

$$P(F) = \boxed{\frac{12}{52}} = \frac{3}{13}$$

(c) a club or a face card?

$\cup$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \boxed{\frac{11}{26}}$$

(d) neither a club nor a face card?

$$P(E^c \cap F^c) = P((E \cup F)^c)$$

$$= 1 - P(E \cup F)$$

$$= 1 - \frac{11}{26}$$

$$= \boxed{\frac{15}{26}}$$

7. A box contains 2 red marbles, 8 yellow marbles, 6 red gumballs, 5 yellow gumballs, and 3 blue jawbreakers. If a sample of 5 objects is randomly chosen from the box (without replacement), what is the probability that

(a) exactly 3 yellow marbles are chosen?

E - event that exactly 3 yellow marbles are chosen.

$$P(E) = \frac{n(E)}{n(S)} = \boxed{\frac{C(8,3)C(16,2)}{C(24,5)}} = \boxed{\frac{40}{253}}$$

$n(S): C(24,5)$

$n(E)$:
$T_1$ - choose 3 yellow marbles $n_1 = C(8,3)$
$T_2$ - choose 2 other objects (not yellow marbles) $n_2 = C(16,2)$

(b) exactly 3 red gumballs and exactly 2 yellow objects are chosen?

F - event that we get exactly 3 red gumballs and 2 yellow objects.

$$P(F) = \frac{n(F)}{n(S)} = \boxed{\frac{C(6,3)C(13,2)}{C(24,5)}} = \frac{65}{1771}$$

(c) exactly 4 yellow marbles or exactly 1 blue jawbreaker is chosen?

$\cup$

A - event that exactly 4 yellow marbles are chosen.

B - - - - - - - 1 blue jawbreaker is chosen

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \overset{\text{and}}{\cap} B)}{n(S)}$$

$$= \boxed{\frac{C(8,4)C(16,1)}{C(24,5)} + \frac{C(3,1)C(21,4)}{C(24,5)} - \frac{C(8,4)C(3,1)}{C(24,5)}}$$

(d) at least 1 yellow marble is chosen?

G - event that at least yellow marble is chosen.

1 or more

1 or 2 or 3 or 4 or 5

$P(G) = 1 - P(G^c)$ where $G^c$ is the event that 0 yellow marbles are chosen.

$$P(G) = 1 - \frac{n(G^c)}{n(S)}$$

$$= \boxed{1 - \frac{C(16,5)}{C(24,5)}}$$

## Rule of Complements

$$P(G) = 1 - P(G^c)$$

$$P(G^c) = 1 - P(G)$$

8. Acme, Inc. ships lightbulbs in lots of 50. Before each lot is shipped, a sample of 8 lightbulbs is selected from the lot for testing. If any of the bulbs is defective, the entire lot is rejected. What is the probability that a lot containing 3 defective lightbulbs will still get shipped?

E - event that the lot will get shipped. (In other words, E is the event that the sample of 8 had no defectives.)

$$P(E) = \frac{n(E)}{n(S)} = \boxed{\frac{C(47,8)}{C(50,8)}}$$

$$= \boxed{\frac{41}{70}}$$

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9. License plates in a certain state are made with 6 symbols using letters and digits. If a person is given a new license plate, what is the probability that the license plate

(a) has 3 letters followed by 3 digits?

E - event that the license plate has 3 letters followed by 3 digits.

$$P(E) = \frac{n(E)}{n(S)} = \frac{26^3 \cdot 10^3}{36^6} \approx \boxed{.0081}$$

$n(S)$: $\underline{36} \cdot \underline{36} \cdot \underline{36} \cdot \underline{36} \cdot \underline{36} \cdot \underline{36} = 36^6$

$n(E)$: $\underbrace{\underline{26} \cdot \underline{26} \cdot \underline{26}}_{\text{letters}} \cdot \underbrace{\underline{10} \cdot \underline{10} \cdot \underline{10}}_{\text{digits}}$

(b) starts with the letter A or B and ends in an odd digit?

F - event that it starts with A or B and ends in an odd digit.

$$P(F) = \frac{n(F)}{n(S)} = \frac{10 \cdot 36^4}{36^6} = \boxed{\frac{5}{648}}$$

$n(F)$: $\underset{\text{A or B}}{\underline{2}}\ \underline{36}\ \underline{36}\ \underline{36}\ \underline{36}\ \underset{\text{odd}}{\underline{5}}$

(c) has no letter or digit repeated?

G - event that no letter or digit is repeated.

$$P(G) = \frac{n(G)}{n(S)} = \frac{P(36,6)}{36^6} \approx \boxed{.6443}$$

$n(G)$: $\underline{36} \cdot \underline{35} \cdot \underline{34} \cdot \underline{33} \cdot \underline{32} \cdot \underline{31} = P(36,6)$

10. Harry, Sally, and 5 of their friends go to the movies and randomly sit in seven adjacent chairs. What is the probability that

(a) Harry and Sally sit on opposite ends from each other?

E - event that Harry & Sally sit on opposite ends.

$$P(E) = \frac{n(E)}{n(S)} = \frac{2!\cdot 5!}{7!} = \boxed{\frac{1}{21}}$$

$n(S)$: $\underline{7}\cdot\underline{6}\cdot\underline{5}\cdot\underline{4}\cdot\underline{3}\cdot\underline{2}\cdot\underline{1} = 7!$

$n(E)$: $\underline{2}\cdot\underline{5}\cdot\underline{4}\cdot\underline{3}\cdot\underline{2}\cdot\underline{1}\cdot\underline{1}$

(b) Sally sits in the middle chair?

F - event that Sally sits in the middle chair.

$$P(F) = \frac{n(F)}{n(S)} = \frac{6!}{7!} = \boxed{\frac{1}{7}}$$

$n(F)$: $\underline{6}\cdot\underline{5}\cdot\underline{4}\cdot\underline{1}\cdot\underline{3}\cdot\underline{2}\cdot\underline{1}$

(c) Harry and Sally sit together?

G - event that Harry and Sally sit together.

$$P(G) = \frac{n(G)}{n(S)} = \frac{6\cdot 2!\cdot 5!}{7!} = \boxed{\frac{2}{7}}$$

$n(G)$: $\underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{2}\cdot\underline{1}\ \underline{1}$

$T_1$ - choose 2 chairs side by side for H. and S. $n_1 = 6$

$T_2$ - arrange H. and S. in their 2 chairs $n_2 = 2!$

$T_3$ - arrange the other 5 people in the remaining chairs $n_3 = 5!$

multiply!

(d) Harry and Sally do not sit together?

$$P(G^c) = 1 - P(G) = 1 - \frac{2}{7} = \boxed{\frac{5}{7}}$$

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11. Three married couples go to the movies. If these 6 people randomly sit in 6 adjacent chairs, what is the probability that each person sits next to his or her spouse (i.e., married couples sit together)?

A - event that married couples sit together.

$P(A) = \frac{n(A)}{n(S)}$

$n(S) : 6!$

$\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}$

$n(A)$

$\overset{3}{\underline{2} \cdot \underline{1}} \cdot \overset{2}{\underline{2} \cdot \underline{1}} \cdot \overset{1}{\underline{2} \cdot \underline{1}}$

$T_1$ - decide the order of the couples. $n_1 = 3!$

$T_2$ - arrange the 1st couple in their chairs $n_2 = 2!$

$T_3$ + $T_4$ are similar $n_3 = 2!$

$n_4 = 2!$

$3!(2!)^3$

$= \frac{3!\,2!\,2!\,2!}{6!}$

$= \boxed{\frac{1}{15}}$

12. A student studying for a vocabulary test knows the meanings of 12 words from a list of 21 words. If the test contains 10 word from the study list, what is the probability that at least 6 of the words on the test are words that the student knows?

E - event that at least 6 of the words on the test are words the student knows.

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \boxed{\frac{C(12,6)C(9,4) + C(12,7)C(9,3) + C(12,8)C(9,2) + C(12,9)C(9,1) + C(12,10)}{C(21,10)}}$$

n(S): Total # of possible tests.

$C(21, 10)$

n(E): Exactly 6 or 7 or 8 or 9 or 10

$C(12,6)C(9,4) + C(12,7)C(9,3) +$ etc

13. Find the probability that in a group of 6 people that at least two of them were born in November. Assume that all months are equally likely.

E - event that at least 2 of the 6 were born in November.

$P(E) = 1 - P(E^c)$, where

$E^c$ is the event that 1 or 0 people were born in Nov.

$$P(E) = 1 - \frac{n(E^c)}{n(S)}$$

$$= 1 - \frac{6 * 11^5 + 11^6}{12^6}$$

$$= \boxed{.0831}$$

$n(S)$:

$\underline{12} \cdot \underline{12} \cdot \underline{12} \cdot \underline{12} \cdot \underline{12} \cdot \underline{12}$

$= 12^6$

$n(E^c)$: Exactly 1 in Nov or None in Nov.

$6 \left( \underline{11} \ \underline{11} \ \underline{11} \ \underline{1} \ \underline{11} \ \underline{11} \right) + \underline{11} \cdot \underline{11} \cdot \underline{11} \cdot \underline{11} \cdot \underline{11} \cdot \underline{11}$

$\boxed{6 \cdot 1 \cdot 11^5 \quad + \quad 11^6}$

$T_1$ - choose which person is the 1 person born in Nov. $n_1 = C(6, 1)$

$T_2$ - make Nov. the chosen person's birth month $n_2 = 1$

multiply!

$T_3$ - assign birth months to the 5 remaining people $n_3 = 11^5$

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