Math 166 - Week in Review #7

Section 7.5 - Conditional Probability and Independent Events

- **Conditional Probability** - the probability of an event occurring given that another event has already occurred.
- We denote "the probability of the event A given that the event B has already occurred" by $P(A|B)$.
- **Conditional Probability of an Event** - If A and B are events in an experiment and $P(B) \neq 0$, then the conditional probability that the event A will occur given that the event B has already occurred is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- **Independent Events** - Two events A and B are independent if the outcome of one does not affect the outcome of the other.
- If A and B are independent events, then $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

  Test for Independence of Two Events - Two events A and B are independent if and only if $P(A \cap B) = P(A)P(B)$.

- NOTE: To determine if two events are independent, you must compute the three probabilities $P(A \cap B)$, $P(A)$, and $P(B)$ *separately*, and then substitute these three numbers into the equation $P(A \cap B) = P(A)P(B)$. If the equality holds, then A and B are independent. If after substituting you find that $P(A \cap B) \neq P(A)P(B)$, then A and B are NOT independent (i.e., A and B are dependent and somehow affect each other).
- Independence of More Than Two Events - If $E_1, E_2, \ldots, E_n$ are independent events, then

$$P(E_1 \cap E_2 \cap \cdots \cap E_n) = P(E_1)P(E_2)\cdots P(E_n)$$

Section 7.6 - Bayes' Theorem

- There is a huge formula in the book for Bayes' Theorem, but you don't have to memorize it!! Just make sure you know the conditional probability formula $P(A|D) = \frac{P(A \cap D)}{P(D)}$, and know how to use a tree diagram (or Venn diagram in some cases) to find these probabilities.

1. A survey was conducted in which 1,000 students at Random University were asked how many hours they are currently taking. The results are given in the table below. Use the table to answer the following questions about a randomly selected student who participated in this survey.

Classification	9 or less	10 to 13	14 to 17	18 or more	Total
Freshman	10	130	140	5	285
Sophomore	15	90	55	20	180
Junior	25	105	80	35	245
Senior	50	110	85	45	290
Total	100	435	360	105	1000

(a) What is the probability that the student is a freshman if he or she is registered for 18 or more hours?

F - event the student is a freshman.
M - — — — — — sophomore
J - — — — — junior
R - — — — — senior

A - event taking 9 or less
B - — — 10-13
C - — — — 14-17
D - — — — 18 or more

$$P(F \mid D) = \frac{P(F \cap D)}{P(D)} = \frac{5/1000}{105/1000} = \boxed{\frac{5}{105}}$$

$$P(F \cap D) = \frac{n(F \cap D)}{n(S)} = \frac{5}{1000}$$ and

$$P(D) = \frac{n(D)}{n(S)} = \frac{105}{1000}$$

(b) What is the probability that a senior is registered for 14-17 hours?

$$P(C \mid R) = \frac{P(C \cap R)}{P(R)} = \frac{85/1000}{290/1000} = \boxed{\frac{85}{290}}$$

(c) What is the probability that the student is a junior given that he or she is registered for 13 hours or less?

E - event that the student is registered for 13 hours or less

$$P(J \mid E) = \frac{P(J \cap E)}{P(E)}$$

$$= \frac{(25+105)/1000}{(100+435)/1000}$$

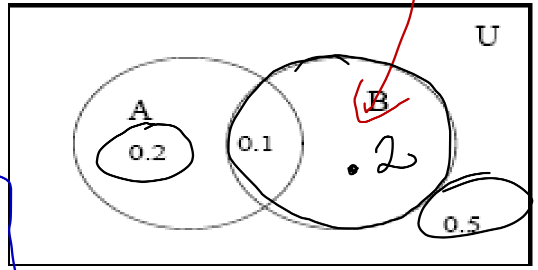
$\frac{n(E)}{n(S)}$

$$= \frac{130/1000}{535/1000}$$

$$= \boxed{\frac{130}{535}}$$

$1-.2-.1-.5$

2. Use the Venn diagram to answer the following.



(a) $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{.1}{.1+.2} = \frac{.1}{.3} = \boxed{\frac{1}{3}}$

(b) $P(B^C|A) = \frac{P(B^c\cap A)}{P(A)} = \frac{.2}{.3} = \boxed{\frac{2}{3}}$

(c) $P(A^C|B^C) = \frac{P(A^c\cap B^c)}{P(B^c)} = \frac{.5}{.2+.5} = \frac{.5}{.7} = \boxed{\frac{5}{7}}$

3. A student has two exams in one day. The probability that he passes the first exam is 0.9, and the probability that he passes the second exam is 0.85. If the probability that the student passes at least one of the two exams is 0.97, are these two events independent?

1 or more

union

$P(A) = .9$, $P(B) = .85$

$P(A \cup B) = .97$

A – event that the student passes the 1st test.

B – – — — — — — – 2nd . . .

Test for Independence

Check: $P(A \cap B) \stackrel{?}{=} P(A)P(B)$

Compute these 3 probab's separately!

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$.97 = .9 + .85 - P(A \cap B)$$

$$P(A \cap B) = .78$$

Check: $P(A \cap B) \stackrel{?}{=} P(A)P(B)$

$$.78 \stackrel{?}{=} (.9)(.85)$$

$$.78 \neq .765$$

$\boxed{\text{Not independent}}$

4. Let S be a sample space for an experiment with events E, F, and G. Use the given information to answer the following questions.

$S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$
$E = \{s_1, s_4, s_5, s_6\}$
$F = \{s_2, s_4, s_6\}$
$G = \{s_3, s_6\}$

Outcome	s_1	s_2	s_3	s_4	s_5	s_6
Probability	$\frac{1}{17}$	$\frac{2}{17}$	$\frac{5}{17}$	$\frac{3}{17}$	$\frac{4}{17}$	$\frac{2}{17}$

(a) $P(F|E) = \dfrac{P(F \cap E)}{P(E)} = \dfrac{\frac{3}{17} + \frac{2}{17}}{\frac{1}{17} + \frac{3}{17} + \frac{4}{17} + \frac{2}{17}} = \dfrac{5/17}{10/17} = \boxed{\frac{1}{2}}$

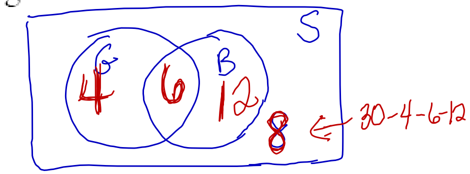
$F \cap E = \{s_4, s_6\}$

(b) $P(G|F) = \dfrac{P(G \cap F)}{P(F)} = \dfrac{\frac{2}{17}}{\frac{2}{17} + \frac{3}{17} + \frac{2}{17}} = \dfrac{2/17}{7/17} = \boxed{\frac{2}{7}}$

$G \cap F = \{s_6\}$

5. A 7th grade class was selected and the following information was collected about the 30 students.

4 students have only glasses.
12 students have only braces.
6 students have glasses and braces.



Determine whether the event that the student has glasses and the event that the student has braces are independent. Justify your answer.

G - event the student has glasses
B - . - - - - - - braces.

Test for Independence

Check: $P(G \cap B) \stackrel{?}{=} P(G)P(B)$

$$P(G \cap B) = \frac{n(G \cap B)}{n(S)} = \frac{6}{30}$$

$$P(G) = \frac{n(G)}{n(S)} = \frac{10}{30}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{18}{30}$$

reduce

$$\frac{6}{30} \stackrel{?}{=} \left(\frac{10}{30}\right)\left(\frac{18}{30}\right)$$

multiply and reduce

$$\frac{1}{5} \stackrel{?}{=} \frac{1}{5} \checkmark$$

These are independent events.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

6. A building on campus has three vending machines: two Coke machines and a snack machine. Based on the model of the machines, the first Coke machine has a 12% chance of breaking down in a particular week, and the second Coke machine has a 4% chance of breaking down in a particular week. The snack machine has a 10% chance of breaking down in a particular week. Assuming independence, find the probability that exactly one machine breaks down.

C_1 - event that the 1st Coke machine breaks down.

C_2 - - - - 2nd - - - - - -

N - - - - snack machine - - -

$$P(C_1 \cap C_2^c \cap N^c) \text{ or } P(C_1^c \cap C_2 \cap N^c) \text{ or } P(C_1^c \cap C_2^c \cap N)$$

$$= P(C_1)P(C_2^c)P(N^c) + P(C_1^c)P(C_2)P(N^c) + P(C_1^c)P(C_2^c)P(N)$$

$$= (.12)(.96)(.9) + (.88)(.04)(.9) + (.88)(.96)(.1)$$

$$= \boxed{.21984}$$

7. Let A, B, and C be three independent events of an experiment with $P(A) = 0.4$, $P(B) = 0.75$, and $P(C) = 0.3$. Calculate each of the following.

(a) $P(A \cap B^C) = P(A)P(B^c) = (.4)(1 - .75) = \boxed{.1}$

(b) $P(A^C \cup C) = P(A^c) + P(C) - P(A^c \cap C)$

$= (1-.4) + .3 - (1-.4)(.3)$

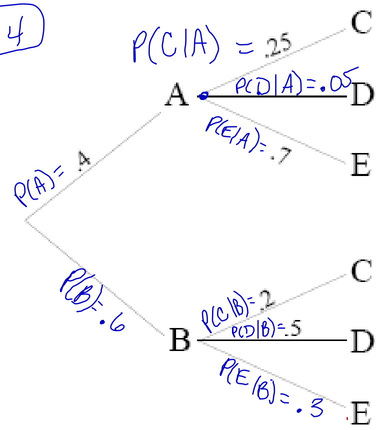
$= .6 + .3 - (.6)(.3)$

$= \boxed{.72}$

(c) $P(B|C) = \dfrac{P(B \cap C)}{P(C)} = \dfrac{(.75)\cancel{(.3)}}{\cancel{(.3)}} = \boxed{.75} = P(B)$

(because of indep.)

8. Fill in the missing probabilities on the tree diagram and then answer the following questions.



(a) $P(B^C) = 1 - P(B) = 1 - .6 = \boxed{.4}$

(b) $P(A \cap D) = (.4)(.05) = \boxed{.02}$

(c) $P(E|A) = \boxed{.7}$ (on the tree)

(d) $P(C) = (.4)(.25) + (.6)(.2) = \boxed{.22}$

(e) $P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$= .6 + .22 - (.6)(.2)$

$= \boxed{.7}$

(f) $P(B|E) = \dfrac{P(B \cap E)}{P(E)} = \dfrac{(.6)(.3)}{(.4)(.7) + (.6)(.3)}$

$= \boxed{\dfrac{9}{23}}$

(g) Are the events B and C independent? Justify your answer.

Test for Indep

Check $P(B\cap C) \overset{?}{=} P(B)P(C)$

$P(B\cap C) = (.6)(.2) = .12$

$P(B) = .6$

$P(C) = .22$ (from (d))

$.12 \overset{?}{=} (.6)(.22)$

$.12 \neq .132$

$\boxed{\text{Not independent}}$

(h) Are the events B and C mutually exclusive? Justify your answer.

means $B\cap C = \emptyset$ (impossible event)

$\hookrightarrow P(B\cap C) = 0$

Check is $P(B\cap C) = 0$?

$P(B\cap C) = (.6)(.2) = .12$

This means there is a chance B and C will occur at the same time, so they are not mutually exclusive.

(i) Are the events A and D independent? Justify your answer.

Check

$P(A\cap D) \overset{?}{=} P(A)P(D)$

$(.4)(.05) \overset{?}{=} (.4)[(.4)(.05)+(.6)(.5)]$

$.02 \overset{?}{=} (.4)(.32)$

$.02 \neq .128$

multiply thru branches on tree

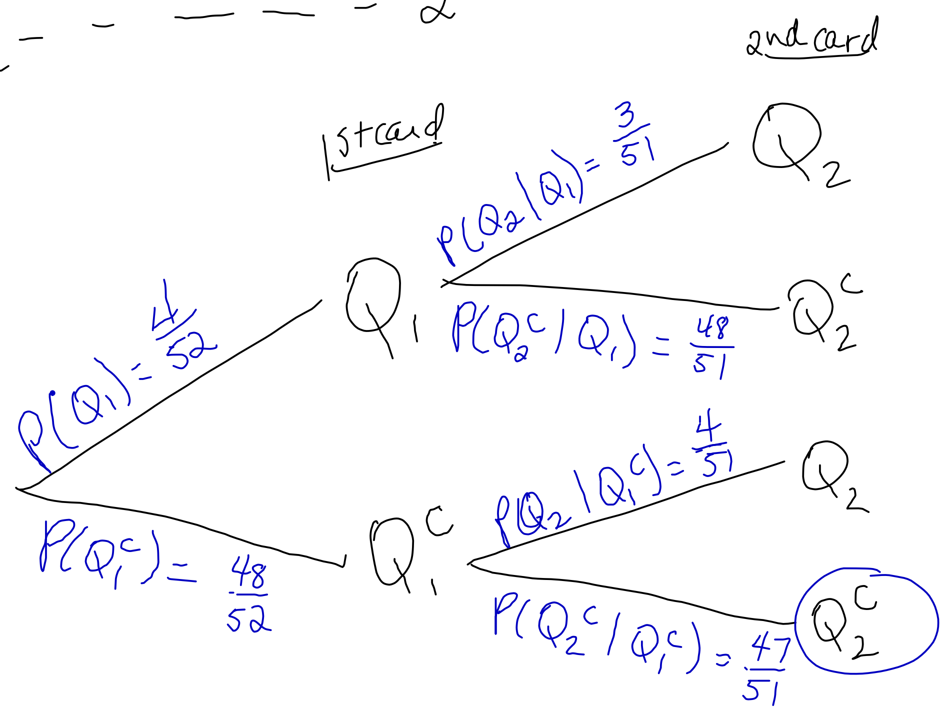
multiply thru branches and add

$\boxed{\text{not independent}}$

9. Two cards are drawn at random (without replacement) from a well-shuffled deck of 52 playing cards. What is the probability that the first card was queen given that the second card drawn was not a queen?

Q_1 - event that the 1st card is a queen

Q_2 - - - - - - 2nd - - - - -



$$P(Q_1 | Q_2^c) = \frac{P(Q_1 \cap Q_2^c)}{P(Q_2^c)} = \frac{\left(\frac{4}{52}\right)\left(\frac{48}{51}\right)}{\left(\frac{4}{52}\right)\left(\frac{48}{51}\right) + \left(\frac{48}{52}\right)\left(\frac{47}{51}\right)}$$

$$= \boxed{\frac{4}{51}}$$

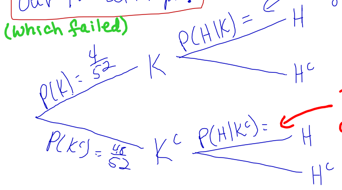
10. Two cards are drawn at random (without replacement) from a well-shuffled deck of 52 playing cards. What is the probability that the first card was a king given that the second card drawn was a heart?

K - event that the 1st card is a king

H - event that the 2^{nd} card is a heart.

Our 1st attempt:

(which failed)



This probab depends on which king we picked!

This probab. depends on whether the 1st card was a heart or not! (K^c doesn't tell us enough.)

2nd attempt

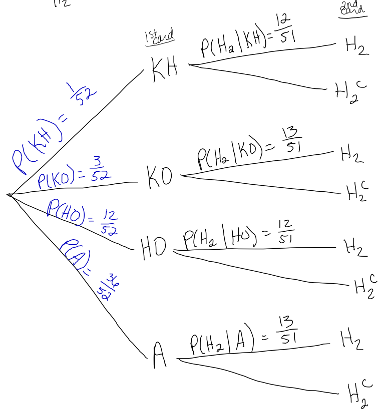
→ KH - event that the 1st card is the king of hearts

KO - - - - - - - 1st card is some other king.

HO - event that the 1st card is some other heart (not king of hearts)

A - - - - - - - - - - any other card.

H_2 - event that the 2^{nd} card is a heart.



$$P\left(\text{1st card is king} \,\middle|\, \substack{\text{2nd was}\\\text{a heart}}\right) = P\left(\substack{\text{1st is}\\\text{a king}} \,\middle|\, H_2\right)$$

$$= \frac{P\left(\substack{\text{1st is a}\\\text{king}} \cap H_2\right)}{P(H_2)}$$

$$= \frac{\left(\frac{1}{52}\right)\left(\frac{12}{51}\right) + \left(\frac{3}{52}\right)\left(\frac{13}{51}\right)}{\left(\frac{1}{52}\right)\left(\frac{12}{51}\right) + \left(\frac{3}{52}\right)\left(\frac{13}{51}\right) + \left(\frac{12}{52}\right)\left(\frac{12}{51}\right) + \left(\frac{36}{52}\right)\left(\frac{13}{51}\right)}$$

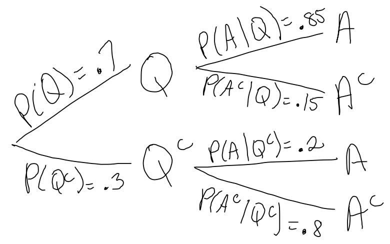
$$= \boxed{\frac{1}{13}}$$

11. The personnel manager at a certain company claims that she approves qualified applicants for a certain job 85% of the time; she rejects an unqualified person 80% of the time. If 70% of all applicants for this job are qualified, find each of the following. (pg. 380 of *Finite Mathematics* by Lial, Greenwell, and Ritchey)

(a) Draw a tree diagram (with probabilities and notation on all branches) representing the above information.

A- event that she approves the applicant

Q- event that the applicant is qualified.



(b) What is the probability that an applicant is approved?

$$P(A) = (.7)(.85) + (.3)(.2)$$

$$= \boxed{\frac{131}{200}} = .655$$

(c) What is the probability that an applicant is qualified if he or she was approved by the personnel manager?

← given

$$P(Q \mid A) = \frac{P(Q \cap A)}{P(A)} = \frac{(.7)(.85)}{131/200} \leftarrow .655$$

$$= \boxed{\frac{119}{131}}$$

(d) What is the probability that an applicant who is unqualifed is approved for the job?

$P(A \mid Q^c) = \boxed{.2}$ (on the tree)

(e) What is the probability that an approved applicant was unqualified?

$$P(Q^c \mid A) = \frac{P(Q^c \cap A)}{P(A)} = \frac{(.3)(.2)}{.655}$$

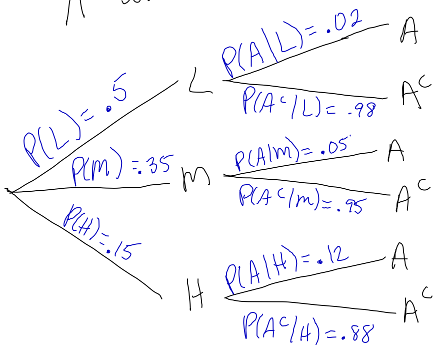
$$= \boxed{\frac{12}{131}}$$

12. An auto insurance company classifies its drivers as low risk, medium risk, or high risk. The table shows the percentage of drivers in these classifications and the probability that a driver in that classification will have an accident during the next year. A driver is selected at random. (This problem is courtesy of Joe Kahlig.)

Classification	Drivers (%)	Accident (%)
low	50	2
medium	35	5
high	15	12

(a) What is the probability that the driver will have an accident in the next year?

L - event that the driver is low risk
M - - - - - - medium risk
H - - - - - - high risk
A - event that the driver has an accident.



$$P(A) = (.5)(.02) + (.35)(.05) + (.15)(.12)$$
$$= \boxed{.0455}$$

(b) What is the probabilty that the driver is rated as a medium risk if he or she has an acident in the next year?

$$P(M|A) = \frac{P(M \cap A)}{P(A)} = \frac{(.35)(.05)}{.0455}$$
$$= \boxed{.3846} = \frac{5}{13}$$

(c) What is the proability that the driver is classified as a high risk but (and) does not have an accident in the next year?

$$P(H \cap A^c) = (.15)(.88) = \boxed{.132}$$

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