

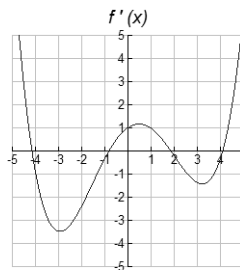
Math 142 - Exam 2 Review

NOTE: Exam 2 covers sections 3.4, 3.5, 3.7, 4.1-4.4, 4.7, 5.1, and 5.2. This review is intended to highlight the material covered on Exam 2 but should not be used as your sole source of practice. Also refer to your instructor's lecture notes, previous week-in-reviews, suggested homework, supplemental homework, and the online homework as additional sources for review and exam preparation.

1. Acme, Inc. has determined the price-demand equation for its model airplane to be $25p + x = 1000$.
 - (a) Find the elasticity of demand, $E(p)$.
 - (b) Find and interpret $E(13)$.
 - (c) If the current price of \$13 per model airplane were increased by 5%, what would be the approximate change in demand?
 - (d) Find and interpret $E(30)$.
 - (e) If the current price is \$30 per model airplane, should Acme increase or decrease this price to produce an increase in revenue?
 - (f) Use elasticity of demand to find the price that maximizes revenue.
2. Let $f(x) = e^x(x+2)^2$. Using calculus techniques, find the intervals on which $f(x)$ is increasing, the intervals on which it is decreasing, and the locations of local extrema.
3. Let $f(x) = 4x - x \ln x$. Using calculus techniques, find the intervals on which $f(x)$ is concave upward, the intervals on which it is concave downward, and the locations of any inflection points.
4. Suppose that the number of students enrolled in a certain university can be modeled by $E(t) = 4t^3 - 87t^2 + 189t + 18,982$ students, where t is the number of years since September 1990, $1 \leq t \leq 17$.
 - (a) Find the relative rate of change of enrollment in September 1995.
 - (b) Find the percentage rate of change of enrollment in September 2003.
5. Let $f(x) = (x^2 - 7)(x^2 - 1)$.
 - (a) Analyze $f(x)$ by finding its domain, x and y -intercepts, and locations of any asymptotes.
 - (b) Find the intervals on which $f(x)$ is increasing, the intervals on which it is decreasing, and the coordinates of all local extrema.
 - (c) Find the intervals on which $f(x)$ is concave upward, the intervals on which it is concave downward, and the coordinates of any inflection points.
 - (d) Sketch a graph of $f(x)$.
6. Suppose that f is a continuous function that satisfies the following.
 - $f(-7) = 0$, $f(7) = 0$, and $f(0) = 3$
 - $f'(-3) = f'(-1) = f'(5) = 0$ and $f'(3)$ does not exist
 - $f'(x) > 0$ on $(-\infty, -3)$ and $(-1, 3)$
 - $f'(x) < 0$ on $(-3, -1)$ and $(3, \infty)$
 - $f''(x) > 0$ on $(-2, 3)$ and $(3, 5)$
 - $f''(x) < 0$ on $(-\infty, -2)$ and $(5, \infty)$

Sketch a possible graph of $f(x)$.
7. At the end of 2002, Bob invested \$2,000 in an account paying 5.4% per year compounded continuously.

- (a) Find the average rate of change of the account's value from the end of 2005 to the end of 2008.
- (b) How quickly is the account's value growing at the end of 2007?
8. Find the first derivative of $g(x) = \frac{4x}{e^x + 3}$.
9. Use the limit definition to find the instantaneous rate of change of $f(x) = 3 - 4x^2$ at $x = 5$.
10. Find the derivative of each of the following.
- (a) $h(x) = 7 \cdot 3^{\ln 7x^2}$
- (b) $j(x) = \log_8 \frac{3x}{x-9}$
11. Acme Pool Supplies has determined the price-demand function for a 50 lb bucket of its chlorine tabs to be $35x + 100p = 17,500$, where x is the number of buckets that can be sold at a unit price of p dollars.
- (a) Approximate the revenue earned from the sale of the 31st bucket of chlorine tabs.
- (b) Find the revenue and marginal revenue from the sale of 55 buckets of chlorine tabs.
- (c) Use your answers in (b) to approximate the revenue earned from the sale of 58 buckets of chlorine tabs.
- (d) Acme Pool Supplies has a fixed cost of \$7,400 and a production cost of \$34 per bucket. Find a model for average cost.
- (e) Find the marginal average profit of producing and selling 42 buckets of chlorine tabs.
12. The following graph represents the first derivative of a function f . Approximate your answers to one decimal place.



- (a) Where is $f(x)$ increasing? Decreasing?
- (b) Where does $f(x)$ have local extrema?
- (c) Where is $f(x)$ concave up? Concave down?
- (d) Where does $f(x)$ have inflection points?
13. Find the derivative of $f(x) = (4x - 3)^7 \ln(2x - 7)$.
14. Find the derivative of $g(x) = \left(\frac{5x}{4x-6}\right)^3$.
15. A rational function f , which has domain $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$, has first derivative $f'(x) = \frac{16x}{(x^2 - 4)^2}$.
- (a) Find the intervals where f is increasing and decreasing. Identify any local extrema.
- (b) Find $f''(x)$ and simplify.
- (c) Find the intervals where f is concave upward and concave downward. Identify any inflection points.