Week in Review #8

Section 7.5: Conditional Probability and Independent Events.
Section 7.6: Bayes' Theorem.

- TO CONVERT CONDITIONAL PROBABILITY TO REGULAR PROBABILITY.
  - $P(B|A) = \dfrac{P(B \cap A)}{P(A)}$
  - probability of the event B occurring knowing that the event A has already occurred.
- A and B are independent events if and only if $P(A \cap B) = P(A)P(B)$

Test for Independence:

If $P(A \cap B) = P(A)P(B)$, the A and B are indpendent events.

1. A clothing company selected 1000 persons at random and surveyed them to determine a relationship between age of purchaser and annual purchases of jeans. The results are given in the table. A person from the survey is selected at random.

(a) What is the probability that the person is under 12 if they purchases 3 or more pairs of jeans annually.

P(under 12 | purchase 3 or more) = $\frac{10}{120}$

Jeans Purchased Annually

Age	0	1	2	3 or More	Totals
Under 12	60	70	30	10	170
12-18	30	100	100	60	290
19-25	70	110	120	30	330
Over 25	100	50	40	20	210
Totals	260	330	290	120	1000

(b) What is the probability that the person purchases 2 pairs of jeans annually if we know they are older than 25.

P(purchase 2 pairs | older than 25) = $\boxed{40/210}$

(c) What is the probability that the person is younger than 19 given they purchase 0 or 1 pair of jeans annually.

P(younger than 19 | purchase 0 or 1 pair)

$= \frac{60+70+30+100}{260+330}$

$= \boxed{\frac{260}{590}}$

2. S is the sample space with events: E, F, and G. Use this information to answer these questions.

$S = \{s_1, s_2, s_3, s_4, s_5, s_6,\}$
$E = \{s_1, s_2, s_5, s_6\}$
$F = \{s_2, s_4, s_5\}$
$G = \{s_3, s_5\}$

outcome	s_1	s_2	s_3	s_4	s_5	s_6
prob.	$\frac{2}{29}$	$\frac{7}{29}$	$\frac{1}{29}$	$\frac{11}{29}$	$\frac{6}{29}$	$\frac{2}{29}$

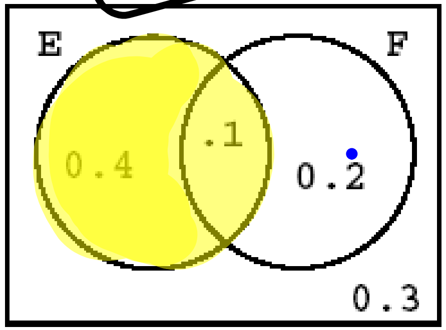
$F \cap E = \{s_2, s_5\}$

(a) $P(F|E) = \dfrac{P(F \cap E)}{P(E)} = \dfrac{\frac{7}{29} + \frac{6}{29}}{\frac{2}{29} + \frac{7}{29} + \frac{6}{29} + \frac{2}{29}} = \dfrac{\frac{13}{29}}{\frac{17}{29}} = \boxed{\frac{13}{17}}$

$G \cap F = \{s_5\}$

(b) $P(G|F) = \dfrac{P(G \cap F)}{P(F)} = \dfrac{6/29}{\frac{7}{29} + \frac{11}{29} + \frac{6}{9}} = \dfrac{\frac{6}{29}}{\frac{24}{29}} = \boxed{\frac{6}{24}}$

3. Use the Venn Diagram to answer the following.

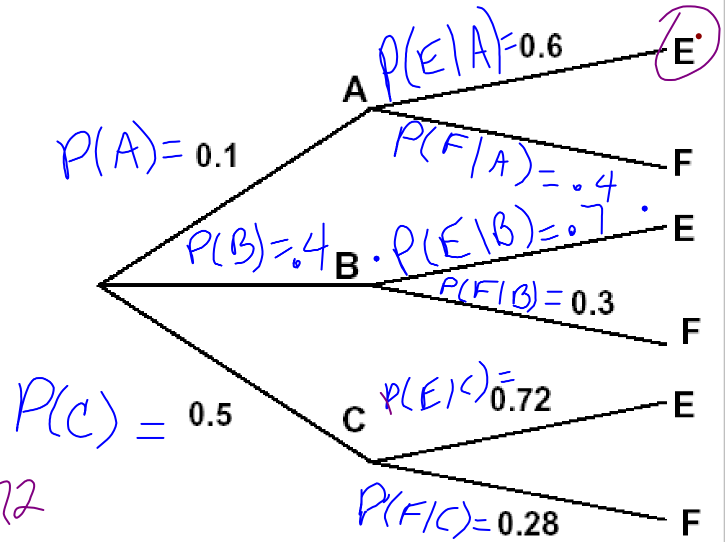


(a) $P(E|F) = \dfrac{P(E\cap F)}{P(F)} = \dfrac{.1}{.1+.2} = \dfrac{.1}{.3} = \boxed{\dfrac{1}{3}}$

(b) $P(F^C|E) = \dfrac{P(F^c\cap E)}{P(E)} = \dfrac{.4}{.4+.1} = \dfrac{.4}{.5} = \dfrac{4}{5}$

4. Fill in the missing values of the tree and then answer the following.

(a) $P(A^C) = 1 - P(A) = 1 - .1 = \boxed{.9}$



(b) $P(B \cap E) = (.4)(.7) = \boxed{.28}$

(c) $P(E|C) = .72$ (on tree)

↳ $\dfrac{P(E\cap C)}{P(C)} = \dfrac{(\cancel{.5})(.72)}{(\cancel{.5})} = .72$

(d) $P(E) = (.1)(.6) + (.4)(.7) + (.5)(.72)$

$= \boxed{.7}$

(e) $P(A \cup F) = P(A) + P(F) - P(A \cap F)$

$= .1 + ((.1)(.4) + (.4)(.3) + (.5)(.28)) - (.1)(.4)$

$= \boxed{.36}$

(f) $P(C|E) = \dfrac{P(C \cap E)}{P(E)} = \dfrac{(.5)(.72)}{.7} = \boxed{\dfrac{18}{35}}$

answer to (d)

(g) Are the events B and E independent? Justify your answer.

Test for independence

$P(B \cap E) \stackrel{?}{=} P(B)P(E)$

$.28 \stackrel{?}{=} (.4)(.7)$

$P(B \cap E) = (.4)(.7) = .28$

$.28 = .28$ ✓

Yes, B and E are Independent.

(h) Are the events A and E independent? Justify your answer.

Test for Independence

$P(A \cap E) \stackrel{?}{=} P(A)P(E)$

$.06 \stackrel{?}{=} (.1)(.7)$

part (d)

$.06 \neq .07$

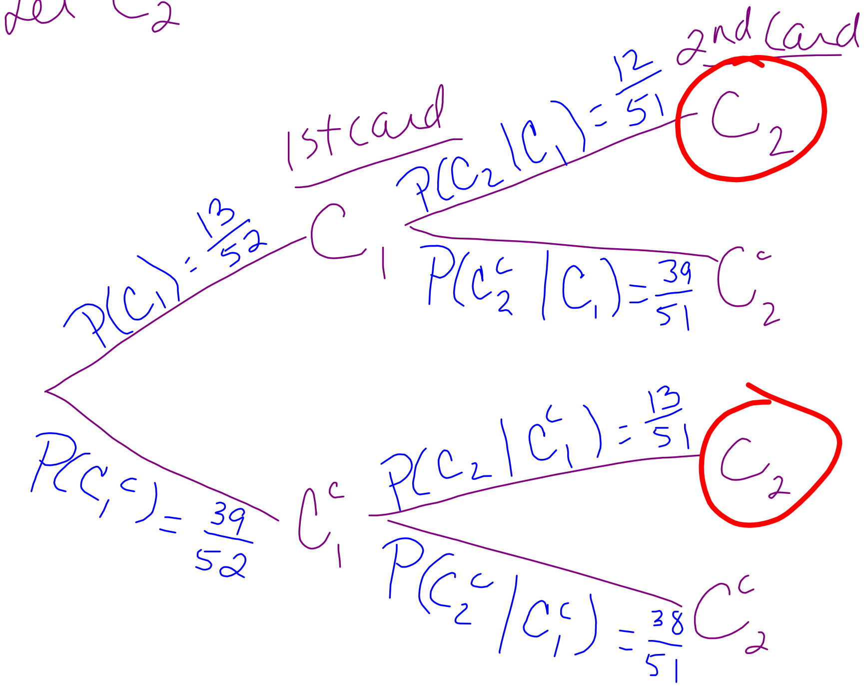
$P(A \cap E) = (.1)(.6) = .06$

Not equal → A and E are NOT independent

5. Two cards are drawn from a standard deck of cards without replacement. What is the probability that the first card is a club if the second card is a club?

Let C_1 be the ~~the~~ event that the 1st card is a club.

Let C_2 . . . . 2nd . . . . .

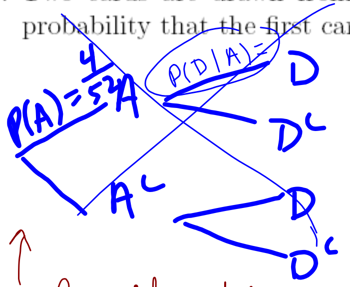


$P(C_1|C_2)$

$$P(C_1|C_2) = \frac{P(C_1 \cap C_2)}{P(C_2)} = \frac{\left(\frac{13}{52}\right)\left(\frac{12}{51}\right)}{\left(\frac{13}{52}\right)\left(\frac{12}{51}\right) + \left(\frac{39}{52}\right)\left(\frac{13}{51}\right)}$$

$$= \boxed{\frac{4}{17}}$$

6. Two cards are drawn from a standard deck of cards without replacement. What is the probability that the first card is an Ace if the second card is a diamond?



This first idea does not work because to find $P(D|A)$, we have to know whether the ace that we drew for the 1st card was a diamond or not.

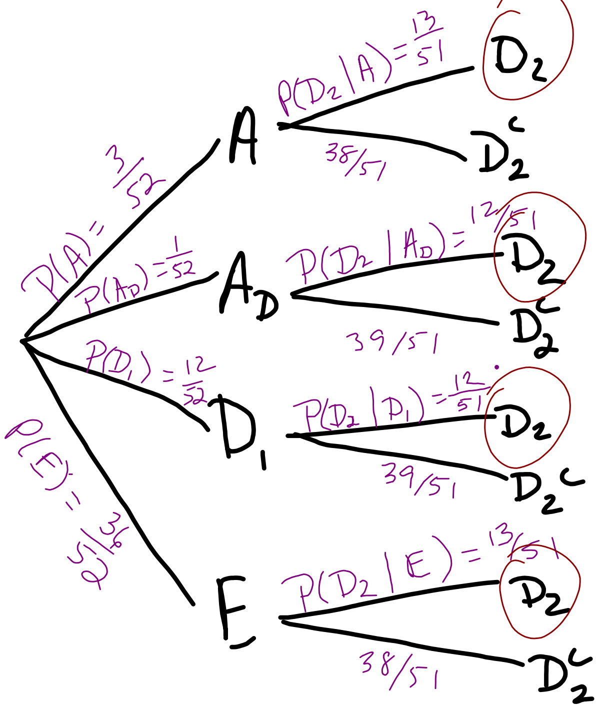
Let A be the event the 1st card is the ace of hearts, clubs, or spades.

Let A_D be the - - - - - - is the ace of Diamonds.

Let D_1 be the event the 1st card is any diamond other than the ace of diamonds.

Let E be the event the card is not an ace and not a diamond.

Let D_2 be the event the 2nd card is a diamond.



$$P(\text{1st card is ace} \mid D_2)$$

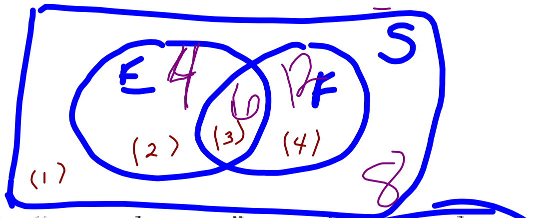
$$= \frac{P(\text{1st is ace} \cap D_2)}{P(D_2)}$$

$$= \frac{\left(\frac{3}{52}\right)\left(\frac{13}{51}\right) + \left(\frac{1}{52}\right)\left(\frac{12}{51}\right)}{\left(\frac{3}{52}\right)\left(\frac{13}{51}\right) + \left(\frac{1}{52}\right)\left(\frac{12}{51}\right) + \left(\frac{12}{52}\right)\left(\frac{12}{51}\right) + \left(\frac{36}{52}\right)\left(\frac{13}{51}\right)}$$

$$= \boxed{\frac{1}{13}}$$

7. A 7th grade class was selected and the following information was collected on the 30 students.

Four of the students only wear glasses.
Twelve of the students only wear braces.
Six of the students wear glasses and braces.
The rest didn't wear glasses and didn't wear braces.



Determine whether the events E="wear glasses" and F="wear braces" are independent or not. Justify your answer

Test for Independence

$$P(E\cap F) \overset{?}{=} P(E)P(F)$$

$$\frac{6}{30} \overset{?}{=} \left(\frac{10}{30}\right)\left(\frac{18}{30}\right)$$

$$\frac{1}{5} \overset{?}{=} \frac{1}{5} \checkmark$$

$$P(E\cap F) = \frac{n(E\cap F)}{n(S)} = \frac{6}{30}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4+6}{30}$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{6+12}{30}$$

Yes, E and F are Independent.

8. A building on campus has three vending machines: two coke machines and a snack machine. Based on the model of the machines, the first coke machine has a 12% chance of breaking down in a particular week and the second coke machine has a 4% chance of breaking down in a particular week. The snack machine has a 10% chance of breaking down in a particular week. Assuming independence, find the probability that exactly one machine breaks down.

↑ This means you can multiply to find probabilities of intersections.

P(Exactly 1 machine breaks down:)

Let C_1 be the event the 1st coke machine breaks down.
Let C_2 . . . 2nd — — — — . .
Let D - — - - snack machine breaks down.

$P(C_1 \cap C_2^c \cap D^c)$ or $P(C_1^c \cap C_2 \cap D^c)$ or $P(C_1^c \cap C_2^c \cap D)$

$P(C_1)P(C_2^c)P(D^c) + P(C_1^c)P(C_2)P(D^c) + P(C_1^c)P(C_2^c)P(D)$

$= (.12)(.96)(.9) + (.88)(.04)(.9) + (.88)(.96)(.1)$

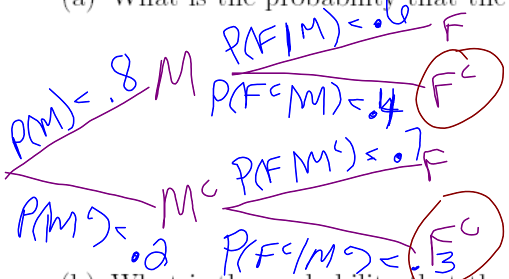
$= \boxed{.21984}$

9. Eighty percent of the workers in a company are men with the rest being women. Of the men, 40% work part time and of the women 70% work full time. A worker is selected at random from the company workforce.

Let M be the event the worker is male

Let F be the event the worker is full time.

(a) What is the probability that the worker works part time?



$$P(F^c) = (.8)(.4) + (.2)(.3)$$

$$= \boxed{.38}$$

(b) What is the probability that the worker is male and works full time?

$$P(M \cap F) = (.8)(.6) = \boxed{.48}$$

(c) What is the probability that ~~the~~ a part time worker is female?

$$P(M^c | F^c) = \frac{P(M^c \cap F^c)}{P(F^c)} = \frac{(.2)(.3)}{.38}$$

given (what we know)

from (a)

$$= \boxed{\frac{3}{19}}$$

10. The following information was compiled regarding married couples living in single-family dwellings. It was found that in 30% of these households, both the husband and the wife worked, and that 10% of these couples were renting. In 50% of the households, only the husband worked, and 20% of these couples were renting. In 15% of the households, only the wife worked, and 70% of these couples were renting. In the households where neither worked, 95% were renting. A couple from this group is selected at random.

(a) Find the probability that this couple is renting.

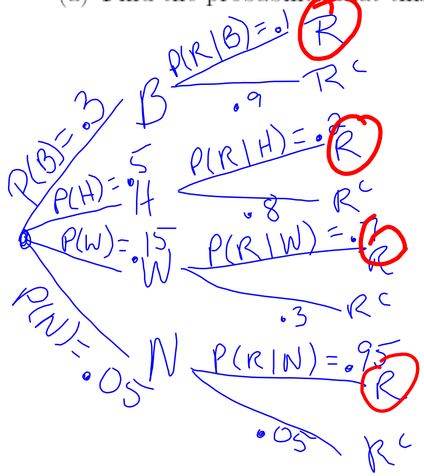
B - both are working

H - only husband works

W - only wife works

N - neither works

R - the couple rents.



$P(R) = (.3)(.1)+(.5)(.2)+(.15)(.7)+(.05)(.95)$

$= \boxed{.2825}$

(b) If the couple is renting, find the probability that only the wife is working.

$$P(W|R) = \frac{P(W \cap R)}{P(R)} = \frac{(.15)(.7)}{.2825}$$

part (a)

$$= \boxed{\frac{42}{113}}$$

11. An auto insurance company classifies its drivers as good risk, medium risk or bad risks. The table shows the percent of the drivers in these classifications and the probability that a driver in that classification will have an accident during the next year. A driver is selected bar random.

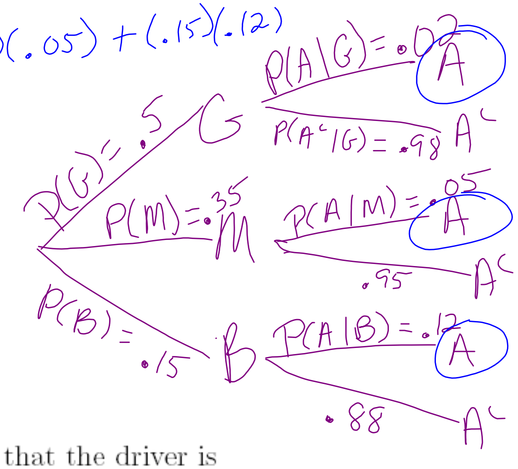
Classification	drivers(%)	Accident(%)
good	50	2
medium	35	5
bad	15	12

Let G be be event the driver is good
Let M . . . . medium
Let B ——— . . . bad
Let A . . . . . . has an accident in the next year.

(a) What is the probability that the driver will have an accident in the next year?

$P(A) = (.5)(.02) + (.35)(.05) + (.15)(.12)$

$= \boxed{.0455}$



(b) What is the probability that the driver is rated as a medium risk if they had an accident in the next year?

$$P(M|A) = \frac{P(M \cap A)}{P(A)} = \frac{(.35)(.05)}{.0455} = \boxed{\frac{5}{13}}$$

↑ part (a)

(c) What is the probability that the driver is rated as a bad risk and they did not have an accident in the next year?

$$P(B \cap A^c) = (.15)(.88) = \boxed{.132}$$

Section 8.1: Distribution of Random variables.

- A random variable is a rule that assigns a number to each outcome of an experiment.
  - finite discrete: takes on a finite number of values(skips values). you can make a list
  - infinite discrete: takes on an infinite number of values(skips values).
  - continuous: takes on any value in an interval.
- probability distribution
- a histogram is a probability distribution represented by a graph(chart).

12. Classify the random variable as finite discrete, infinite discrete or continuous and give the values of the random variable.

(a) You toss a coin and X = the number of tosses until the first head occurs.

outcome	H	TH	TTH	TTTH	
Possible Values of X	1	2	3	4	. . .

Infinite Discrete.

(b) A football team plays twelve games in a regular season and X = the number of games the team wins.

Possible values of X = 0, 1, 2, 3, ..., 12 finite discrete

(c) X = the temperature of a fish tank in your house.

Continuous (we cannot make a list of possible values of X without skipping possibilities)

Possible values of X may be $32 \le X \le 212$ (any reasonable interval is ok.)

(d) X = the number of minutes that you slept in your math class on a particular class day.

Possible values of X. $0 \le X \le 50$ (if you are in a MWF class)

Continuous

$0 \le X \le 75$ (if you are in a TR class)

13. A box has 2 green, 2 red and 5 yellow balls. A sample of 6 balls ~~are~~ is drawn without replacing the balls drawn. Let the random variable X be the number of yellow balls drawn.

these are not possible values of X because we are getting a sample of 6 balls and there are only 4 non-yellow balls available.

(a) Give the range of values that the random variable X may assume.

$X = \cancel{0}, \cancel{1}, \boxed{2, 3, 4, 5}$

(b) Find the probability distribution of X.

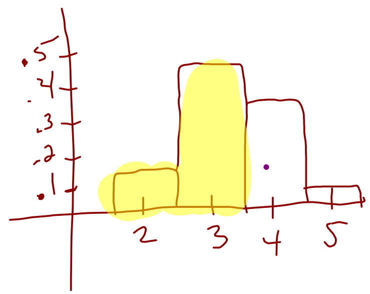
Value of X	2	3	4	5
$P(X=x)$	$\frac{C(5,2)C(4,4)}{C(9,6)}$	$\frac{C(5,3)C(4,3)}{C(9,6)}$	$\frac{C(5,4)C(4,2)}{C(9,6)}$	$\frac{C(5,5)C(4,1)}{C(9,6)}$

$P(X=2) = \frac{n(X=2)}{n(S)}$

$P(X=3)$ $P(X=4)$ $P(X=5)$

(c) Draw the histogram of X.

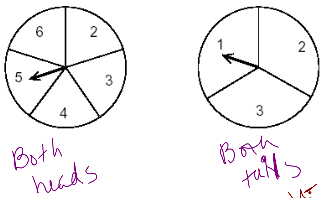
Value of X	2	3	4	5
$P(X=x)$	.119	.476	.357	.048



(d) $P(X = 4) =$ $.357$

(e) $P(X < 4) =$ $.119 + .476 = \boxed{.595}$

14. You pay $2.00 to play a game. The game consists of flipping two coins. If both coins are heads, then you get to spin the spinner on the left for the dollar amount that you win. If both coins are tails, then you get to spin the spinner on the right for the dollar amount that you win. All other results for the coins means that you lose the game. Assume that the sections in each respective spinner are equal. Let the random variable X be your net winnings when you play the game one time. What is the probability distribution for this game.



X = net winnings

X = winnings − cost to play

Net Winnings

2−2 = $0

3−2 = $1

4−2 = $2

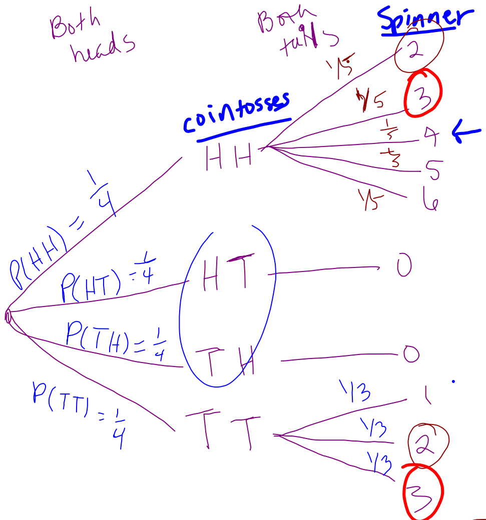
5−2 = $3

6−2 = $4

1−2 = −$1

0−2 = −$2

possible values of X.



Values of X	−2	−1	0	1	2	3	4
P(X=x)	$\boxed{\frac{1}{2}}$	$(\frac{1}{4})(\frac{1}{3})$ $= \boxed{\frac{1}{12}}$	$(\frac{1}{4})(\frac{1}{5})+(\frac{1}{4})(\frac{1}{3}) = 2/15$		$(\frac{1}{4})(\frac{1}{5})$		

$\frac{1}{4}+\frac{1}{4}$

P(X = −2)

P(X = −1)

$(\frac{1}{4})(\frac{1}{5})$

$(\frac{1}{4})(\frac{1}{5})$

$(\frac{1}{4})(\frac{1}{5})+(\frac{1}{4})(\frac{1}{3})$

final Answer:

Value of X	−2	−1	0	1	2	3	4
P(X=x)	$\frac{1}{2}$	$\frac{1}{12}$	$\frac{2}{15}$	$\frac{2}{15}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{1}{20}$