Math 166 - Week in Review #8

Section 7.5 - Conditional Probability and Independent Events

- **Conditional Probability** - the probability of an event occurring given that another event has already occurred.
- We denote "the probability of the event A given that the event B has already occurred" by $P(A|B)$.
- **Conditional Probability of an Event** - If A and B are events in an experiment and $P(B) \neq 0$, then the conditional probability that the event A will occur given that the event B has already occurred is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- **Independent Events** - Two events A and B are independent if the outcome of one does not affect the outcome of the other.
- If A and B are independent events, then $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

  Test for Independence of Two Events - Two events A and B are independent if and only if $P(A \cap B) = P(A)P(B)$.
- NOTE: To determine if two events are independent, you must compute the three probabilities $P(A \cap B)$, $P(A)$, and $P(B)$ *separately*, and then substitute these three numbers into the equation $P(A \cap B) = P(A)P(B)$. If the equality holds, then A and B are independent. If after substituting you find that $P(A \cap B) \neq P(A)P(B)$, then A and B are NOT independent (i.e., A and B are dependent and somehow affect each other).
- Independence of More Than Two Events - If $E_1, E_2, \ldots, E_n$ are independent events, then

$$P(E_1 \cap E_2 \cap \cdots \cap E_n) = P(E_1)P(E_2)\cdots P(E_n)$$

Section 7.6 - Bayes' Theorem

- There is a huge formula in the book for Bayes' Theorem, but you don't have to memorize it!! Just make sure you know the conditional probability formula $P(A|D) = \frac{P(A \cap D)}{P(D)}$, and know how to use a tree diagram (or Venn diagram in some cases) to find these probabilities.

Section 8.1 - Distributions of Random Variables

- A **random variable** is a rule that assigns a number to each oucome of a chance experiment.
- **Finite Discrete Random Variable** - A random variable is called finite discrete if it assumes only finitely many values. (You can write ALL possible values of the random variable in a list that stops.)
- **Infinite Discrete Random Variable** - A random variable is said to be infinite discrete if it takes on infinitely many values, which may be arranged in a sequence. (You can write all the possible values of the random variable in a list of numbers that has a pattern and goes on forever.)
- **Continuous Random Variable** - A random variable is called continuous if the values it may assume comprise an interval of real numbers. (For a continuous random variable, it is not possible to write an all-inclusive list of values.)
- **Histogram** - a graphical representation of a probability distribution of a random variable.

  Steps for Making a Histogram
  1. Locate the values of the random variable on the number line.
  2. Centered above each value of the random variable, make a rectangle with width 1 and height equal to the probability associated with that value of the random variable.

1. A survey was conducted in which 1,000 students at Random University were asked how many hours they are currently taking. The results are given in the table below. Use the table to answer the following questions about a randomly selected student who participated in this survey.

	Classification	9 or less (A)	10 to 13 (B)	14 to 17 (C)	18 or more (D)	Total
F =	Freshman	10	130	140	5	285
S =	Sophomore	15	90	55	20	180
J =	Junior	25	105	80	35	245
R =	Senior	50	110	85	45	290
	Total	100	435	360	105	1000

(a) What is the probability that the student is a freshman if he or she is registered for 18 or more hours?

$$P(F \mid D) = \frac{P(F\cap D)}{P(D)} = \frac{5/1000}{105/1000} = \boxed{\frac{5}{105}}$$

given info

$$P(F\cap D) = \frac{n(F\cap D)}{n(S)} = \frac{5}{1000}$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{105}{1000}$$

(b) What is the probability that a senior is registered for 14-17 hours?

$$P(C \mid R) = \frac{85}{290}$$

given

(can also use formula)

1

(c) What is the probability that the student is a junior given that he or she is registered for 13 hours or less?

$$P(J \mid E) = \frac{P(J\cap E)}{P(E)} = \frac{130/1000}{535/1000} = \frac{130}{535}$$

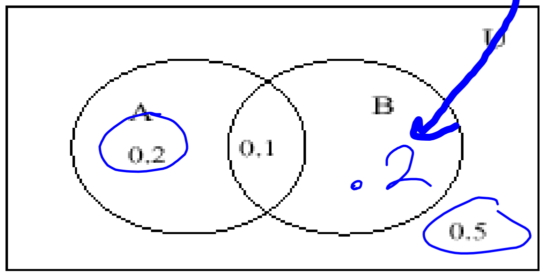
E - event that the student is registered for 13 hrs or less.

$$P(J\cap E) = \frac{n(J\cap E)}{n(S)} = \frac{130}{1000}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{535}{1000}$$

2. Use the Venn diagram to answer the following.

$1 - .2 - .1 - .5$



(a) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.1}{.1 + .2} = \dfrac{.1}{.3} = \boxed{\dfrac{1}{3}}$

(b) $P(B^C|A) = \dfrac{P(B^c \cap A)}{P(A)} = \dfrac{.2}{.2 + .1} = \dfrac{.2}{.3} = \boxed{\dfrac{2}{3}}$

(c) $P(A^C|B^C) = \dfrac{P(A^c \cap B^c)}{P(B^c)} = \dfrac{.5}{.2 + .5} = \dfrac{.5}{.7} = \boxed{\dfrac{5}{7}}$

1 or more ← union

3. A student has two exams in one day. The probability that he passes the first exam is 0.9, and the probability that he passes the second exam is 0.85. If the probability that the student passes at least one of the two exams is 0.97, are these two events independent?

A - event that he passes the 1st exam

B - 2nd - - - - - - .

Test for Indep.

Check: $P(A \cap B) \stackrel{?}{=} P(A)P(B)$

$P(A) = .9$

$P(B) = .85$

$P(A \cup B) = .97$

given in the problem

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$.97 = .9 + .85 - P(A \cap B)$$
$$.78 = P(A \cap B)$$

Now substitute:

$$P(A \cap B) \stackrel{?}{=} P(A)\,P(B)$$
$$.78 \stackrel{?}{=} (.9)(.85)$$
$$.78 \neq .765$$

A and B are not independent.

4. Let S be a sample space for an experiment with events E, F, and G. Use the given information to answer the following questions.

$S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$.
$E = \{s_1, s_4, s_5, s_6\}$.
$F = \{s_2, s_4, s_6\}$.
$G = \{s_3, s_6\}$.

$F \cap E = \{s_4, s_6\}$

Outcome	s_1	s_2	s_3	s_4	s_5	s_6
Probability	$\frac{1}{17}$	$\frac{2}{17}$	$\frac{5}{17}$	$\frac{3}{17}$	$\frac{4}{17}$	$\frac{2}{17}$

(a) $P(F|E) = \dfrac{P(F \cap E)}{P(E)} = \dfrac{\frac{3}{17} + \frac{2}{17}}{\frac{1}{17} + \frac{3}{17} + \frac{4}{17} + \frac{2}{17}} = \dfrac{5/17}{10/17} = \boxed{\frac{1}{2}}$

(b) $P(G|F) = \dfrac{P(G \cap F)}{P(F)} = \dfrac{2/17}{\frac{2}{17} + \frac{3}{17} + \frac{2}{17}} = \dfrac{2/17}{7/17} = \boxed{\frac{2}{7}}$

$G \cap F = \{s_6\}$

5. A 7th grade class was selected and the following information was collected about the 30 students.

 4 students have only glasses.
 12 students have only braces.
 6 students have glasses and braces.

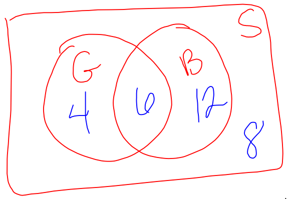
Determine whether the event that the student has glasses and the event that the student has braces are independent. Justify your answer.

G – event the student has glasses

B – - — — - – braces

Test for Indep.

Check: $P(G \cap B) \overset{?}{=} P(G)P(B)$



1) $P(G) = \frac{n(G)}{n(S)} = \frac{4+6}{30} = \frac{1}{3}$

2) $P(B) = \frac{n(B)}{n(S)} = \frac{18}{30}$

3) $P(G \cap B) = \frac{n(G \cap B)}{n(S)} = \frac{6}{30}$

Substitute:

$$P(G \cap B) \overset{?}{=} P(G)P(B)$$

$$\frac{6}{30} \overset{?}{=} \left(\frac{1}{3}\right)\left(\frac{18}{30}\right)$$

reduce ↓

$$\frac{1}{5} \overset{?}{=} \frac{1}{5} \checkmark$$

G and B are indep. events.

6. Let A, B, and C be three independent events of an experiment with $P(A) = 0.4$, $P(B) = 0.75$, and $P(C) = 0.3$. Calculate each of the following.

(a) $P(A \cap B^C) = P(A)P(B^c)$

$= (.4)(1-.75)$

$= \boxed{.1}$

(b) $P(A^C \cup C) = P(A^c) + P(C) - P(A^c \cap C)$ because of indep.

$= (1-.4) + .3 - P(A^c)P(C)$

$= .6 + .3 - (.6)(.3)$

$= \boxed{.72}$

(c) $P(B|C) = P(B) = \boxed{.75}$

equal because independence means C does not affect B, or you could use the formula

$P(B|C) = \frac{P(B \cap C)}{P(C)}$

$= \frac{(.75)\cancel{(.3)}}{\cancel{.3}}$

$= .75$

7. A building on campus has three vending machines: two Coke machines and a snack machine. Based on the model of the machines, the first Coke machine has a 12% chance of breaking down in a particular week, and the second Coke machine has a 4% chance of breaking down in a particular week. The snack machine has a 10% chance of breaking down in a particular week. Assuming independence, find the probability that exactly one machine breaks down.

C_1 – event that the 1st Coke machine breaks down.

C_2 – " " " 2nd " " " " .

W – event that the snack machine breaks down.

$P(C_1 \cap C_2^c \cap W^c)$ or $P(C_1^c \cap C_2 \cap W^c)$ or $P(C_1^c \cap C_2^c \cap W)$

$P(C_1)P(C_2^c)P(W^c) + P(C_1^c)P(C_2)P(W^c) + P(C_1^c)P(C_2^c)P(W)$

$= (.12)(.96)(.9) + (.88)(.04)(.9) + (.88)(.96)(.1)$

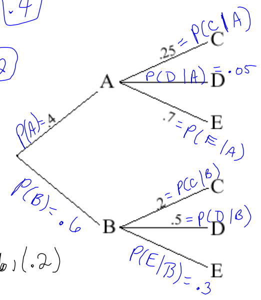
$= \boxed{.21984}$

8. Fill in the missing probabilities on the tree diagram and then answer the following questions.

(a) $P(B^C) = 1 - P(B) = 1 - .6 = \boxed{.4}$

(b) $P(A \cap D) = (.4)(.05) = \boxed{.02}$

(c) $P(E|A) = \boxed{.7}$ (on the tree)



(d) $P(C) = (.4)(.25) + (.6)(.2)$

$= \boxed{.22}$

(e) $P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$= .6 + .22 - (.6)(.2)$

$= \boxed{.7}$

(f) $P(B|E) = \dfrac{P(B \cap E)}{P(E)} = \dfrac{(.6)(.3)}{(.4)(.7) + (.6)(.3)}$

$= .3913 = \boxed{\dfrac{9}{23}}$

(g) Are the events B and C independent? Justify your answer.

check $P(B \cap C) \stackrel{?}{=} P(B)P(C)$

$(.6)(.2) = (.6)(.22)$

$.12 \neq .132$

B and C are not indep.

(h) Are the events B and C mutually exclusive? Justify your answer.

Check → Is $P(B \cap C) = 0$?

events that cannot happen at the same time

In general, ($A \cap B = \emptyset$ for mut. excl. events)

$P(B \cap C) = (.6)(.2) = .12 \neq 0$

there is a chance B and C happen together → B and C are NOT mutually exclusive.

(i) Are the events A and D independent? Justify your answer.

Check $P(A \cap D) \stackrel{?}{=} P(A)P(D)$

$.02 \stackrel{?}{=} (.4)\left((.4)(.05) + (.6)(.5)\right)$

from (b)

$.02 \stackrel{?}{=} (.4)(.32)$

$.02 \stackrel{?}{=} .128$ NO!

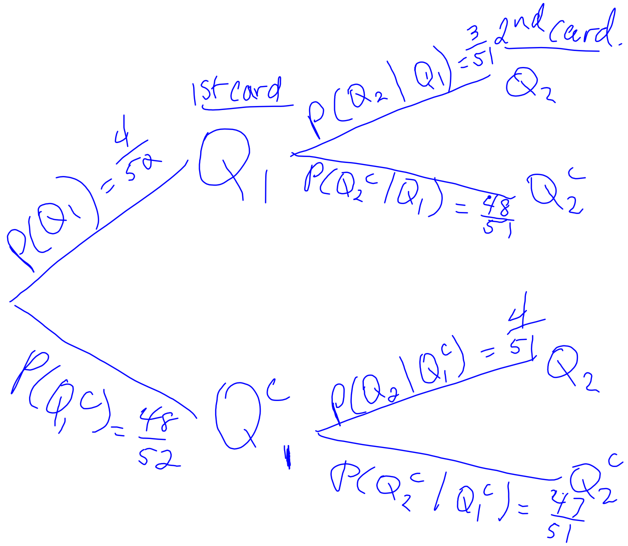
NOT indep.!

$P(Q_1 | Q_2^c)$

9. Two cards are drawn at random (without replacement) from a well-shuffled deck of 52 playing cards. What is the probability that the first card was queen given that the second card drawn was not a queen?

Q_1 - event that 1st card is a queen

Q_2 - - - 2nd - - - -



$$P(Q_1 | Q_2^c) = \frac{P(Q_1 \cap Q_2^c)}{P(Q_2^c)}$$

$$= \frac{\left(\frac{4}{52}\right)\left(\frac{48}{51}\right)}{\left(\frac{4}{52}\right)\left(\frac{48}{51}\right) + \left(\frac{48}{52}\right)\left(\frac{47}{51}\right)}$$

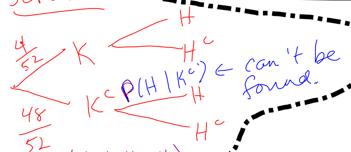
$$= \boxed{\frac{4}{51}}$$

10. Two cards are drawn at random (without replacement) from a well-shuffled deck of 52 playing cards. What is the probability that the first card was a king given that the second card drawn was a heart?

$P(1^{st}\text{ card is king} \mid 2^{nd}\text{ card is a heart})$

Find this

Scratch



(This is a failed 1st attempt.)

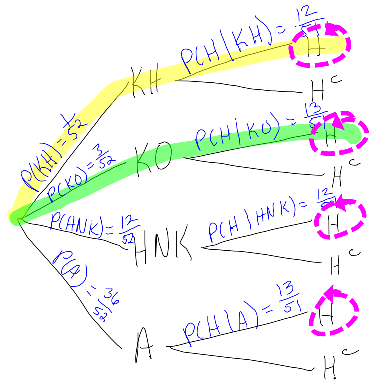
KH – event that 1st card is the king of hearts

KO – – – – – – – – – is a king other than the king of hearts

HNK – – – – – – – – – is a heart, but not the king of hearts.

A – – – – – – 1st card is any other card not mentioned above (not kings and not hearts)

H – event that the 2nd card is a heart.



The paths highlighted in yellow and green are the only two paths in which the 1st card is a king and the 2nd card is a heart.

$$P(1^{st}\text{ is king} \mid 2^{nd}\text{ is heart}) = \frac{P(1^{st}\text{ is king} \cap 2^{nd}\text{ is heart})}{P(2^{nd}\text{ is heart})}$$

(yellow path) (green path)

$$= \frac{\left(\frac{1}{52}\right)\left(\frac{12}{51}\right) + \left(\frac{3}{52}\right)\left(\frac{13}{51}\right)}{\left(\frac{1}{52}\right)\left(\frac{12}{51}\right) + \left(\frac{3}{52}\right)\left(\frac{13}{51}\right) + \left(\frac{12}{52}\right)\left(\frac{12}{51}\right) + \left(\frac{36}{52}\right)\left(\frac{13}{51}\right)}$$

all paths ending in H $\longrightarrow$

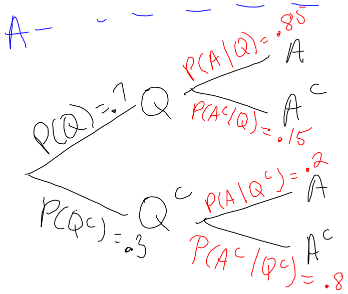
$$= \boxed{\frac{1}{13}}$$

11. The personnel manager at a certain company claims that she approves qualified applicants for a certain job 85% of the time; she rejects an unqualified person 80% of the time. If 70% of all applicants for this job are qualified, find each of the following. (pg. 380 of *Finite Mathematics* by Lial, Greenwell, and Ritchey)

(a) Draw a tree diagram (with probabilities and notation on all branches) representing the above information.

Q - event that the applicant is qualified.

A - " " " " " " is accepted



(b) What is the probability that an applicant is approved?

$$P(A) = (.7)(.85) + (.3)(.2)$$

$$= \boxed{.655} = \frac{131}{200}$$

(c) What is the probability that an applicant is qualified if he or she was approved by the personnel manager?

given

$$P(Q \mid A) = \frac{P(Q \cap A)}{P(A)} = \frac{(.7)(.85)}{.655}$$

given

$$= \boxed{\frac{119}{131}}$$

(d) What is the probability that an applicant who is unqualifed is approved for the job?

given

$$P(A \mid Q^c) = \boxed{.2}$$

(on tree)

(e) What is the probability that an approved applicant was unqualified?

$$P(Q^c \mid A) = \frac{P(Q^c \cap A)}{P(A)}$$

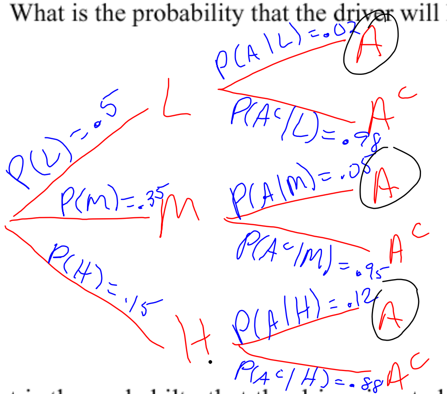
$$= \frac{(.3)(.2)}{.655} = \boxed{\frac{12}{131}}$$

12. An auto insurance company classifies its drivers as low risk, medium risk, or high risk. The table shows the percentage of drivers in these classifications and the probability that a driver in that classification will have an accident during the next year. A driver is selected at random. (This problem is courtesy of Joe Kahlig.)

Classification	Drivers (%)	Accident (%)
low	50	2
medium	35	5
high	15	12

L - low risk
M - medium risk
H - high risk
A - accident in the next year.

(a) What is the probability that the driver will have an accident in the next year?



$P(A) = (.5)(.02) + (.35)(.05) + (.15)(.12)$

$= \boxed{.0455}$

(b) What is the probabilty that the driver is rated as a medium risk if he or she has an acident in the next year?

$$P(M|A) = \frac{P(M \cap A)}{P(A)}$$

$$= \frac{(.35)(.05)}{.0455}$$

$$= \boxed{\frac{5}{13}} = .3846$$

(c) What is the proability that the driver is classified as a high risk but does not have an accident in the next year?

and $\cap$

$$P(H \cap A^c) = (.15)(.88)$$

$$= .132 = \boxed{\frac{33}{250}}$$

Discrete - you can make a list
Finite discrete - the list stops

Continuous - you cannot make an all-inclusive list.

13. Classify each of the following random variables as either finite discrete, infinite discrete, or continuous, and give the values of the random variable.

(a) Cast a die until a 5 lands up. Let X denote the number of throws in one trial of the experiment.

Value of $X = 1, 2, 3, 4, \ldots$ Infinite Discrete.

(b) A farmer plants 10 watermelon seeds. Let X denote the number of the seeds that sprout.

$X = 0, 1, 2, 3, \ldots, 10$ finite discrete

(c) Let X denote the weight of my ~~cat~~ Mouse in pounds.

continuous. $X > 0$

(d) Let X denote the number of minutes that a person waits in line at a grocery store check-out lane.

continuous

$X \geq 0$

(e) X = # of hours Bob watches TV in a day.

Continuous

$0 \leq X \leq 24$

14. A box contains 2 pens, 2 pencils, and 5 highlighters. A sample of 6 items is drawn from the box without replacement. Let the random variable X denote the number of pencils in the sample.

$X = \#$ of pencils

(a) What type of random variable is X, and what are the possible values of X?

Values of $X = 0, 1, 2$ finite discrete

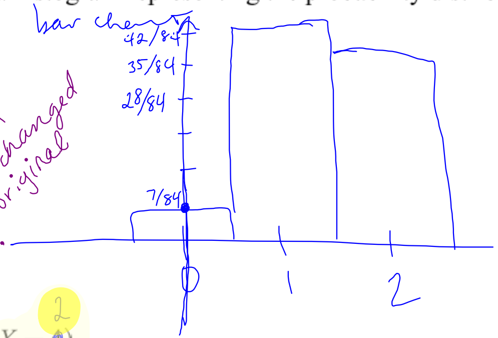
(b) Find the probability distribution of X.

Table

Value of X	0	1	2
$P(X=x)$	$P(X=0) = \frac{n(X=0)}{n(S)} = \frac{C(7,6)}{C(9,6)} = \frac{7}{84}$	$\frac{C(2,1)C(7,5)}{C(9,6)} = \frac{42}{84}$	$\frac{C(2,2)C(7,4)}{C(9,6)} = \frac{35}{84}$

(c) Draw a histogram representing the probability distribution of X.

bar chart



Note: Parts (d) and (e) have been changed from the original printout.

(d) Find $P(X = \cancel{0}\,2)$.

$P(X=2) = \frac{35}{84}$ ← in probab distrib.

(e) Find $P(X < \cancel{0}\,2)$.

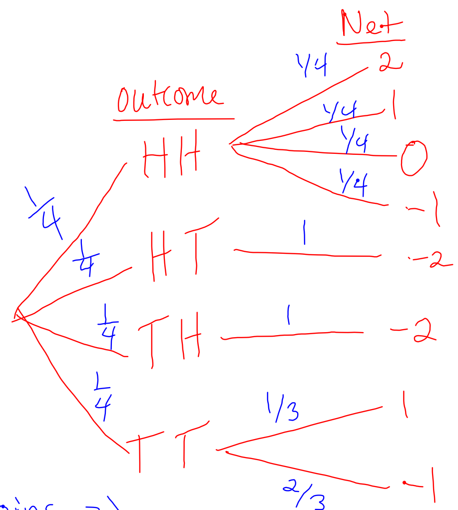
$$P(X<2) = \frac{7}{84} + \frac{42}{84} = \boxed{\frac{49}{84}}$$

15. A game consists of flipping two fair coins. If both coins land heads, then you draw 1 card from a bag that contains 4 cards, each marked with the number 1, 2, 3, or 4, and you win this amount in dollars. If both coins land tails, then you draw 1 block from a bucket that contains 3 blocks, two of which are labeled with a 1 and one of which is labeled with a 3, and you win this amount in dollars. Otherwise, you win nothing. It costs \$2 to play this game. Let X denote the net winnings of a person who plays this game once. Find the probability distribution of X.

winnings − cost to play

net!

Value of X	$P(X=x)$
2	$P(X=2) = (\frac{1}{4})(\frac{1}{4}) = \frac{1}{16}$
1	$P(X=1) = (\frac{1}{4})(\frac{1}{4}) + (\frac{1}{4})(\frac{1}{3}) = \frac{7}{48}$
0	$(\frac{1}{4})(\frac{1}{4}) = \frac{1}{16}$
−1	$(\frac{1}{4})(\frac{1}{4}) + (\frac{1}{4})(\frac{2}{3}) = \frac{11}{48}$
−2	$(\frac{1}{4})(1) + (\frac{1}{4})(1) = \frac{1}{2}$



(fair coins → uniform)

Note: Since the problem asked you to find a probability distribution, your answer is the entire table on the left above.