Math 166 - Week in Review #8

Section 5.1 - Introduction to Matrices

- The **order** (size) of a matrix is always *number of rows* × *number of columns*.
- c_{ij} represents the entry of the matrix C in row i and column j.
- To add and subtract matrices, they must be the same size.
- When adding or subtracting matrices, add or subtract corresponding entries.
- A scalar product is computed by multiplying each entry of a matrix by a scalar (a number).
- Transpose - The rows of the matrix A become the columns of A^T.
- The **zero matrix** of order $m \times n$ is the matrix O with m rows and n columns, all of whose entries are zero.

Section 5.2 - Multiplication of Matrices

- The matrix product AB can be computed only if the number of columns of A equals the number of rows of B.
- If $C = AB$, then c_{ij} is computed by multiplying the i^{th} row of A by the j^{th} column of B.
- Identity Matrix - Denoted by I_n, the identity matrix is the $n \times n$ matrix with 1's down the main diagonal (from upper left corner to lower right corner) and 0's for all other entries.
- If A is $m \times n$, then $AI_n = A$ and $I_m A = A$.
- In general, matrix multiplication is not commutative.

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1. Let $A = \begin{bmatrix} 4 & x \\ 3 & -7 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 & 0 \\ 8 & k & -6 \end{bmatrix}$, $C = \begin{bmatrix} 4 & -5 \\ 0 & b \\ 7 & -10 \end{bmatrix}$, and $D = \begin{bmatrix} 5 & -1 & 3a \\ 2 & 6 & 4 \end{bmatrix}$. Compute each of the following:

   (a) $B + 3D$
   (b) $2C + B$
   (c) $4D - 3C^T$
   (d) $4a_{21} - 2c_{32} + 7d_{13}$
   (e) DB
   (f) $B^T DA$
   (g) CD^T
   (h) BB^T
   (i) A^2

2. Solve for x and y:

$$3\begin{bmatrix} 2 & x \\ 5y & -1 \end{bmatrix} - \begin{bmatrix} -6 & 1 \\ 3y & -5 \end{bmatrix}^T = \begin{bmatrix} 12 & -7 \\ -2x & 2 \end{bmatrix}$$

3. The times (in minutes) required for assembling, testing, and packaging large and small capacity food processors are shown in the following table:

	Assembling	Testing	Packaging
Large	45	15	10
Small	30	10	5

   (a) Define a matrix T that summarizes the above data.

   (b) Let $M = \begin{bmatrix} 100 & 200 \end{bmatrix}$ represent the number of large and small food processors ordered, respectively. Find MT and explain the meaning of its entries.

   (c) If assembling costs \$3 per minute, testing costs \$1 per minute, and packaging costs \$2 per minute, find a matrix C that, when multiplied with T, gives the total cost for making each size of food processor.

4. If $A = \begin{bmatrix} 4 & 0 & k \\ -9 & m & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & j & 8 \\ 5 & n & -6 \end{bmatrix}$, and if $C = B^T A$, then find

   (a) c_{32}

   (b) c_{13}

5. Acme Flowers is a florist shop with three locations–one in San Antonio (SA), one in Dallas (D), and one in Houston (H). Each shop makes three standard arrangments A, B, and C. The matrix M below shows the number of each type of arrangement ordered in the month of January. The matrix N below shows the number of roses (R), carnations (C), and chrysanthemums (M) used in each type of arrangment.

$$M = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} SA & D & H \end{array} \\ \begin{bmatrix} 18 & 20 & 16 \\ 12 & 17 & 10 \\ 13 & 11 & 9 \end{bmatrix} \end{array} \qquad N = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} R & C & M \end{array} \\ \begin{bmatrix} 5 & 10 & 2 \\ 7 & 6 & 3 \\ 9 & 12 & 5 \end{bmatrix} \end{array}$$

How should these matrices be multiplied to produce a matrix T that gives the total number of each type of flower needed at each location to meet January's orders?