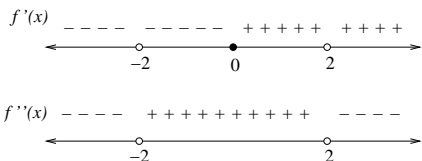## Math 142 - Week in Review #8

1. Sketch the graph of a function f that satisfies the following:

   - Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
   - Vertical asymptotes: $x = -2$ and $x = 2$
   - Horizontal asymptote: $y = -3$
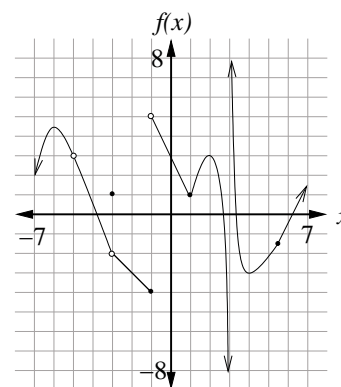   - x-intercepts: none; y-intercept: $(0, 1)$

   $f'(x)$ $\quad ----\quad -----\quad +++++\quad ++++$

   $\qquad\qquad\qquad -2 \qquad\quad 0 \qquad\quad 2$

   $f''(x)$ $\quad ----\quad ++++++++++\quad ----$

   $\qquad\qquad\qquad -2 \qquad\qquad\qquad 2$

2. Sketch the graph of a function f that satisfies the following:

   - Domain: All real numbers
   - x and y-intercept: $(0, 0)$
   - Vertical asymptotes: none
   - $\lim\limits_{x \to -\infty} f(x) = -\infty$ and $\lim\limits_{x \to \infty} f(x) = 0$
   - $f'(x) > 0$ on $(-\infty, 2)$
   - $f'(x) < 0$ on $(2, \infty)$
   - $f''(x) > 0$ on $(4, \infty)$
   - $f''(x) < 0$ on $(-\infty, 4)$

3. Use the given graph to find the absolute extrema (locations and values) of $f(x)$ on each of the intervals below.

   (a) $[0, 2]$
   (b) $[0, 3]$
   (c) $[1, 5]$
   (d) $(3.5, \infty)$
   (e) $(-5, -3)$

4. Find the absolute extrema (locations and values) of $f(x) = x^2 e^{-0.4x}$ on

   (a) $[-2, 8]$
   (b) $[3, 12]$

5. Find all local extrema of each of the given functions on its domain. Use the Second Derivative Test when it applies.

   (a) $f(x) = x^4 - 8x^3 - 32x^2 + 10$

   (b) $g(x) = 0.5x^2 - 8x + 9\ln(x + 2)$ given that $g'(x) = \dfrac{x^2 - 6x - 7}{x + 2}$ and $g''(x) = \dfrac{x^2 + 4x - 5}{(x + 2)^2}$

(c) $h(x) = 2x^5 - 15x^4 - 90x^3 + 75$

6. Each of the following functions has one absolute extremum on the provided interval. Find the location and value of the absolute extremum and classify it as an absolute maximum or absolute minimum.

(a) $f(x) = \dfrac{e^x}{x}$ on $(0,\infty)$ given that $f''(x) = \dfrac{e^x(x^2 - 2x + 2)}{x^3}$

(b) $g(x) = 5x\ln x - 15x$ on $(0,\infty)$

(c) $h(x) = 20 - 3x - \dfrac{12}{x^2}$ on $(0,\infty)$

7. Bob would like to build a rectangular corral with four parallel partitions. If he plans to use 1,200 feet of fencing, what dimensions will maximize the total area of the corral?

8. Now Bob wants to fence in a rectangular area of 4,232 ft^2 in his back yard. He would like for the fence to extend the same distance to the left and right of the back side of his house, which is 60 feet long. Material for the sides of the fence that are parallel to the back side of the house costs $2 per foot, and material for the other two sides costs $4 per foot. Find the dimensions of the fenced area that minimizes cost.

9. Acme Water Supply wants to make a closed cylindrical water tank that has a volume of $225,000\pi$ cubic feet. The material for the top and bottom of the tank costs $10 per square foot, and the material for the side costs $6 per square foot. Find the dimensions (height and radius) of the water tank that minimizes cost.

10. The owner of a luxury motor yacht that sails among the 4,000 Greek islands charges $600 per person per day if exactly 20 people sign up for the cruise. However, if more than 20 people sign up for the cruise, the price of each fare is reduced by $4 for each additional passenger. Assuming at least 20 people sign up for the cruise, determine how many passengers will result in the maximum revenue for the owner of the yacht. What is the maximum revenue? What would be the fare per passenger in this case? (courtesy Jenn Whitfield)

11. *Inventory Control* - Acme Jelly Company expects a uniform annual demand of 49,000 jars of its jelly. Acme orders its jars from the Emca Bottling Company. For each order of jars placed, Emca charges a fee of $150, and Acme's cost of storing empty jars for a year is $0.30 per jar. How many jar orders should Acme place per year and how many jars should be in each shipment if the total cost from ordering and storage is to be minimized?

12. Apply the graphing strategy to sketch the graph of $f(x) = \dfrac{2x^2 + x - 15}{x^2 - 9}$.