Math 142 - Week in Review #9

1. If $f(x) = 4x^3 - 2x + \sqrt{7}$, give three different functions that are antiderivatives of $f(x)$.

2. Compute each of the following indefinite integrals and give the appropriate units for each.

   (a) $\int \left(3e^t + \frac{15}{t} - 5t\right) dt$ births per year, where t is in years

   (b) $\int \left(57 - 3x^{-5} + 2x^{-1}\right) dx$ miles per hour, where x is in hours

   (c) $\int \left(5\sqrt{x} - \sqrt{11} + \frac{4}{\sqrt[5]{x}}\right) dx$ students per section per year, where x is in years

3. Compute each of the following indefinite integrals.

(a) $\int 7x^6\sqrt{5+x^7}\,dx$

(b) $\int (x^9+4x)e^{x^{10}+20x^2}\,dx$

(c) $\int \frac{8t-7}{4t^2-7t}\,dt$

(d) $\int \frac{6m}{\sqrt[3]{m-3}}\,dm$

(e) $\int t^2\sqrt{t+4}dt$

(f) $\int \frac{4}{p\ln p}dp$

4. Find $f(x)$ if $f'(x) = 9x^{-1} + 5x^{-2} - 7$ and $f(3) = 10$.

5. The research department of Acme, Inc. has determined the marginal cost function for one particular item to be $C'(x) = 0.12e^{0.04x}$ dollars per item, where x is the number of items produced. If Acme's fixed costs amount to \$3,000, find a model for the total production cost of this item.

6. The rate at which a particular plant grows is given by $r(t) = \frac{1}{2}t^2 + 4$ mm per day, where t is the number of days since the plant was potted in fresh soil, $0 \leq t \leq 5$.

   (a) Compute L_3 to estimate $\int_0^3 r(t)dt$, i.e., to estimate the area under $r(t)$ on the interval $[0,3]$.

   (b) Now compute a midpoint sum with 6 rectangles of equal width to estimate $\int_0^3 r(t)dt$.

   (c) Give an interpretation to your answers in (a) and (b).

7. Given that $\int_2^7 f(x)dx = -2$, $\int_2^7 g(x)dx = 8$, and $\int_7^9 g(x)dx = 5$, find each of the following.

(a) $\int_2^7 (8g(x) - f(x))\,dx$

(b) $\int_2^9 10g(x)dx$

(c) $\int_2^2 7f(x)dx$

(d) $\int_7^2 (f(x) + g(x))\,dx$

8. Find the average value of $k(x) = 4x^2 - 6x$ on $[2,5]$.

9. Compute each of the following by hand.

(a) $\int_{-2}^{3}(7x-8e^{x})dx$

(b) $\int_{1}^{4}\frac{x}{(x^{2}-9)^{5}}dx$

(c) $\int_{0}^{a}5t(8t^{4}-7t^{-3})dt$

(d) $\int_{-1}^{2}(x-5)(x^2-10x)^3dx$

10. The rate at which the concentration of a particular drug in the blood stream increases when taken daily can be modeled by

$$r(t)=\frac{2.2}{t}\mu\text{g/mL per day}$$

where t is the number of days since the daily regimen was started, $1\leq t\leq 17$.

(a) Find the average rate of change of the concentration of the drug in the blood stream from taking the second dose through taking the eighth dose.

(b) If five days after the regimen was started, the concentration of this drug in the blood stream was 4.5 μg/mL, find a model for the concentration of the drug in the bloodstream.

(c) Find the average concentration of the drug in the bloodstream from taking the third dose through taking the tenth dose.

11. The temperature of a cup of coffee can be modeled by $T(x) = 70 + 130e^{-0.05x}$°F where x is the number of minutes since the cup of coffee was poured.

    (a) What is the average temperature of the coffee during the first 1.25 hours since it was poured?

    (b) What is the average rate of change of the coffee's temperature from 0.75 hour to 1.5 hours after it was poured?

12. Acme Widget Company's marginal profit is given by $P'(x) = 35e^{-0.01x}$ dollars per widget, where x is the number of widgets produced per day.

    (a) If the current production level is 250 widgets per day and the manager wishes to increase production to 275 widgets per day, how will this production increase affect profit?

    (b) Find a model for profit if the profit earned by selling 120 widgets is \$300.