## Math 141 - Exam 3 Review

1. A bag contains 2 red, 1 blue, and 3 green marbles. One marble is chosen randomly from the bag.

   (a) Give a non-uniform sample space for this experiment. Then write the event that the marble is green.

   S = {red, blue, green}   E = {green}

   Colors do not have the same probab. of being selected.

   (b) Give a uniform sample space for this experiment. Then write the event that the marble is green.

   Let $r_1$ be the 1st red marble, $b_1$ be the blue marble, and $g_1$ be the 1st green marble, ...

   $S = \{r_1, r_2, b_1, g_1, g_2, g_3\}$, $E = \{g_1, g_2, g_3\}$

   Individual marbles have the same probab of being selected.

2. An experiment consists of casting a 4-sided die and flipping a coin.

   (a) Give an appropriate sample space for this experiment.

   S = {1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T}

   (b) Let $E$ be the event that a 3 is rolled on the die, and let $F$ be the event that the coin lands tails. Are these events mutually exclusive? Justify your answer.

   E = {3H, 3T}

   F = {1T, 2T, 3T, 4T}

   $E \cap F = \{3T\}$

   Since E and F have an element in common, they are NOT mutually exclusive

3. Are mutually exclusive events and independent events the same thing?

   No! E+F Events are mutually exclusive if $P(E \cap F) = 0$. (can't happen at the same time)

   Events are independent if $P(E \cap F) = P(E)P(F)$. (the outcome of one does not affect the other)

4. Jack and Jill are two weather forcasters in Gonzales. The probability that Jack accurately predicts the weather on any given day is 0.68, and the probability that Jill accurately predicts the weather on any given day is 0.72. If the probability that at least one of them is correct on any given day is 0.89, are Jack and Jill making their weather predictions independently?

   ← union

   A = event that Jack is correct

   B = " " Jill " "

   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

   $.89 = .68 + .72 - P(A \cap B)$

   $P(A \cap B) = .51$

   Test for Indep

   $P(A \cap B) \stackrel{?}{=} P(A)P(B)$?

   $.51 = (.68)(.72)$? → $.51 = .4896$ No!

   Not independent!

5. Is the following statement correct? "The probability that Kurt spends less than \$15 on a new DVD is 0.4. Therefore the probability that Kurt spends more than \$15 on a new DVD is 0.6."

No. ~~If all we know is the prob.~~
we cannot assume that the probab. that Kurt
will buy a DVD that costs exactly \$15 is 0.

6. A fair 6-sided die and a fair 8-sided die are rolled. What is the probability that

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | (1,1) | (1,2) | | | | |
| 2 | (2,1) | (2,2) | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | | | | | (8,6) |

(a) the sum of the dice is 4 or 7?

E = Sum is 4 or 7

$P(E) = \frac{n(E)}{n(S)} = \frac{3+6}{8 \cdot 6} = \boxed{\frac{9}{48}}$

(b) the sum of the dice is 7 or at least one 3 is showing?

F = sum is 7 or at least one 3 is showing.

$P(F) = \frac{n(F)}{n(S)} = \boxed{\frac{17}{48}}$

(c) the sum of the dice is 6 if the 8-sided die shows an even number?

A - sum is 6
B - 8-sided die shows even

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/48}{24/48} = \boxed{\frac{2}{24}}$

(d) the sum of the dice is 12 provided that exactly one 6 is showing?

C = sum is 12
D = exactly one 6 is showing

$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{0/48}{12/48} = \boxed{0}$

7. Madison has 5 red, 7 yellow, and 4 blue crayons in her desk drawer. If she selects two at random, what is the probability that she will get two of the same color?

E - event that she gets 2 of the same color.

$P(E) = \frac{n(E)}{n(S)} = \boxed{\frac{C(5,2)+C(7,2)+C(4,2)}{C(16,2)}}$

2 red or 2 yellow or 2 blue

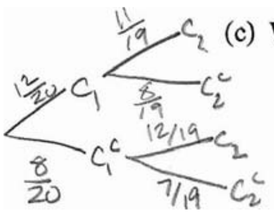
8. A local business employs 12 cashiers, 3 shift managers, and 5 stockers. Two employees are selected at random to attend a workshop.

   (a) What is the probability that the first employee selected is a cashier?

   $\frac{12}{20}$

   (b) Assuming that the first employee selected is a cashier, what is the probabilty that the second employee selected is a cashier?

   $\frac{11}{19}$




   (c) What is the probability that neither the first nor the second employee selected is a cashier?

   $C_1 = $ 1st is cashier

   $C_2 = $ 2nd is cashier

   $P(C_1^c \cap C_2^c) = P(C_1^c)P(C_2^c \mid C_1^c)$

   $= \left(\frac{8}{20}\right)\left(\frac{7}{19}\right)$

9. A manufacturer of automobiles receives 500 car radios from each of three different suppliers. The shipment from supplier A contains 5 defective radios, the shipment from supplier B contains 7 defective radios, and the shipment from supplier C contains 2 defective radios. As a means of quality control, one radio is selected at random from each of the shipments. What is the probability that

   Note: you may use a tree diagram, or you may use the fact that the types of radios coming from each supplier are independent.

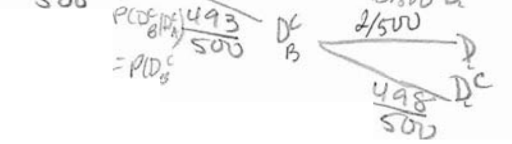
   (a) all of the radios selected are working properly?

   by independence

   $P(\text{1st works} \cap \text{2nd works} \cap \text{3rd works})$

   $= P(\text{1st works})\,P(\text{2nd works})\,P(\text{3rd works})$

   $= \left(\frac{495}{500}\right)\left(\frac{493}{500}\right)\left(\frac{498}{500}\right) \approx 0.9722$




   (b) at least one of the radios selected is defective?

   $G = $ 1 or more

   $P(G) = 1 - P(G^c)$ no defectives

   $= 1 - 0.9722 = \boxed{0.0278}$

   (c) exactly one of the selected radios is defective?

   $P(D_A \cap D_B^c \cap D_C^c)$ or $P(D_A^c \cap D_B \cap D_C^c)$ or $P(D_A^c \cap D_B^c \cap D_C)$

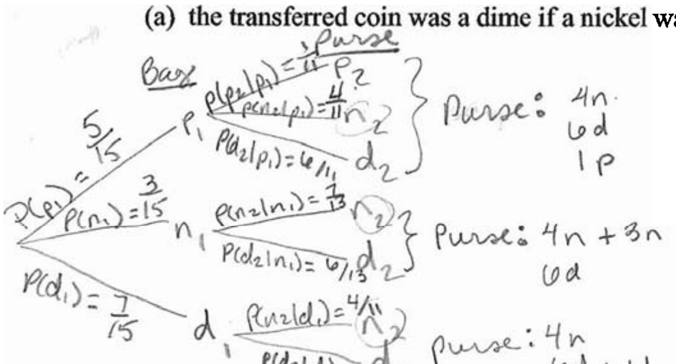
   $\left(\frac{5}{500}\right)\left(\frac{493}{500}\right)\left(\frac{498}{500}\right) + \left(\frac{495}{500}\right)\left(\frac{7}{500}\right)\left(\frac{498}{500}\right) + \left(\frac{495}{500}\right)\left(\frac{493}{500}\right)\left(\frac{2}{500}\right)$

   $= \boxed{0.0275}$

10. A bag contains 5 pennies, 3 nickels, and 7 dimes. A purse contains 4 nickels and 6 dimes. A coin is drawn from the bag and transferred to the purse, but if a nickel is selected from the bag, then all of the nickels in the bag are transferred to the purse. A coin is then drawn from the purse. The type of coin drawn from each of the bag and purse is recorded. What is the probability that

$p_1$ = event that penny is drawn from bag
$n_1$ = - - - nickel - - bag
$d_1$ = - - dime - - bag
$p_2$ = - - penny - - purse
$n_2$ = - nickel - - purse
$d_2$ dime - - purse

(a) the transferred coin was a dime if a nickel was selected from the purse?




$$P(d_1|n_2)=\frac{P(d_1\cap n_2)}{P(n_2)}=\frac{\left(\frac{7}{15}\right)\left(\frac{4}{11}\right)}{\left(\frac{5}{15}\right)\left(\frac{4}{11}\right)+\left(\frac{3}{15}\right)\left(\frac{7}{13}\right)+\left(\frac{7}{15}\right)\left(\frac{4}{11}\right)}$$

$$=\frac{28/165}{57/143}=\boxed{\frac{364}{855}}\ (a)$$

(b) both coins are pennies?

$$P(p_1\cap p_2)=\left(\frac{5}{15}\right)\left(\frac{1}{11}\right)=\frac{5}{165}=\boxed{\frac{1}{33}}$$

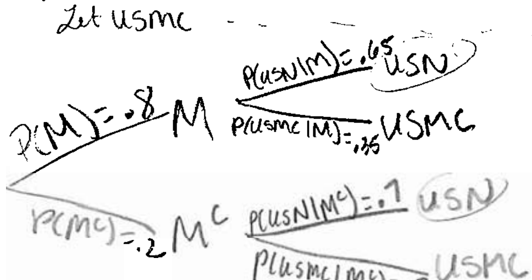
(c) the coin drawn from the purse is a penny if the coin drawn from the bag was a nickel?

$P(p_2|n_1)=0$ (not possible)

11. A naval academy has a student body that is 80% male. 35% of the males and 30% of the females plan to seek a commission in the United States Marine Corps, and all other students plan to seek a commission in the United States Navy.

(a) What is the probability that a student at this academy is male or plans to seek a commission in the Navy?

Let M be the event the student is male
Let USN be the event - - - plans to go to Navy.
Let USMC - - - - - Marine Corps.




$$P(M\cup USN)=P(M)+P(USN)-P(M\cap USN)$$
$$=.8+((.8)(.65)+(.2)(.7))-(.8)(.65)$$
$$=\boxed{0.94}$$

(b) What is the probability that a student who plans to seek a commission in the Marine Corps is female?

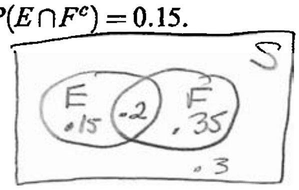
$$P(M^c|USMC)=\frac{P(M^c\cap USMC)}{P(USMC)}=\frac{(.2)(.3)}{(.8)(.35)+(.2)(.3)}=.1765=\boxed{3/17}$$

(c) What is the percentage of students at this academy who plan to join the Marine Corps?

$$P(USMC)=(.8)(.35)+(.2)(.3)=.34$$

$\boxed{34\%}$

12. Let $E$ and $F$ be two events of an experiment with $P(E) = 0.35$, $P(F) = 0.55$, and $P(E \cap F^c) = 0.15$.




(a) Find $P(E \cap F)$.

$P(E \cap F) = .2$

(b) What is the probability that exactly one of these two events occurs?

$.15 + .35 = \boxed{.5}$

(c) Are $E$ and $F$ mutually exclusive?

No b/c $P(E \cap F) = .2$, which means it is possible for them to occur at the same time.

(d) Are $E$ and $F$ independent?

Test for Independence

$P(E \cap F) \stackrel{?}{=} P(E)P(F)$

$.2 \stackrel{?}{=} (.35)(.55)$

$.2 \stackrel{?}{=} .1925$ No! Not independent

(e) Find $P(E|F)$.

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{.2}{.55} = \frac{20}{55} = \boxed{\frac{4}{11}}$

(f) Find the probability that at least one of the two events occurs.

union

$P(E \cup F) = .15 + .2 + .35 = \boxed{.7}$

13. Classify each of the following random variables and give the possible values they each may assume.

(a) $X$ = the number of times a coin is flipped until tails appears.

$X = 1, 2, 3, 4, \ldots$

Infinite Discrete

(b) $X$ = the number of cards drawn (without replacement) from a standard deck of 52 playing cards until a red card is drawn.

$X = 1, 2, 3, \ldots, 27$

finite discrete

(c) $X$ = the weight of a newborn baby. (in pounds)

$0 < X \leq 30$ (approximate interval)

Continuous

(d) $X$ = the number of hours my cat Mouse sleeps in one day.

$0 \leq X \leq 24$

continuous

(e) $X$ = the number of times my phone rings in one hour.

$X$ = finite discrete

$X = 0, 1, 2, \ldots, n$ where $n$ = the maximum # of rings in one hour

14. 16 people are selected at random. What is the probability that at least 2 of the people in this group

(a) were born in the same week? (There are 52 weeks in a year.)

E - at least 2 were born in same week : $E^c$ = none born in same week

$P(E) = 1 - P(E^c)$

$= 1 - \frac{n(E^c)}{n(S)}$

$= 1 - \frac{P(52,16)}{52^{16}}$

$n(E^c)$: $\underline{52}\ \underline{51}\ \underline{50}\ \underline{49} \cdots \underline{37}$ } 16 blanks

$n(S) =$ $\underline{52}\ \underline{52}\ \underline{52}\ \underline{52} \cdots \underline{52}$ } 16 blanks

(b) were born in the same month?

G = at least 2 in same month

$G^c$ = none in same month

$P(G) = 1 - P(G^c) = 1 - \frac{0}{n(S)} = \boxed{1}$

15. The odds against it snowing in College Station next winter are 17 to 2. What is the probability that it will snow in College Station next winter?

odds in favor = 2 to 17

against = 17 to 2

$P(E) = \frac{2}{2+17} = \boxed{\frac{2}{19}}$

16. Two cards are drawn at random from a standard deck of 52 playing cards. What are the odds that the second card drawn is a king given that the first card drawn was a queen?

1st find probab.

$P(2^{nd}$ is king $| 1^{st}$ is queen$) = \frac{4}{51}$ ← probab.

odds in favor $= \frac{4/51}{1 - 4/51} = \frac{4}{47}$ $\boxed{4 \text{ to } 47}$

17. A probability distribution has a mean of 100 and a standard deviation of 4. Use Chebychev's Theorem to estimate the probability that an outcome of the experiment lies between 90 and 110. (NOTE: This topic is not covered in all classes.)

$\mu = 100$ , $\sigma = 4$

☆ $P(90 \le X \le 110) \geq 1 - \frac{1}{k^2} = 1 - \frac{1}{(2.5)^2} = \boxed{0.84}$

100 to 110 → 10

90 to 100 → 10

$\mu \pm 10$

since $\sigma = 4$, $10 = 4k$

$\frac{10}{4} = k$ $k = \frac{5}{2} = 2.5$

18. Fred wants to purchase a 10-year term life insurance policy that will pay his beneficiary $100,000 in the event that Fred does not survive the next 10 years. Using life insurance tables, he determines that the probability that he will live another 10 years is 0.97. What is the minimum amount that he can expect to pay for his premium?

This amount makes the insurance company's expected gain = 0

Let $X$ = the insurance co's gain

Want $E(X)=0$.

need probab. distrib.

| | $X$ | $P(X=x)$ |
|---|---|---|
| (lives) | $A$ | $.97$ |
| (do not live) | $A-100{,}000$ | $.03$ |

$E(X) = A(.97) + (A-100\,000)(.03)$

want $E(X) = 0$: $E(X) = .97A + .03A - 3000$

$0 = A - 3000$

$\boxed{A = \$3000}$

19. Suppose you roll two fair 6-sided dice and take the sum of the numbers landing up. You will win twice what you paid if the sum is 7 or 11. You lose what you paid if the sum is 2, 3, or 12. For any other sum, you win \$5. The game costs \$10 to play. Let $X$ denote the ~~cost~~ net winnings of someone who plays once.

(a) What is the expected net winnings?

$X$ = net Winnings

$E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n$

$E(X) = 10\left(\frac{8}{36}\right) + (-10)\left(\frac{4}{36}\right) + (-5)\left(\frac{24}{36}\right) = \boxed{-\$2.22}$

| Win | Outcome | Value of $X$ | $P(X=x)$ |
|---|---|---|---|
| \$20 | 7 or 11 | 10 | $P(\text{sum} = 7 \text{ or } 11) = \frac{8}{36}$ |
| \$0 | 2, 3, 12 | −10 | $P(\text{sum} = 2, 3, \text{or } 12) = \frac{4}{36}$ |
| \$5 | all others | −5 | $P(\text{any other sum}) = \frac{24}{36}$ |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

(b) How much should be charged to make this game fair?

| Value of $X$ | | $P(X=x)$ |
|---|---|---|
| $2A - A$ | $= A$ | $\frac{8}{36}$ |
| $0 - A$ | $-A$ | $4/36$ |
| $5 - A$ | $5-A$ | $24/36$ |

$36\left[0 = E(X) = \left[A\left(\frac{8}{36}\right) + (-A)\left(\frac{4}{36}\right) + (5-A)\left(\frac{24}{36}\right)\right]\right]36$

$0 = 8A - 4A + (5-A)24$

$0 = 4A + 120 - 24A$

$0 = -20A + 120$

$20A = 120$

$\boxed{A = \$6}$

20. A cashier at a convenience store kept a record of the number of items purchased by each customer on one day. The data she collected are summarized in the following table:

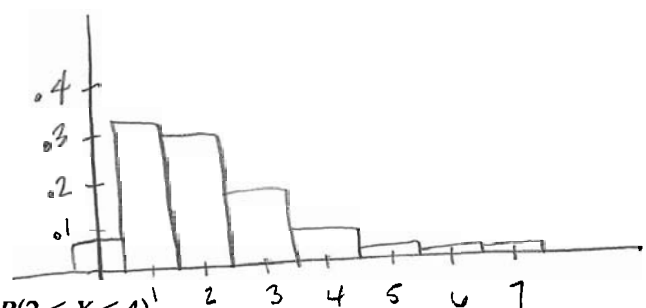
| Number of items purchased | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Number of customers | 5 | 25 | 22 | 13 | 7 | 2 | 1 | 1 |

Total = 76

(a) Write the probability distribution of the number of items purchased.

| X = # of items purchased | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(X=x)$ | $\frac{5}{76}$ | $\frac{25}{76}$ | $\frac{22}{76}$ | $\frac{13}{76}$ | $\frac{7}{76}$ | $\frac{2}{76}$ | $\frac{1}{76}$ | $\frac{1}{76}$ |
| | .07 | .33 | .29 | .17 | .09 | .03 | .01 | .01 |

(b) Draw a histogram associated with the probability distribution found in part (a).




(c) Find $P(2 \le X \le 4)$.

$P(X=2)+P(X=3)+P(X=4) =$

$\frac{22}{76} + \frac{13}{76} + \frac{7}{76} = \boxed{\frac{42}{76}}$

(d) Find $P(X \le 5)$.

$1 - P(X > 5) = 1 - (P(X=6) + P(X=7))$

$= 1 - \left(\frac{1}{76} + \frac{1}{76}\right) = \frac{74}{76}$

(e) How many items could a customer on that day be expected to buy?

or $E(X) = 0 \cdot \frac{5}{76} + 1 \cdot \frac{25}{76} + \cdots + 7\left(\frac{1}{76}\right)$ ← Expected Value → can use $\bar{X}$ in 1var Stats $L_1, L_2$

$\boxed{E(X) = 2.0921}$

(f) Compute the mean, median, mode, standard deviation, and variance for the frequency chart. Be sure to label all answers.

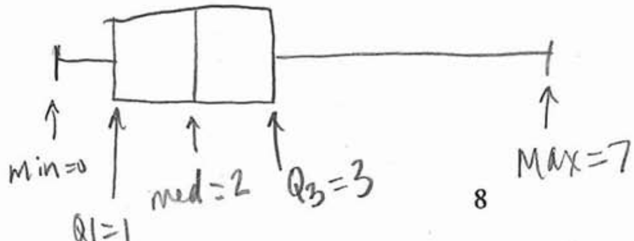
NOTE: this is a sample!

mean $= \bar{X} = 2.0921$ Standard dev $= S_x = 1.3873$

median $= 2$ variance $= S_x^2 = 1.9247$

mode $= 1$

(g) Draw a box and whisker graph with labels for these data. Are there any outliers? (NOTE: This topic is not covered in all classes.)




outliers = data points that are more than 1.5 IQR's above Q3 or below Q1.

$Q1 - 3 = 1 - 3 = -2$

no data pts less than -2

$IQR = Q3 - Q1 = 3 - 1 = 2$

$1.5(IQR) = (1.5)(2) = 3$

$Q3 + 3 = 3 + 3 = 6$

There is one data point at X=7 (outlier)

*****Each of the following problems comes from Section 8.4. Not all instructors are including this section on Exam 3.*****

21. A student takes a 10 question multiple choice exam, each question of which has 5 answer choices (1 correct, 4 incorrect). Being unprepared for the exam, the student randomly guesses at each question.

1 1 1 1 1 1 4 4 4 4

(a) binompdf(10, 1/5, 6) = .0055

(a) What is the probability that the student gets exactly 6 questions correct?

$P(X=6) = \dfrac{n(X=6)}{n(S)} = \dfrac{C(10,6)(1^6)(4^4)}{5^{10}} = \boxed{.0055}$

(this method is by hand)

(a) or BINOM PRGM $n=10$, $p=1/5$, option 1, $r=6$ $\boxed{.0055}$

(b) What is the probability that the student gets at least 60% of the questions correct?

b) $P(X \geq 6) = \boxed{.0064}$ ← either BINOM PRGM $n=10$, $p=1/5$, option 2, lower R = 6, upper R = 10 $P(X \geq 6)$

or binomcdf(10, 1/5, 10) − binomcdf(10, 1/5, 5) = $\boxed{.0064}$

(c) What is the probability that the student gets the first 4 correct and the last 6 incorrect?

C = correct

$C^c$ = not correct

$P(C_1 \cap C_2 \cap C_3 \cap C_4 \cap C_5^c \cap C_6^c \cdots \cap C_{10}^c)$

Indep. $(\frac{1}{5})(\frac{1}{5})(\frac{1}{5})(\frac{1}{5})(\frac{4}{5})(\frac{4}{5}) \cdots (\frac{4}{5}) = (\frac{1}{5})^4(\frac{4}{5})^6 = \boxed{.000419}$

(d) How many questions should the student expect to get correct?

$E(X) = np = 10(\frac{1}{5}) = 2$

(e) Find the variance and standard deviation for the number of questions answered correctly.

$\text{var}(X) = npq = (10)(\frac{1}{5})(\frac{4}{5}) = 1.6$

$\sigma_X = \sqrt{npq} = \sqrt{1.6} = \boxed{1.2649}$

22. A company manufactures one product. For quality control, a random sample of 6 items is selected from a each lot of products made by this company before the lot is shipped. If any defective items are found in the sample, the entire lot is rejected. If 2.3% of the items produced by this company are defective, what is the probability that a lot will be shipped?

P(shipped) = P(no defectives)

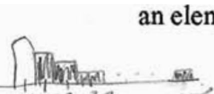
BINOM program

$n=6$, $p=0.023$, option 1, $r=0$

or binompdf(6, 0.023, 0)

$= \boxed{.8697}$

23. A psychological study has determined that 4.8% of all kindergarteners have Attention Deficit Disorder (ADD). In an elementary school with 115 kindergarteners, find the probability that more than 30% have ADD.




more than 30% of 115 = .3(115) = 34.5 so want $P(X > 34.5) = P(X \geq 35)$

BINOM program: $n=115$, $p=.048$, option 2, lower R = 35, upper R = 115

or 1 − binomcdf(115, .048, 34) = 1 − 1 = 0

$P(X \geq 35) = 6.066 \times 10^{-19}$

$\approx 0$

24. The police department of a certain town estimates that 23% of all drivers in their town do not wear their seatbelts. If 60 cars are stopped at random, what is the probability that more than 90% of the drivers are wearing their seatbelts?

$n = 60$

"success" = wearing seatbelt

$p = 1 - .23 = .77$

Indep. Trials? yes

Want prob. of more than 90% of 60

(.9)(60) = 54, so more than 54 successes → $P(X \geq 55)$

Binom program $\boxed{P(X \geq 55) = .0028}$

$n = 60$

$p = .77$

option 2

lower R = 55

upper R = 60

or binomcdf(60, .77, 60) − binomcdf(60, .77, 54)