Math 142 - Week in Review #9

1. If $f(x) = 4x^3 - 2x + \sqrt{7}$, give three different functions that are antiderivatives of $f(x)$.

$F(x) = x^4 - x^2 + \sqrt{7} \cdot x$ Check: $F'(x) = 4x^3 - 2x + \sqrt{7}$

$F(x) = x^4 - x^2 + \sqrt{7} \cdot x + 9$

$F(x) = x^4 - x^2 + \sqrt{7}x - 5$

Note: Any constant can be added.

2. Compute each of the following indefinite integrals and give the appropriate units for each.

(a) $\int \left(3e^t + \frac{15}{t} - 5t\right) dt$ births per year, where t is in years

$= \boxed{3e^t + 15\ln|t| - \frac{5}{2}t^2 + C \text{ births}}$

(b) $\int (57 - 3x^{-5} + 2x^{-1})\, dx$ miles per hour, where x is in hours

$= 57x - 3\left(\frac{x^{-4}}{-4}\right) + 2\ln|x| + C$ miles

$= \boxed{57x + \frac{3}{4}x^{-4} + 2\ln|x| + C \text{ miles}}$

(c) $\int \left(5\sqrt{x} - \sqrt{11} + \frac{4}{\sqrt[5]{x}}\right) dx$ students per section per year, where x is in years

$= \int \left(5x^{\frac{1}{2}} - \sqrt{11} + 4x^{-\frac{1}{5}}\right) dx$

$= 5\left(\frac{x^{3/2}}{\frac{3}{2}}\right) - \sqrt{11}\,x + 4\left(\frac{x^{\frac{4}{5}}}{\frac{4}{5}}\right) + C$ students per section

$= \boxed{\frac{10}{3}x^{3/2} - \sqrt{11}\,x + 5x^{4/5} + C \text{ students per section}}$

For 3a, b, c, and f, the "fix it" method is on the left, and substitution is on the right.

3. Compute each of the following indefinite integrals.

(a) $\int 7x^6\sqrt{5+x^7}\,dx$

$= \int (5+x^7)^{\frac{1}{2}}(7x^6)\,dx$

$= \boxed{\frac{2}{3}(5+x^7)^{3/2} + C}$

or

$u = 5+x^7$

$du = 7x^6\,dx$

$= \int u^{1/2}\,du$

$= \frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{2}{3}(5+x^7)^{\frac{3}{2}} + C}$

(b) $\int (x^9+4x)e^{x^{10}+20x^2}\,dx$

$= \frac{1}{10}\int (10x^9+40x)e^{x^{10}+20x^2}\,dx$

$= \boxed{\frac{1}{10}e^{x^{10}+20x^2} + C}$

or

$u = x^{10}+20x^2$

$du = (10x^9+40x)\,dx$

$\frac{1}{10}\int e^{x^{10}+20x^2}(10)(x^9+4x)\,dx$

$= \frac{1}{10}\int e^u\,du$

$= \frac{1}{10}e^u + C = \boxed{\frac{1}{10}e^{x^{10}+20x^2} + C}$

(c) $\int \frac{8t-7}{4t^2-7t}\,dt$

$= \boxed{\ln|4t^2-7t| + C}$

or $\int \frac{8t-7}{4t^2-7t}\,dt$

$u = 4t^2-7t$

$du = (8t-7)\,dt$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C = \boxed{\ln|4t^2-7t| + C}$

(d) $\int \frac{6m}{\sqrt[3]{m-3}}\,dm$

$u = m-3 \Rightarrow m = u+3$

$du = dm$

$= \int \frac{6m}{\sqrt[3]{u}}\,du$

$= 6\int \frac{u+3}{u^{1/3}}\,du$

$= 6\int \left(\frac{u}{u^{1/3}} + 3u^{-1/3}\right)du$

$= 6\int \left(u^{2/3} + 3u^{-1/3}\right)du$

$6\left[\left(\frac{u^{5/3}}{5/3}\right) + 3\left(\frac{u^{2/3}}{2/3}\right)\right] + C$

$= 6\left(\frac{3}{5}u^{5/3} + \frac{9}{2}u^{2/3}\right) + C$

$= \boxed{\frac{18}{5}(m-3)^{5/3} + 27(m-3)^{\frac{2}{3}} + C}$

(e) $\int t^2\sqrt{t+4}\,dt$

$u = t+4 \Rightarrow t = u-4$
$du = dt$

$= \int t^2 \sqrt{u}\,du$

$= \int (u-4)^2 u^{1/2}\,du$

$= \int (u^2-8u+16)u^{1/2}\,du$

$= \int \left(u^{\frac{5}{2}} - 8u^{3/2} + 16u^{1/2}\right)du$

$= \frac{2}{7}u^{\frac{7}{2}} - 8\left(\frac{2}{5}u^{5/2}\right) + 16\left(\frac{2}{3}u^{3/2}\right) + C$

$= \boxed{\frac{2}{7}(t+4)^{\frac{7}{2}} - \frac{16}{5}(t+4)^{\frac{5}{2}} + \frac{32}{3}(t+4)^{\frac{3}{2}} + C}$

(f) $\int \frac{4}{p\ln p}\,dp$

$= 4\int\left(\frac{1}{p}\right)\left(\frac{1}{\ln p}\right)dp$

$= \boxed{4\ln|\ln p| + C}$

or $4\int\left(\frac{1}{p}\right)\left(\frac{1}{\ln p}\right)dp$

$u = \ln p$
$du = \frac{1}{p}\,dp$

$= 4\int \frac{1}{u}\,du$

$= 4\ln|u| + C$

$= \boxed{4\ln|\ln p| + C}$

4. Find $f(x)$ if $f'(x) = 9x^{-1} + 5x^{-2} - 7$ and $f(3) = 10$.

$f(x) = \int f'(x)dx = \int (9x^{-1} + 5x^{-2} - 7)\,dx$

$f(x) = 9\ln|x| + 5\left(\frac{x^{-1}}{-1}\right) - 7x + C$

$f(x) = 9\ln|x| - \frac{5}{x} - 7x + C$

$f(3) = \underbrace{9\ln 3 - \frac{5}{3} - 7(3)}_{} + C = 10$

$-12.7792 + C = 10$

$C = 22.7792$

$\boxed{f(x) = 9\ln|x| - \frac{5}{x} - 7x + 22.7792}$

5. The research department of Acme, Inc. has determined the marginal cost function for one particular item to be $C'(x) = 0.12e^{0.04x}$ dollars per item, where x is the number of items produced. If Acme's fixed costs amount to \$3,000, find a model for the total production cost of this item.

$C(x)$

$C(x) = \int C'(x)dx$

(can let $u = 0.04x$
$du = 0.04dx$)

$= \int 0.12e^{0.04x}\,dx$

$= \frac{0.12}{0.04}\int 0.04e^{0.04x}\,dx$

$C(x) = 3e^{0.04x} + C$

$C(0) = 3000$

$C(0) = 3e^{0.04(0)} + C = 3000$

$\Rightarrow 3 + C = 3000$

$C = 2997$

$C(x) = 3e^{0.04x} + 2997$ dollars,
where x is the number of items produced

6. The rate at which a particular plant grows is given by $r(t) = \frac{1}{2}t^2 + 4$ mm per day, where t is the number of days since the plant was potted in fresh soil, $0 \le t \le 5$.

(a) Compute L_3 to estimate $\int_0^3 r(t)dt$, i.e., to estimate the area under $r(t)$ on the interval $[0,3]$.

Left sum with 3 rectangles

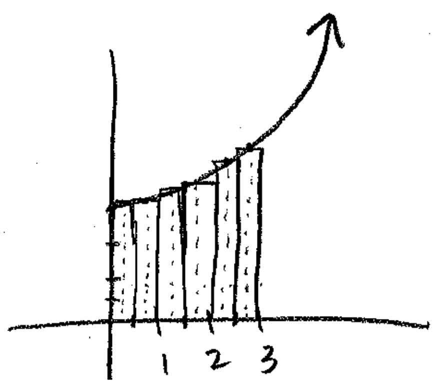
width $= \Delta x = \frac{3-0}{3} = 1$

Left Endpt	Height of Rect.	Width	Area of Rect
0	$r(0) = 4$	1	$4 * 1 = 4$
1	$r(1) = 4.5$	1	4.5
2	$r(2) = 6$	1	+ 6
		Total:	14.5

$\boxed{\text{so } \int_0^3 r(t)dt \approx 14.5}$

(b) Now compute a midpoint sum with 6 rectangles of equal width to estimate $\int_0^3 r(t)dt$.

width $= \frac{3-0}{6} = \frac{1}{2}$



Midpt	height	Width	Area
0.25	$r(0.25) = \frac{129}{32}$	$\frac{1}{2}$	$\left(\frac{129}{32}\right)\left(\frac{1}{2}\right) = \frac{129}{64}$
0.75	$r(0.75) = 137/32$	$\frac{1}{2}$	$\left(\frac{137}{32}\right)\left(\frac{1}{2}\right) = \frac{137}{64}$
1.25	$r(1.25) = 153/32$	$\frac{1}{2}$	$\left(\frac{153}{32}\right)\left(\frac{1}{2}\right) = \frac{153}{64}$
1.75	$r(1.75) = 177/32$	$\frac{1}{2}$	$\left(\frac{177}{32}\right)\left(\frac{1}{2}\right) = \frac{177}{64}$
2.25	$r(2.25) = 209/32$	$\frac{1}{2}$	$\left(\frac{209}{32}\right)\left(\frac{1}{2}\right) = \frac{209}{64}$
2.75	$r(2.75) = \frac{249}{32}$	$\frac{1}{2}$	$\left(\frac{249}{32}\right)\left(\frac{1}{2}\right) = \frac{249}{64}$

$+$

$\frac{527}{32} = \boxed{16.46875}$

so $\int_0^3 r(t)dt \approx \boxed{16.46875}$

(c) Give an interpretation to your answers in (a) and (b).

theses areas give an approximation for the amount that the plant has grown (in mm) during the first 3days once it was potted in fresh soil.

7. Given that $\int_2^7 f(x)dx = -2$, $\int_2^7 g(x)dx = 8$, and $\int_7^9 g(x)dx = 5$, find each of the following.

(a) $\int_2^7 (8g(x) - f(x))\,dx$ $= \int_2^7 8g(x)dx - \int_2^7 f(x)dx$

$= 8\int_2^7 g(x)dx - \int_2^7 f(x)dx$

$= 8(8) - (-2) = \boxed{66}$

(b) $\int_2^9 10g(x)dx$

$= 10\int_2^9 g(x)dx = 10\left(\int_2^7 g(x)dx + \int_7^9 g(x)dx\right)$

$= 10(8+5)$

$= \boxed{130}$

(c) $\int_2^2 7f(x)dx$

$= \boxed{0}$ (since the limits of integration are the same)

(d) $\int_7^2 (f(x) + g(x))\,dx$ $= \int_7^2 f(x)dx + \int_7^2 g(x)dx$

$= -\int_2^7 f(x)dx - \int_2^7 g(x)dx$

$= -(-2) - (8) = \boxed{-6}$

8. Find the average value of $k(x) = 4x^2 - 6x$ on $[2,5]$.

$[a,b]$

$\frac{1}{b-a}\int_a^b \underbrace{f(x)}_{k(x)}dx$ in general

$\frac{1}{5-2}\int_2^5 (4x^2-6x)dx = \frac{1}{3}\text{fnInt}(4x^2-6x, X, 2, 5)$

$= \frac{1}{3}(93) = \boxed{31}$

* Please note the changes made to #9b and #9c.

9. Compute each of the following by hand.

(a) $\int_{-2}^{3}(7x-8e^x)dx$

$$=\left[7\left(\frac{x^2}{2}\right)-8e^x\right]\Big|_{-2}^{3}$$

$$=\left[\frac{7}{2}(3)^2-8e^3\right]-\left[\frac{7}{2}(-2)^2-8e^{-2}\right]$$

$$=\frac{63}{2}-8e^3-14+8e^{-2}$$

$$=\boxed{\frac{35}{2}-8e^3+8e^{-2}}$$

(b) $\int_1^4 \frac{x}{(x^2+9)^5}dx$ $u=x^2+9$

change (was a −) $du=2x\,dx$

$$=\frac{1}{2}\int_1^4 \frac{2x}{(x^2+9)^5}dx$$

$$=\frac{1}{2}\int_{10}^{25}\frac{1}{u^5}du$$ when $x=1$, $u=(1)^2+9=10$; when $x=4$, $u=(4)^2+9=25$

$$=\frac{1}{2}\int_{10}^{25}u^{-5}du$$

$$=\frac{1}{2}\left(\frac{u^{-4}}{-4}\right)\Big|_{10}^{25}=-\frac{1}{8}\left(u^{-4}\Big|_{10}^{25}\right)=-\frac{1}{8}\left(25^{-4}-10^{-4}\right)=\boxed{1.218\times10^{-5}}$$

(c) $\int_1^a 5t(8t^4-7t^{-3})dt$, $a>0$

change (was a 0)

$$=\int_1^a\left(40t^5-35t^{-2}\right)dt$$

$$=\left[40\left(\frac{t^6}{6}\right)-35\left(\frac{t^{-1}}{-1}\right)\right]\Big|_1^a$$

$$=\left[\frac{20}{3}t^6+\frac{35}{t}\right]\Big|_1^a$$

$$=\left(\frac{20}{3}a^6+\frac{35}{a}\right)-\left(\frac{20}{3}(1)^6+\frac{35}{1}\right)$$

$$=\boxed{\frac{20}{3}a^6+\frac{35}{a}-\frac{125}{3}}$$

(d) $\int_{-1}^{2}(x-5)(x^2-10x)^3dx$

$u=x^2-10x$
$du=(2x-10)dx$

when $x=-1$, $u=(-1)^2-10(-1) = 1+10=11$

when $x=2$, $u=(2)^2-10(2) = 4-20 = -16$

$$=\frac{1}{2}\int_{-1}^{2}(2x-10)(x^2-10x)^3dx$$

$$=\frac{1}{2}\int_{11}^{-16}u^3du$$

$$=-\frac{1}{2}\int_{-16}^{11}u^3du$$

$$=-\frac{1}{2}\left[\frac{u^4}{4}\Big|_{-16}^{11}\right]$$

$$=-\frac{1}{8}\left[u^4\Big|_{-16}^{11}\right]$$

$$=-\frac{1}{8}\left(11^4-(-16)^4\right)$$

$$=\boxed{6361.875}$$

10. The rate at which the concentration of a particular drug in the blood stream increases when taken daily can be modeled by

$$r(t)=\frac{2.2}{t}\mu\text{g/mL per day}$$

where t is the number of days since the daily regimen was started, $1 \le t \le 17$.

$\frac{1}{b-a}\int_a^b r(t)dt$ $r(t)$

(a) Find the average rate of change of the concentration of the drug in the blood stream from taking the second dose through taking the eighth dose.

$b=8$ $a=2$

$$\text{Avg rate of change} = \frac{1}{8-2}\int_2^8 \frac{2.2}{t}dt$$

$$= \frac{1}{6}\text{fnInt}(2.2/t, t, 2, 8) = \boxed{0.5083\ \mu g/mL \text{ per day}}$$

(b) If five days after the regimen was started, the concentration of this drug in the blood stream was 4.5 μg/mL, find a model for the concentration of the drug in the bloodstream.

$$c(t)=\int\frac{2.2}{t}dt$$

$$c(t)=2.2\ln|t|+C$$

$$c(5)=2.2\ln(5)+C=4.5$$

$$C=0.9592$$

$\boxed{c(t)=2.2\ln|t|+0.9592\ \mu g/mL,}$ where t is the # of days since the start of the regimen, $1\le t\le 17$.

(c) Find the average concentration of the drug in the bloodstream from taking the third dose through taking the tenth dose.

$b=10$ $\frac{1}{b-a}\int_a^b c(t)dt$ $a=3$

$$=\frac{1}{10-3}\int_3^{10}(2.2\ln t+0.9592)dt$$

$$=\frac{1}{7}\text{fnInt}(2.2\ln t+0.9592, t, 3, 10)$$

$$=\boxed{4.9601\ \mu g/mL}$$

11. The temperature of a cup of coffee can be modeled by $T(x) = 70 + 130e^{-0.05x}$°F where x is the number of minutes since the cup of coffee was poured.

(a) What is the average temperature of the coffee during the first 1.25 hours since it was poured?

1.25 hrs = 75 min

$$\frac{1}{75-0}\int_0^{75}\left(70+130e^{-0.05x}\right)dx$$

$$= \boxed{103.8514\,°F}$$

(b) What is the average rate of change of the coffee's temperature from 0.75 hour to 1.5 hours after it was poured?

r.o.c.

45 min 90 min

Method 1

r.o.c. means deriv.

$$T'(x) = 130(-0.05)e^{-0.05x}$$

$$\text{r.o.c} = T'(x) = -6.5e^{-0.05x}$$

$$\text{Avg r.o.c} = \frac{1}{90-45}\int_{45}^{90}\left(-6.5e^{-0.05x}\right)dx$$

$$= \boxed{-0.2724\,°F \text{ per min.}}$$

Method 2

$$\text{Avg r.o.c.} = \frac{T(b)-T(a)}{b-a}$$

$$= \frac{T(90)-T(45)}{90-45}$$

$$= \frac{71.4442-83.7019}{45}$$

$$= \boxed{-0.2724\,°F \text{ per min}}$$

12. Acme Widget Company's marginal profit is given by $P'(x) = 35e^{-0.01x}$ dollars per widget, ~~where x is the number~~ of widgets produced per day.

(a) If the current production level is 250 widgets per day and the manager wishes to increase production to 275 widgets per day, how will this production increase affect profit? Change in profit

$$\text{Change in profit} = \int_{250}^{275} P'(x)\,dx = \text{fnInt}(35e^{-0.01x}, x, 250, 275)$$

$$= \boxed{\$63.55}$$

If production/sales increases from 250 to 275 widgets, profit will increase by \$63.55.

(b) Find a model for profit if the profit earned by selling 120 widgets is \$300.

$$P(x) = \int P'(x)\,dx$$

$$= \int 35e^{-0.01x}\,dx$$

$u = -0.01x$

$du = -0.01dx$

$$= \frac{35}{-0.01}\int e^{-0.01x}(-0.01)\,dx$$

$$= \frac{35}{-0.01}\int e^u\,du$$

$$= -3500e^u + C$$

$$P(x) = -3500e^{-0.01x} + C$$

$$P(120) = -3500e^{-0.01(120)} + C = 300$$

$$C = 1354.1797$$

$P(x) = -3500e^{-0.01x} + 1354.1797$ dollars where x is the number of widgets produced and sold per day