<u>**Math 166 - Exam 3 Review**</u>

NOTE: For reviews of the other sections on Exam 3, refer to the first page of WIR #7 and #8. Exam 3 covers Sections 7.5, 7.6, and 8.1-8.6.

<u>Section 8.4 - Binomial Distribution</u>

- A *binomial experiment* has the following properties:
  1. The number of trials in the experiment is fixed.
  2. There are two outcomes of the experiment: "success" and "failure."
  3. The probability of success in each trial is the same.
  4. The trials are independent of each other.

- **Notation:** In a binomial experiment it is customary to denote the probability of a success by the letter p and the probability of failure by the letter q.

- <u>Computation of Probabilities in Bernoulli Trials</u> - In a binomial experiment in which the probability of success in any trial is p, the probability of exactly r successes in n independent trials is given by $C(n,r)p^r q^{n-r}$.

- If X is a binomial random variable associated with a binomial experiment consisting of n trials with probability of success p and probability of failure q, then the **mean** (expected value), **variance**, and **standard deviation** of X are

$$\begin{aligned} \mu &= E(x) = np \\ \text{Var}(X) &= \sigma_x{}^2 = npq \\ \sigma_x &= \sqrt{npq} \end{aligned}$$

<u>Section 8.5 - The Normal Distribution</u>

- <u>Properties of the Normal Curve</u>
  1. The normal curve is completely determined by μ and σ. (σ determines the sharpness or flatness of the curve.)
  2. The curve has a peak at $x = \mu$.
  3. The curve is symmetric with respect to the vertical line $x = \mu$.
  4. The curve always lies above the x-axis but approaches the x-axis as x extends indefinitely in either direction.
  5. The area under the curve and above the x-axis is 1.
  6. For any normal curve, 68.27% of the area under the curve lies within 1 standard deviation from the mean, 95.45% of the area lies within 2 standard deviations of the mean, and 99.73% of the area lies within 3 standard deviations of the mean.

- The *standard* normal random variable Z has mean 0 and standard deviation 1.

<u>Section 8.6 - Applications of the Normal Distribution</u>

- When approximating binomial probabilities by using the normal curve, first draw and shade a piece of a histogram corresponding to the probability you are being asked to find, and then use appropriate lower and upper bounds (adjust by 0.5) under the normal curve with $\mu = np$ and $\sigma = \sqrt{npq}$ to approximate the probability.

1. Are mutually exclusive events and independent events the same thing?

events that cannot occur at the same time = mutually excl. events

$(P(A \cap B) = 0)$

Indep events = events that don't affect each other

$$P(A \cap B) = P(A)P(B)$$

2. Jack and Jill are two weather forcasters in Gonzales. The probability that Jack accurately predicts the weather on any given day is 0.68, and the probability that Jill accurately predicts the weather on any given day is 0.72. If the probability that at least one of them is correct on any given day is 0.89, are Jack and Jill making their weather predictions independently?

A - event that Jack accurately predicts weather.

B - — — — Jill — — — — — .

Test for Independence

$$P(A \cap B) \stackrel{?}{=} P(A)P(B)$$

$$.51 \stackrel{?}{=} (.68)(.72)$$

$$.51 \neq .4896$$

$\boxed{\text{Not independent}}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$.89 = .68 + .72 - P(A \cap B)$$

$$P(A \cap B) = .51$$

3. A manufacturer of automobiles receives 500 car radios from each of three different suppliers. The shipment from supplier A contains 5 defective radios, the shipment from supplier B contains 7 defective radios, and the shipment from supplier C contains 2 defective radios. As a means of quality control, one radio is selected at random from each of the shipments. What is the probability that

use independence!

(a) all of the radios selected are working properly?

W_A - event that the radio from supplier A works

W_B - - - - - - - - - - B - - .

W_C - - - - - - - C - - .

$$P(W_A \cap W_B \cap W_C) = \left(\frac{495}{500}\right)\left(\frac{493}{500}\right)\left(\frac{498}{500}\right)$$

$$= \boxed{.9722}$$

(b) at least one of the radios selected is defective?

$$1 - P(\text{no defectives})$$

$$= 1 - P(\text{all work}) = 1 - .9722 = \boxed{.0278}$$

(c) exactly one of the selected radios is defective?

$$P(W_A^c \cap W_B \cap W_C) + P(W_A \cap W_B^c \cap W_C) + P(W_A \cap W_B \cap W_C^c)$$

$$= \left(\frac{5}{500}\right)\left(\frac{493}{500}\right)\left(\frac{498}{500}\right) + \left(\frac{495}{500}\right)\left(\frac{7}{500}\right)\left(\frac{498}{500}\right) + \left(\frac{495}{500}\right)\left(\frac{493}{500}\right)\left(\frac{2}{500}\right)$$

$$= \boxed{.0275}$$

4. The odds against it snowing in College Station next winter are 17 to 2. What is the probability that it will snow in College Station next winter?

won't will

$$P(\text{will snow}) = \frac{2}{17+2} = \boxed{\frac{2}{19}}$$

5. A student takes a 10 question multiple choice exam, each question of which has 5 answer choices (1 correct, 4 incorrect). Being unprepared for the exam, the student randomly guesses at each question. X = number of successes

(a) What is the probability that the student gets exactly 6 questions correct?

Binomial

$n = 10$

"success" - answering correctly

$p = \frac{1}{5}$

Indep?

$P(X=6) = \text{binompdf}(10, \frac{1}{5}, 6)$

$= \boxed{.0055}$

(b) What is the probability that the student gets at least 60% of the questions correct?

(at least 6 questions correct)

$P(X \geq 6) = P(6 \leq X \leq 10)$

$= \text{binomcdf}(10, \frac{1}{5}, 10) - \text{binomcdf}(10, \frac{1}{5}, 5)$

$= \boxed{.0064}$

(c) What is the probability that the student gets the first 4 correct and the last 6 incorrect? use independence.

C_1 - get the 1st question correct etc.

$P(C_1 \cap C_2 \cap C_3 \cap C_4 \cap C_5^c \cap C_6^c \cap \cdots \cap C_{10}^c) =$

$(\frac{1}{5})(\frac{1}{5})(\frac{1}{5})(\frac{1}{5})(\frac{4}{5})(\frac{4}{5}) \cdots (\frac{4}{5}) = (\frac{1}{5})^4 (\frac{4}{5})^6 = \boxed{4.1943 \times 10^{-4}}$

(d) How many questions should the student expect to get correct?

$E(X) = np = 10 * \frac{1}{5} = \boxed{2}$

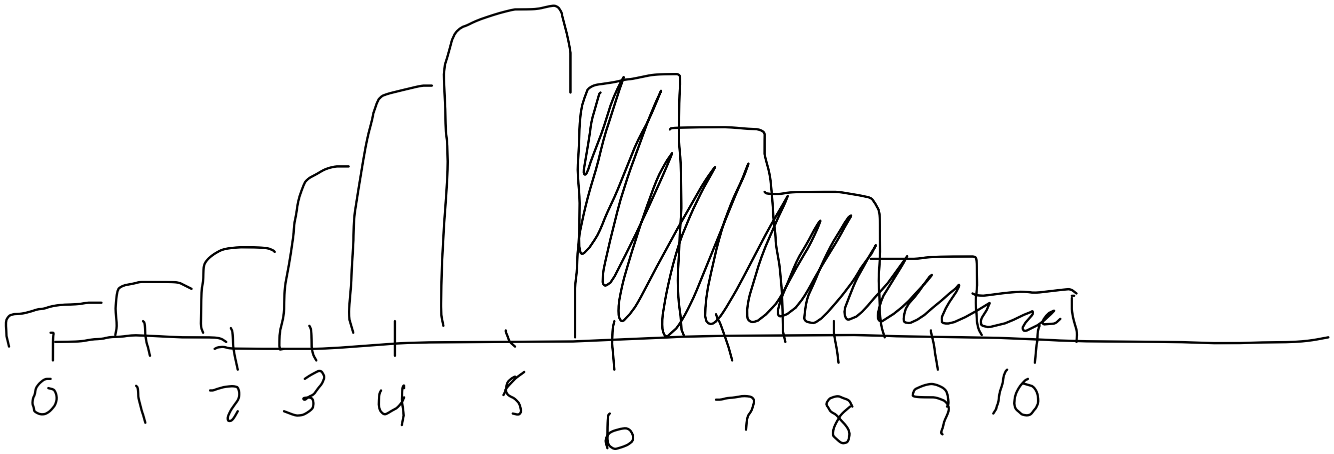
(e) Find the variance and standard deviation for the number of questions answered correctly.

q = probab of failure

$\text{variance} = \sigma_X^2 = npq = 10(\frac{1}{5})(1 - \frac{1}{5})$

$= \boxed{1.6}$

$\text{std. dev} = \sigma_X = \sqrt{npq} = \sqrt{1.6} \approx \boxed{1.2649}$



$$\text{binomcdf}(10, \tfrac{1}{5}, 10) - \text{binomcdf}(10, \tfrac{1}{5}, 5)$$

$$= 1 - \text{binomcdf}(10, \tfrac{1}{5}, 5) = \boxed{.0064}$$

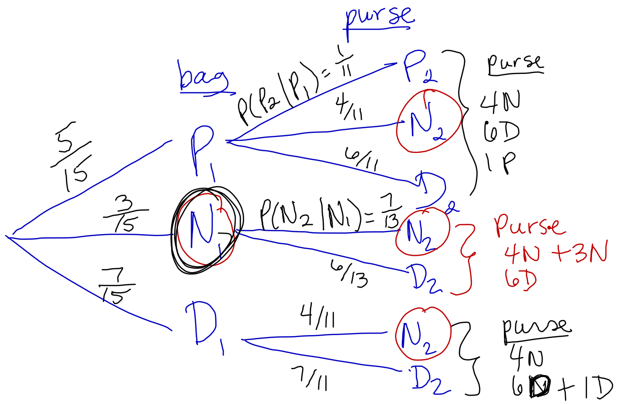
6. A bag contains 5 pennies, 3 nickels, and 7 dimes. A purse contains 4 nickels and 6 dimes. A coin is drawn from the bag and transferred to the purse, but if a nickel is selected from the bag, then all of the nickels ~~in the bag~~ are transferred to the purse. A coin is then drawn from the purse. The type of coin drawn from each of the bag and purse is recorded. What is the probability that

P_1 - penny out of bag
N_1 - nickel out of bag
D_1 - dime out of bag

(a) the transferred coin was a dime if a nickel was selected from the purse?

given

P_2 - penny out of purse
N_2 - nickel — —
D_2 - dime — — — —



$$P(D_1|N_2) = \frac{P(D_1 \cap N_2)}{P(N_2)} = \frac{\left(\frac{7}{15}\right)\left(\frac{4}{11}\right)}{\left(\frac{5}{15}\right)\left(\frac{4}{11}\right) + \left(\frac{3}{15}\right)\left(\frac{7}{13}\right) + \left(\frac{7}{15}\right)\left(\frac{4}{11}\right)}$$

$$= \boxed{\frac{364}{855}}$$

(b) both coins are pennies?

$$P(P_1 \cap P_2) = \left(\frac{5}{15}\right)\left(\frac{1}{11}\right) = \boxed{\frac{1}{33}}$$

(c) the coin drawn from the purse is a penny if the coin drawn from the bag was a nickel?

$$P(P_2 \mid N_1) = 0 \quad (\text{not possible})$$

7. Classify each of the following random variables and give the possible values they each may assume.

(a) X = the number of times a coin is flipped until tails appears.

$X = 1, 2, 3, 4, \ldots$ Infinite Discrete

(b) X = the number of cards drawn without replacement from a standard deck of 52 playing cards until a red card is drawn.

$X = 1, 2, 3, \ldots 27$ finite discrete

(c) X = the weight of a newborn baby.

$X > 0$ continuous

(d) X = the number of hours my cat Mouse sleeps in one day.

$0 \leq X \leq 24$ continuous

$0, 0.1, 0.2,$

(e) X = the number cards drawn with replacement from a standard deck of 52 playing cards until a club is drawn.

$X = 1, 2, 3, 4, \ldots$ Infinite discrete

8. A naval acadamy has a student body that is 80% male. 35% of the males and 30% of the females plan to seek a commission in the United States Marine Corps, and all other students plan to seek a commission in the United States Navy.

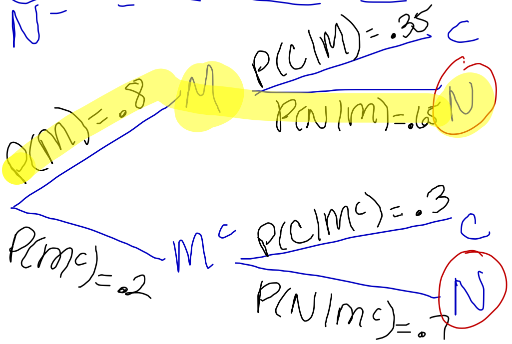
(a) What is the probability that a student at this academy is male or plans to seek a commission in the Navy?

union

M - event that the student is male

C - " " " " " is planning to go to the USMC

N - " " " " " " " US Navy



$$P(M \cup N) = P(M) + P(N) - P(M \cap N)$$

$$= .8 + [(.8)(.65) + (.2)(.7)] - (.8)(.65)$$

$$= \boxed{.94}$$

(b) What is the probability that a student who plans to seek a commission in the Marine Corps is female?

given

$$P(M^c | C) = \frac{P(M^c \cap C)}{P(C)} = \frac{(.2)(.3)}{(.8)(.35) + (.2)(.3)}$$

$$= \boxed{3/17}$$

(c) What is the percentage of students at this academy who plan to join the Marine Corps?

$$P(C) = (.8)(.35) + (.2)(.3) = .34$$

$$\boxed{34\%}$$

9. Let E and F be two events of an experiment with $P(E) = 0.35$, $P(F) = 0.55$, and $P(E \cap F^c) = 0.15$.

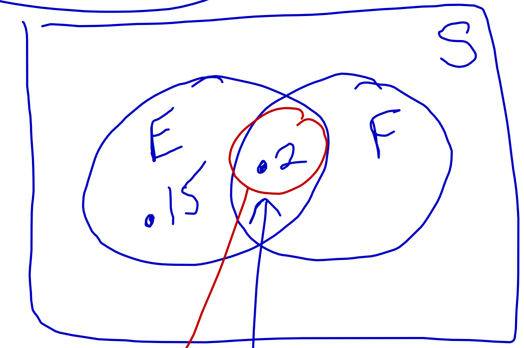
(a) Are E and F independent?

Test

$$P(E \cap F) \stackrel{?}{=} P(E)P(F)$$

$$.2 \stackrel{?}{=} (.35)(.55)$$

$$.2 \neq .1925$$



$.35 - .15 = .2$

$\boxed{\text{not independent}}$

(b) Find $P(E|F)$.

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{.2}{.55} = \boxed{\frac{4}{11}}$$

10. Two cards are drawn at random from a standard deck of 52 playing cards. What are the odds that the second card drawn is a king given that the first card drawn was a queen?

$P(E) = \frac{4}{51}$

odds: $\frac{P(E)}{P(E^c)} = \frac{4/51}{1 - 4/51} = \frac{4/51}{47/51} = \frac{4}{47}$

answer: $\boxed{4 \text{ to } 47}$

11. Fred wants to purchase a 10-year term life insurance policy that will pay his beneficiary \$100,000 in the event that Fred does not survive the next 10 years. Using life insurance tables, he determines that the probability that he will live another 10 years is 0.97. What is the minimum amount that he can expect to pay for his premium?

↖ the amt that would make the insurance company's expected net gain = 0.

a = amt that should be charged so that the co's $\underbrace{\text{expected}}_{E(X)}$ $\underbrace{\text{net gain}}_{X}$ = 0.

X = the insurance company's net gain.

	Value of X	$P(X=x)$
lives	a	.97
dies	$a-100{,}000$	.03

$$E(X) = .97a + .03(a-100{,}000) = 0$$

$$.97a + .03a - 3000 = 0$$

$$\boxed{a = \$3000}$$

12. The following game costs $3 per play: A bag contains 2 gold coins and 28 silver coins. The player grabs two coins from the bag without replacement. A player wins $20 for getting the two gold coins and $5 for getting one gold and one silver coin. There is no prize for selecting two silver coins.

(a) Find the expected net winnings of a person playing this game once. Round to 2 decimal places.

$E(X)$

X = net winnings

outcome	Win	Values of X	$P(X=x)$
2 gold	$20 - 3 =$	17	$P(X=17) = P(2\text{gold}) = \frac{n(2\text{gold})}{n(S)} = \frac{C(2,2)}{C(30,2)} = \frac{1}{435}$
1 gold, 1 silver	$5 - 3 =$	2	$\frac{C(2,1)C(28,1)}{C(30,2)} = 56/435$
2 silver	$0 - 3 =$	-3	$\frac{C(28,2)}{C(30,2)} = 126/145$

$$E(X) = 17\left(\frac{1}{435}\right) + 2\left(\frac{56}{435}\right) + (-3)\left(\frac{126}{145}\right)$$

$$E(X) = \boxed{-\$2.31}$$

(b) How much should be charged to make this game fair? Round to 2 decimal places.

$E(X) = 0$

a = am.t charged to make the game fair.

Value of X	$P(X=x)$
$20-a$	$1/435$
$5-a$	$56/435$
$-a$	$126/145$

$$\frac{1}{435}(20-a) + \frac{56}{435}(5-a) - \frac{126}{145}a = 0$$

$$\boxed{a = \$0.69}$$

13. Random Elementary did a survey of its 55 fourth graders to find out how many siblings each of them have. The following table summarizes this information:

Number of Siblings	0	1	2	3	4	5
Number of Students	10	21	15	3	5	1

(a) Define an appropriate random variable X for this data, and then find $P(X > 2)$.

X = the # of siblings that a 4th grader at Random Elem has.

$P(X>2) = \frac{3+5+1}{55} = \boxed{\frac{9}{55}}$

(b) Find $P(1 \leq X < 4)$.

$P(X=1) + P(X=2) + P(X=3) = \frac{21}{55} + \frac{15}{55} + \frac{3}{55} = \boxed{\frac{39}{55}}$

(c) Find $E(X)$ and interpret its meaning.

$\boxed{E(X) = 1.5455}$

(d) Compute the mean, median, mode, standard deviation, and variance for the data. Be sure to label all answers.

μ = mean = 1.5455
median = 1
mode = 1

std. dev = σ_X = 1.2183
Variance = σ_X^2 = 1.4843

(e) All of the students in the fifth grade class were surveyed in the same way, and it was found that mean number of siblings was 1.7845 with a standard deviation of 1.0043. Which data set has a greater amount of spread (or dispersion) about its mean?

4th graders since their std. dev. is higher

14. A company manufactures one product. For quality control, a random sample of 6 items is selected from a each lot of products made by this company before the lot is shipped. If any defective items are found in the sample, the entire lot is rejected. If 2.3% of the items produced by this company are defective, what is the probability that a lot will be shipped?

E - event that the lot is shipped

$P(E) = P(\text{no defectives in the sample})$

$n = 6$

"success" = getting a defective

$p = .023$

Indep? close enough

$P(X = 0)$

$= \text{binompdf}(6, .023, 0)$

$= \boxed{.8697}$

15. A psychological study has determined that 4.8% of all kindergarteners have Attention Deficit Disorder (ADD). In an elementary school with 115 kindergarteners, find the probability that more than 3% have ADD.

$n = 115$

"success" - having ADD

$p = .048$

Indep ✓

3.45

$P(\text{more than } 3\%) = P(\text{more than } .03 \times 115)$

$= P(X > 3.45)$

$= P(X \geq 4)$

$= 1 - P(X \leq 3)$

$= 1 - \text{binomcdf}(115, .048, 3)$

$= \boxed{.8075}$

binomcdf(115, .045, 115) -
binomcdf(115, .048, 3)

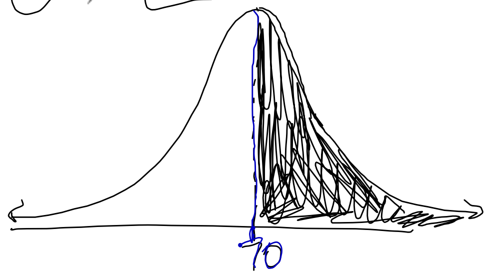
16. The police department of a certain town estimates that 23% of all drivers in their town do not wear their seatbelts. If 60 cars are stopped at random, what is the probability that more than 54 of the drivers are wearing their seatbelts?

Answer: $\boxed{.0028}$

work = 1 − binomcdf(60, .77, 54)

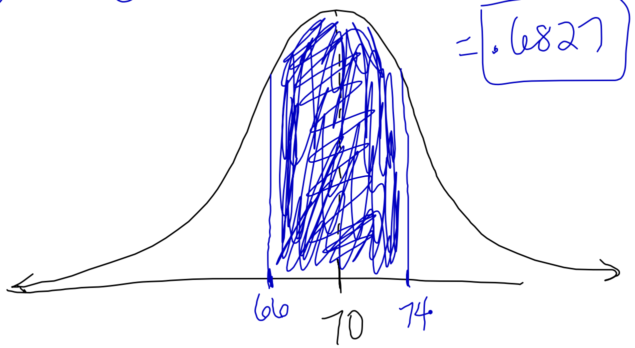
17. Let X be a normal random variable with $\mu = 70$ and $\sigma = 4$. By first sketching a normal curve and shading an appropriate area under the curve, find each of the following probabilities.

(a) $P(X > 70) = \boxed{.5}$ = shaded area

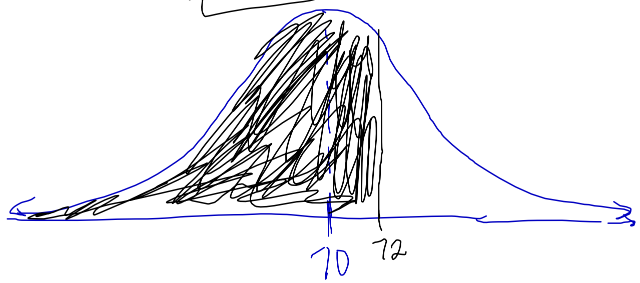


μ = mean
σ = std. dev.

(b) $P(66 < X < 74)$ = shaded area = normalcdf(66, 74, 70, 4)
$= \boxed{.6827}$



(c) $P(X \le 72)$ = normalcdf(−1E99, 72, 70, 4)
$= \boxed{.6915}$



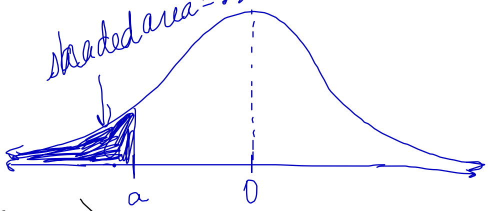
← $\mu = 0$ and $\sigma = 1$

18. Let Z be the standard normal random variable. Find the value of a if

(a) $P(Z < a) = 0.253$ = area under the normal curve

a # on the x-axis

shaded area = .253

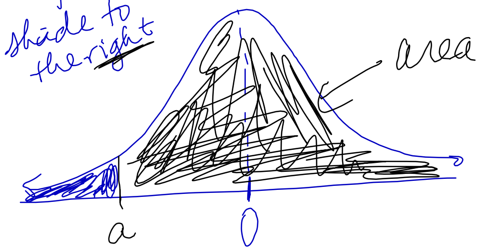


$a = \text{invnorm}(.253, 0, 1) = \boxed{-.66651}$

(b) $P(Z \geq a) = 0.713$ ← = shaded area

shade to the right

area = .713



$a = \text{invnorm}(1 - .713, 0, 1)$
$= \boxed{-.5622}$

(c) $P(-a < Z < a) = 0.35$

area = .35

area = .325

area = .325

$1 - .35 = .65 = 2$ white areas

$.65/2 = .325$

find $a = \text{invnorm}(.35 + .325, 0, 1)$

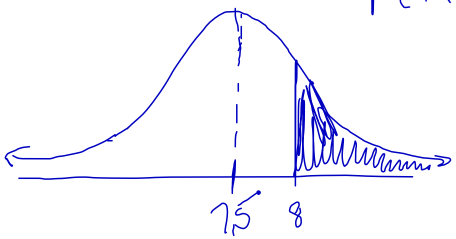
$a = \boxed{.4538}$

19. At a certain hospital, the weights of babies at birth are normally distributed with a mean of 7.5 pounds and a standard deviation of 1.1 pounds.

(a) What is the probability that a randomly selected newborn at this hospital weighs more than 8 pounds?

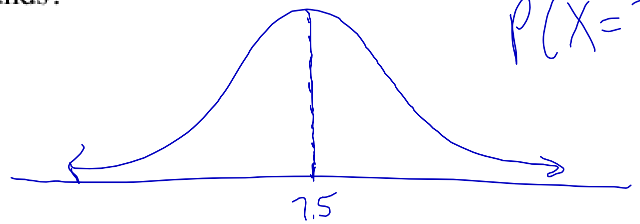
$P(X>8) = \text{normalcdf}(8, 1E99, 7.5, 1.1)$

$= \boxed{.3247}$

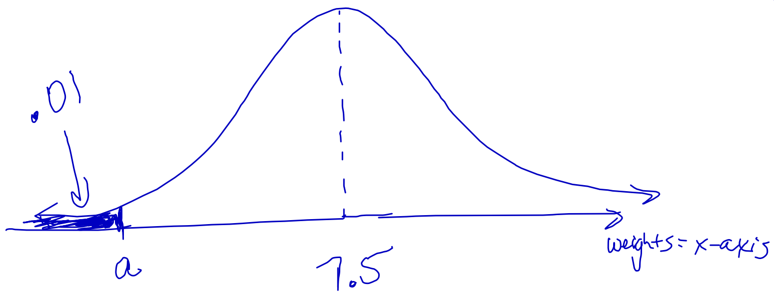


(b) What is the probability that a randomly selected newborn at this hospital weighs exactly 7.5 pounds?

$P(X=7.5) = 0$



(c) Only 1% of all babies born at this hospital weigh less than <u>a</u> pounds.



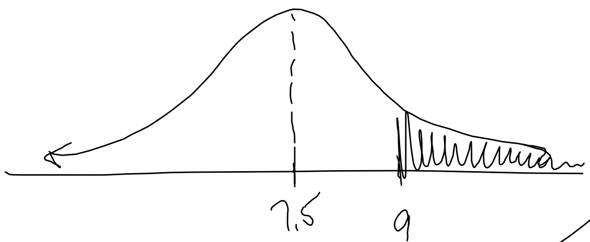
$a = \text{invnorm}(.01, 7.5, 1.1) = \boxed{4.9410 \text{ lbs}}$

(d) 25% of all babies born at this hospital weigh more than $\underline{8.2419}$ pounds.

(e) If you randomly access records of 1,000 newborns born at this hospital, how many of those babies would you expect to weigh more than 9 pounds at birth?

1st find $P(X>9) = \text{normalcdf}(9, 1E99, 7.5, 1.1)$

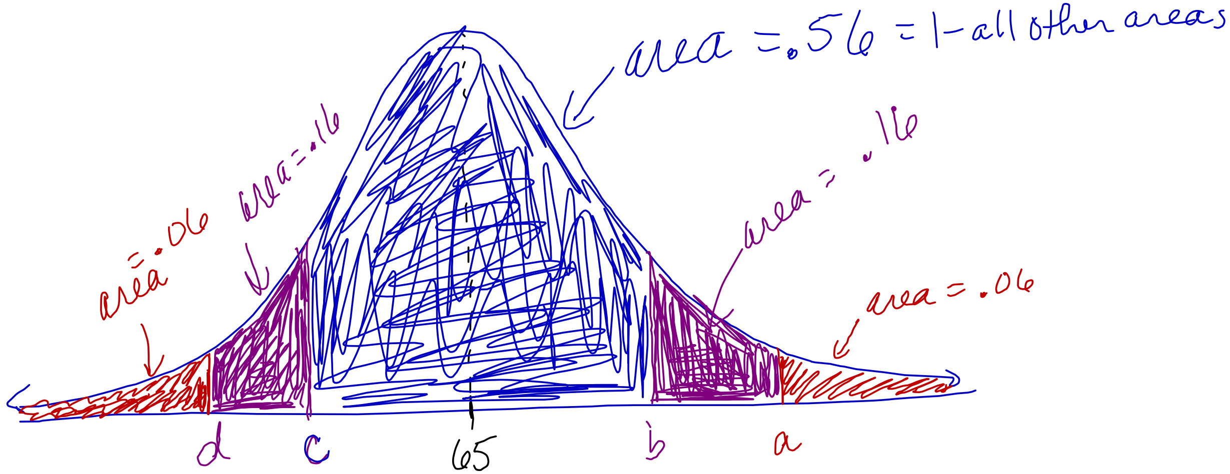
$= .0863$



$1000 * .0863 = 86.3$

physically, 86 babies

20. A physics instructor gave an exam to her class that had an average of 65 and standard deviation of 13. She decided to assign grades as follows: the top 6% and the bottom %6 will receive A's and F's, respectively. The next 16% in either direction will be given B's and D's, and the remaining students will receive C's. Assuming that the grades on the exam are normally distributed, find the cutoffs for each grade level.



a = invnorm(1 − .06, 65, 13) = 85.2121 = cutoff for A's

b = invnorm(1 − .06 − .16, 65, 13) = 75.0385 = cutoff for B's

c = invnorm(.06 + .16, 65, 13) = 54.9615 = cutoff for C's

d = invnorm(.06, 65, 13) = 44.7879 = cutoff for D's

21. In the large city of Winchestertonfieldville, 40% of the drivers exceed the speed limit by 20 mph or more. Use the normal approximation to the binomial distribution for each of the following. Find the probability that among 375 drivers,

(a) at least 195 exceed the speed limit by at least 20 mph.

$n=375$

$P(X \geq 195) = P(195 \leq X \leq 375)$

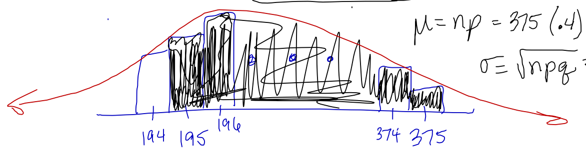
"success" = exceed speed limit by 20 mph or more.

$p = .4$

$P(195 \leq X \leq 375) \approx \text{normalcdf}(194.5, 375.5, 375*.4, \sqrt{375*.4*.6}) = \boxed{1.363 \times 10^{-6}}$

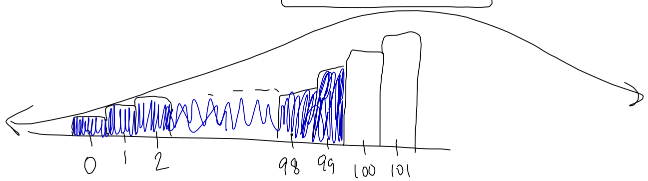
$\mu = np = 375(.4)$

$\sigma = \sqrt{npq} =$



(b) fewer than 100 exceed the speed limit by 20 mph or more.

$P(X < 100) \approx \text{normalcdf}(-.5, 99.5, 375(.4), \sqrt{375(.4)(.6)}) = \boxed{5.1096 * 10^{-8}}$



(c) between 231 and 283 drivers do not exceed the speed limit by 20 mph or more.

not inclusive

Let Y = # of drivers who do <u>not</u> exceed speed limit by 20 mph or more.

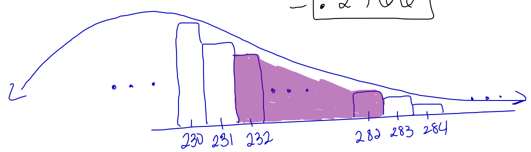
want $P(231 < Y < 283)$

$n = 375$

"success" = <u>not</u> exceeding by more than 20 mph

$p = 1 - .4 = .6$

$P(231 < Y < 283) \approx \text{normalcdf}(231.5, 282.5, 375*.6, \sqrt{375*.6*.4}) = \boxed{.2466}$



22. Fun Trip Ships, Inc. has determined that 7% of the people who book passage on one of their cruises do not arrive for check-in at embarkation. The *Rey del Sol* cruise ship can accommodate 1,320 passengers. If Fun Trip Ships, Inc. books reservations for 1,400 passengers on this ship, what is the probability that the cruise is overbooked? Use the normal approximation to the binomial distribution.

P(overbooked) = ?

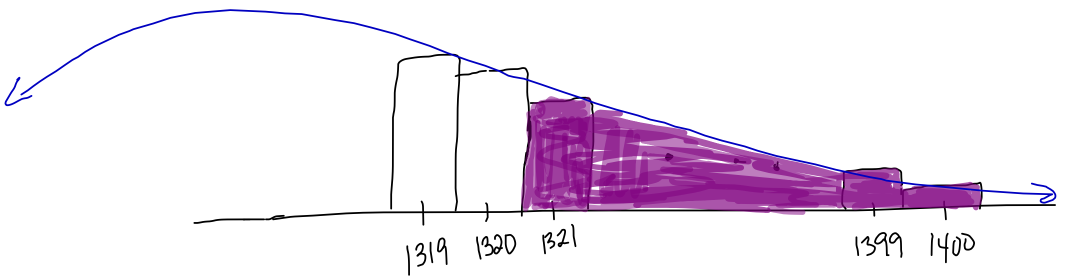
overbooked means more than 1320 $\underbrace{\text{arrived for check-in}}_{\text{"success"}}$

$n = 1400$

$p = 1 - .07 = .93$

$P(X > 1320) \approx \text{normalcdf}(1320.5, 1400.5, (1400)(.93), \sqrt{1400 * .93 * .07})$

$= \boxed{.0263}$



23. A probability distribution has a mean of 100 and a standard deviation of 4. Use Chebychev's Theorem to estimate the probability that an outcome of the experiment lies between 90 and 110.

$\mu = 100$

$\sigma = 4$

$$P(90 \le X \le 110) \approx 1 - \frac{1}{k^2} = 1 - \frac{1}{(2.5)^2} = .84$$

$$\begin{array}{r} 100 \\ -90 \\ \hline 10 \end{array} \qquad \begin{array}{r} 110 \\ -100 \\ \hline 10 \end{array}$$

difference

difference $= \sigma k$

$10 = 4k$

$2.5 = k$

at least .84