Review for Final Exam

**Disclaimer**: This review is more heavily weighted on Chapter 5 (finance), although some problems from other chapters will be included. Please also take a look at the previous Week in Reviews for more practice problems on other chapters.

1. A bank deposit paying simple interest at the rate of $6\%$/year grew to a sum of $\$1300$ in $8$ months. Find the principal.

\[
F = P \left(1 + rt\right) = 1300 = P \left(1 + 0.06 \times \frac{8}{12}\right) \Rightarrow 1300 = 1.04P \Rightarrow P = \frac{1300}{1.04} = \$1,250
\]

2. Determine the simple interest rate at which $\$1200$ will grow to $\$1274$ in $8$ months.

\[
t = 8 \text{ months} = \frac{8}{12} \text{ years}
\]

\[
F = P \left(1 + rt\right) \Rightarrow \frac{1274}{1200} = \frac{1200}{1200} \left(1 + r \left(\frac{8}{12}\right)\right)
\]

\[
1.0617 = 1 + r \left(\frac{66.7}{12}\right) \Rightarrow r = \frac{0.0617}{66.7} = 0.925 \%
\]
3. Bethany, Joshua, Carol, Ray, and Julia have five different savings plans for a Christmas fund. Each person contributes exactly the same amount of money throughout the year, but they do it in different ways. All scenarios receive 3% annual interest, and all involve investing for exactly one year. In each case below, compute the amount of money available at the end of the year. **ROUND ALL DOLLAR AMOUNTS TO TWO DECIMAL PLACES.**

**Bethany** invests $104 per month; her funds are compounded monthly.

\[
N = 12, \quad PMT = 104, \quad I = 3\% \quad \Rightarrow \quad FV = 1265.30
\]

Joshua invests $24 per week; his funds are compounded weekly.

\[
N = 52, \quad PMT = 24, \quad I = 3\% \quad \Rightarrow \quad FV = 1266.54
\]

Carol invests $1248 at the beginning of the year; her money is compounded monthly.

\[
N = 12, \quad PMT = 0, \quad I = 3\% \quad \Rightarrow \quad FV = 1285.96
\]

Ray invests $1248 at the beginning of the year, and receives only simple annual interest.

\[
F = P \left(1 + rt\right) = 1248 \left(1 + 0.03 \times 1\right) = 1.03 \times 1248 = 1285.44
\]

Julia invests $1248 at the beginning of the year; her interest is compounded continuously.

\[
F = Pe^{rt} = 1248 \left(e^{0.03 \times 1}\right) = 1286.00
\]
4. Which lender would you choose for borrowing a loan: one with 6.75% annual interest rate compounded monthly or one with 7% annual interest rate compounded quarterly?

Effective rates

\[ \text{Eff}(6.75, 12) \approx 6.9678 \]  \text{better for borrowing}

\[ \text{Eff}(7, 4) \approx 7.1857 \]

5. The Crown Colony Condo Association is required to set aside funds to replace its common area carpet. Current cost of the carpet is $20,000, and it will need to be replaced in 6 years. The cost of carpet is expected to increase by 2% annually.

(a) What will the carpet cost in 6 years?

\[ \begin{align*}
N &= 6 \\
I &= 2\% \\
PMT &= 0 \\
FV &= 22,523.25 \\
PV &= -20,000 \\
P/Y &= C/Y = 1
\end{align*} \]

\[ F = P \left(1 + \frac{r}{n}\right)^{nt} \]

(b) The condo can invest in an annuity at 5% semiannually. What payment should be made semiannually?

\[ \begin{align*}
N &= 4 \times 2 \\
I &= 5\% \\
PMT &= -1632.65 \\
PV &= 0 \\
FV &= 22,523.25 \\
P/Y &= C/Y = 2
\end{align*} \]

6. Sharon at age 65, can expect to live for 25 years. If she can invest at 10% compounded monthly, how much does she need now to guarantee $500 every month for 25 years?

\[ \begin{align*}
N &= 25 \times 12 \\
I &= 10\% \\
PMT &= 500 \\
FV &= 0 \\
PV &= -55,023.62 \\
P/Y &= C/Y = 12
\end{align*} \]
7. Mr. Smith obtained a 25-year mortgage on a house. The monthly payments are $2247.57 based on a 7% interest rate. How much did he borrow? How much interest will be paid?

\[ \begin{align*}
N &= 25 \times 12 \\
I &= 7\% \\
P &= 0 \\
\frac{P}{Y} &= C/Y = 12 \\
\text{Interest} &= \frac{(2247.57)(2247.57) - 318001.72}{\text{loan}} \\
&= $352,269.28
\end{align*} \]

8. A couple buys a house for $375,000 using a down payment of $80,000. They can amortize the balance at 6.5% compounded monthly for 30 years.

(a) What is the monthly payment?

\[ \begin{align*}
N &= 30 \times 12 \\
I &= 6.5\% \\
P &= 0 \\
P_Y &= C/Y = 12 \\
\text{loan} = 375,000 - 80,000 &= 295,000 \\
\text{monthly pmt.} = \frac{1864.60}{1864.60}
\end{align*} \]

(b) What is the total interest paid?

\[ (30 \times 12)(1864.60) - 295,000 = \frac{376,256}{376,256} \]

(c) What is their outstanding principal and equity after 6 years?

\[ \begin{align*}
N &= 24 \times 12 \\
I &= 6.5\% \\
P &= 0 \\
P_Y &= C/Y = 12 \\
\text{outstanding principal} = \frac{271,592.74}{271,592.74} \quad \text{item value} = \text{outstanding balance} = \frac{375,000 - 271,592.74}{375,000 - 271,592.74} = \frac{103,407.26}{103,407.26}
\end{align*} \]

(d) How much of the 86th payment goes toward interest and how much goes toward the principal?

\[ \begin{align*}
\text{Interest in each pmt.} &= \text{(Periodic rate)} \times \text{Outstanding balance} \\
&= \left(\frac{6.5\%}{12}\right)(266,308.00) = 1442.50 \\
\text{Interest} &= \frac{1864.60 - 1442.50}{1864.60 - 1442.50} = \frac{422.10}{422.10}
\end{align*} \]
9. Five years ago, Diane secured a bank loan of $300,000 to help finance the purchase of a loft in the San Francisco Bay area. The term of the mortgage was 30 years, and the interest rate was 9% per year compounded monthly on the unpaid balance. Because the interest rate for a conventional 30-year home mortgage has now dropped to 5% per year compounded monthly, Diane is thinking of refinancing her property. (Round your answers to the nearest cent.)

(a) What is Diane’s current monthly mortgage payment?

\[ N = 12 \times 30 \quad \rightarrow \quad PMT = -2413.87 \]

\[ PV = 300,000 \quad FV = 0 \quad P/Y = C/Y = 12 \]

\[ \text{monthly payment} \quad \$2,413.87 \]

(b) What is Diane’s current outstanding balance? 30 - 5 = 25 years left

\[ N = 25 \times 12 \quad PMT = -2413.87 \]

\[ I = 9\% \quad FV = 0 \quad PV = 287,640.66 \quad P/Y = C/Y = 12 \]

\[ \text{outstanding balance} \quad \$287,640.66 \]

(c) If Diane decides to refinance her property by securing a 30-year home mortgage loan in the amount of the current outstanding principal at the prevailing interest rate of 5% per year compounded monthly, what will be her monthly mortgage payment? Use the rounded outstanding balance.

\[ N = 30 \times 12 \quad \rightarrow \quad PMT = -1544.12 \]

\[ PV = 287,640.66 \quad FV = 0 \quad P/Y = C/Y = 12 \]

\[ \text{monthly payment} \quad \$1,544.12 \]

(d) How much less would Diane’s monthly mortgage payment be if she refinances? Use the rounded values from parts (a)-(c).

\[ 2413.87 - 1544.12 = \$869.75 \]

(e) How much would Diane save if she refines now?

\[ (25 \times 12)(2413.87) - (30 \times 12)(1544.12) \]

\[ = \$168,227.80 \]

\[ \text{she saves} \quad \$168,227.80 \]

Dec 8-4:06 PM
10. A quality colored pencils manufacturer determines that when the price of a box of pencils is $15.00, the quantity demanded is 1200. When the price is $22.50, the quantity demanded decreases by 100. The supplier is not willing to supply any pencil boxes at a price of $10.00, but will supply 300 pencil boxes at a price of $17.50. What is the market equilibrium for this quality pencil manufacturer?

\[ m = \frac{\Delta Q}{\Delta P} = \frac{1200 - 300}{22.50 - 10.00} = 20 \]

\[ P = \frac{20}{0.25} = 80 \]

\[ Q = 300 \]

11. Find the equation of the line that passes through the x-intercept of the line \(2x - 6y = 18\) and is perpendicular to the line that passes through the points (1, 5) and (3, -4).

\[ \text{Slope of } 2x - 6y = 18 \text{ is } m = \frac{2}{-6} = -\frac{1}{3} \]

\[ \text{Slope of line through (1, 5) and (3, -4) is } m = \frac{\Delta y}{\Delta x} = \frac{-4 - 5}{3 - 1} = -\frac{9}{2} \]

\[ \text{Perpendicular slope } m' = -\frac{1}{m} = -\frac{1}{-\frac{9}{2}} = \frac{2}{9} \]

\[ y - 5 = \frac{2}{9}(x - 1) \]

\[ y = \frac{2}{9}x - \frac{2}{9} + 5 \]

\[ y = \frac{2}{9}x + \frac{37}{9} \]
12. Solve the following systems of linear equations:

(a) $-x + 3y + 2z = 1$
    $x - 2y + z = -5$
    $2x - y = 13$

\[
\begin{bmatrix}
-1 & 3 & 2 \\
1 & -2 & 1 \\
2 & -1 & 0
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

$z = 3.18$

\[
\begin{align*}
x &= 9.27 \\
y &= 5.55
\end{align*}
\]

unique

(b) $2x + 4y - 3z = 10$
    $-x + 2y - 3z = 4$
    $3x - 6y + 9z = 12$

\[
\begin{bmatrix}
2 & 4 & -3 \\
-1 & 2 & -3 \\
3 & -6 & 9
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 1.25 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

no solutions!

(c) $x - 2y + 3z = -4$
    $2x + 6y - 2z = 2$
    $3x + 9y - 3z = 3$

\[
\begin{bmatrix}
1 & -2 & 3 \\
2 & 6 & -2 \\
3 & 9 & -3
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 1.4 \\
0 & 1 & -0.8 \\
0 & 0 & 0
\end{bmatrix}
\]

$z = t$

\[
\begin{align*}
x &= 1.4t - 2 \\
y &= 0.8t + 1
\end{align*}
\]

infinitely many solutions

\[(x, y, z) = (-1.4t - 2, 0.8t + 1, t)\]

for all real # $t$
13. A potter makes only vases and ash trays. (He’s a beginner.) Each vase requires 6 pieces of clay and takes 8 hours to make. Each ash tray requires 4 pieces of clay and takes 2 hours to make. The potter makes a profit of $12 on each vase and $3 on each ash tray. He knows that the number of ash trays made should be at most twice the number of vases made. The potter only has available 42 pieces of clay each week, but wants to work at least 24 hours each week. How many vases and ash trays should the potter make each week in order to maximize profit? Is there any leftover clay at this level of production? What would be the level of production that minimizes profit?

Objective Function

\[ P = 12x + 3y \]

Optimize subject to:

\[ \begin{align*}
6x + 4y &\leq 42 \\
8x + 2y &\geq 24 \\
y &\leq 2x
\end{align*} \]

\[ \text{Graph} \]

C: x-intercept of L2: \[ 6x + 4y = 42 \Rightarrow x = \frac{42}{6} = 7 \] \( (7,0) \)

D: \( (0,0) \) of L3: \[ 8x + 2y = 24 \Rightarrow x = \frac{24}{8} = 3 \] \( (3,0) \)

A: \( \begin{bmatrix} 4 \end{bmatrix} = 2 \)

\[ 8x + 2y = 24 \Rightarrow 8x + 2(2x) = 24 \Rightarrow 12x = 24 \Rightarrow x = 2 \Rightarrow y = 0 \] \( (2,4) \)

B: \( \begin{bmatrix} 2 \end{bmatrix} = 2 \)

\[ 6x + 4y = 42 \Rightarrow 6x + 4(2x) = 42 \Rightarrow 14x = 42 \Rightarrow x = 3 \Rightarrow y = 6 \] \( (3,6) \)

Vertices:

\[ \begin{array}{c|c|c}
\text{Vertices} & \text{Profit} & \text{Comment} \\
\hline
(2,4) & 36 & \text{Maximized} \ ($36 \text{ for } x=2, y=0 \text{ no ash trays) } \\
(3,6) & 54 & \text{Clay consumed} = 6(7) + 4(0) = 42 \text{ available, no leftovers} \\
(7,0) & 84 & \text{Minimized on line segment between } (2,4) \text{ and } (3,0) \text{ (P=54)} \\
(3,0) & 36 & \end{array} \]
14. A survey of 60 customers of an ice cream parlor was done asking which toppings were their favorite, among fruit (F), nuts (N) and sprinkles (S). Six customers only liked nuts, 32 liked sprinkles, 7 liked all three, 7 liked fruit and sprinkles but not nuts, 13 liked nuts and sprinkles, 30 customers liked exactly one of the toppings, 21 liked fruit or nuts but not sprinkles. What is the probability that a customer in this group liked exactly 2 of the toppings?

\[ P(E) = \frac{n(E)}{\text{total #}} = \frac{3 + 7 + 6}{60} = \frac{16}{60} = \frac{4}{15} \]

Remark: To answer the question, it is not necessary to find some other numbers like 12. Only the boxed numbers are needed.
15. Suppose I have 16 DVDs. 5 are comedies, 6 are dramas, and 5 are action movies. My DVD case only holds 9 DVDs. I know that I want 3 comedies, 4 dramas, and 2 action movies in the DVD case. How many ways are there to arrange 9 DVDs in my DVD case if I want the comedies together, the dramas together, and the action movies together?

\[
\# \text{Permutations of } 3 \text{ groups (comedies, dramas, and action movies)} = P(3,3) = 3!
\]
\[
\# \text{Permutations of comedies} = P(5,3)
\]
\[
\# \text{Permutations of dramas} = P(6,4)
\]
\[
\# \text{Permutations of action movies} = P(5,2)
\]

Answer: \( 3! \cdot P(5,3) \cdot P(6,4) \cdot P(5,2) = 2,592,000 \)

16. A bag contains domino tiles: 10 are blue, 9 green, 8 red, and 7 white. A sample of 8 tiles is taken from the bag without looking. What is the probability that the sample contains

(a) exactly 3 green domino tiles or exactly 4 white ones?

\[
P(3G \cup 4W) = P(3G) + P(4W) - P(3G \cap 4W)
\]
\[
= \frac{C(9,3) \cdot C(25,5)}{C(34,8)} + \frac{C(7,4) \cdot C(27,4)}{C(34,8)} - \frac{C(9,3) \cdot C(7,4) \cdot C(18,1)}{C(34,8)}
\]

\[
= \frac{875}{3162} \approx 0.2767
\]

(b) at least 1 red domino tile?

\[
P(\text{no red}) = 1 - \frac{C(26,8)}{C(34,8)} \approx 0.9140
\]
17. Three machines produce widgets:
Machine A makes 40% of the widgets; 6% of these are defective.
Machine B makes 50% of the widgets; 3% of these are defective.
Machine C (the oldest machine) makes 10% of the widgets; 15% of these are defective.
One widget is selected at random.

(a) What is the probability that the widget is defective?

\[
P(D) = (0.4)(0.06) + (0.5)(0.03) + (0.1)(0.15) = 0.054
\]

(b) What is the probability that widget is defective and it was made by machine A?

\[
P(D \cap A) = P(A) \cdot P(D|A) = (0.4)(0.06) = 0.024
\]

(c) If the widget is found to be defective, what is the probability that it was made by machine B?

\[
P(B|D) = \frac{P(B \cap D)}{P(D)} = \frac{0.5 \times 0.03}{0.054} = \frac{5}{18} \approx 0.2778
\]

(Bayes' Theorem)
18. The rates of on-time flights for commercial jets are continuously tracked by the U.S. Department of Transportation. Recently, Southwest Air had the best rate with 80% of its flights arriving on time. A test is conducted by randomly selecting 10 Southwest flights and observing whether they arrive on time.

(a) Find the probability that exactly 2 flights arrive late.

\[ \text{"Success": arriving late } \rightarrow \text{Probability } = 0.2 \]

\[ \binom{10}{2} p^2 (1-p)^8 = \binom{10}{2} (0.2)^2 (0.8)^8 \approx 0.3020 \]

(b) Find the probability that at least 2 flights but fewer than 8 flights arrive on time.

\[ \text{"Success": arriving on time } \rightarrow \text{Probability } = 0.8 \]

\[ P(2 \leq X < 8) = \text{binomcdf}(10, 0.8, 7) - \text{binomcdf}(10, 0.8, 1) \approx 0.3222 \]

(c) What is the expected number, variance and standard deviation of on-time flights in this test?

\[ E(X) = np = (10)(0.8) = 8 \]

\[ \text{Var}(X) = npq = 10 (0.8)(0.2) = 1.6 \]

\[ \sigma_X = \sqrt{\text{Var}(X)} = \sqrt{1.6} \approx 1.265 \]
19. A simple game consists of rolling a pair of dice. The game costs $2 to play. If a double is rolled, you win $4. If the sum of the dice is 9, you win $7. If exactly one two is rolled, you win $1. Otherwise, you win nothing. Let $X$ be the net winnings of a person who plays this game. Find the expected value, standard deviation, and variance of $X$.

\[ n(S) = 6 \times 6 = 36 \]

"Double" = \{ (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) \} \implies P(\text{double}) = \frac{6}{36}

"Sum of 9" = \{ (3,6), (4,5), (5,4), (6,3) \} \implies P(\text{sum of 9}) = \frac{4}{36}

"Exactly one 2" = \{ (2,1), (2,3), (2,4), (2,5), (2,6), (1,2), (3,2), (4,2), (5,2), (6,2) \}

\implies P(\text{exactly one 2}) = \frac{10}{36}

<table>
<thead>
<tr>
<th>$X$</th>
<th>4-2</th>
<th>7-2</th>
<th>1-2</th>
<th>0-2</th>
<th>\rightarrow L_1</th>
</tr>
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<tbody>
<tr>
<td>Prob</td>
<td>6/36</td>
<td>4/36</td>
<td>10/36</td>
<td>16/36</td>
<td>\rightarrow L_2</td>
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\begin{align*}
\text{Double} & \uparrow \quad \text{sum of 9} & \uparrow \quad \text{exactly one 2} & \uparrow \quad \text{all other cases: probabilities must add up to 1.}
\end{align*}

1-var stats $L_1, L_2 \implies E(X) = \bar{X} = -0.28, \sigma \approx 2.3287, \text{Var}(X) = \sigma^2 \approx 5.4228$

Dec 8-4:09 PM
20. The combined math/verbal scores for a standardized college admission test is normally distributed with mean 760 and standard deviation 250.

(a) What score corresponds to the 80th percentile?

\[ P(X \leq a) = 0.8 \Rightarrow a = \text{inv Norm}(0.8, 760, 250) \approx 970.41 \]

(b) What symmetric interval about the mean contains 76% of the scores?

\[ P(X \leq A) = \frac{1 - 0.75}{2} = 0.12 \Rightarrow A = \text{inv Norm}(0.12, 760, 250) = 466.25 \]

\[ B = A + 2(\mu - A) = 466.25 + 2(760 - 466.25) = 1053.75 \]

(c) A college requires a minimum score of 1010 for admission. What percentage of students meet the requirement for entrance to this college?

\[ P(X \geq 1010) = \text{Normalcdf}(1010, 1E99, 760, 250) \]

\[ \approx 0.1587 \]

\[ \Rightarrow 15.87\% \text{ of students meet the requirement}. \]