Week in Review, Sections 3.1-3.3

1. Write a system of inequalities whose solution set is the inside of the triangle with vertices (2, -4), (-2, -1) and (4, 6).

\[ A \text{ to } B : \]
\[ m = \frac{-3}{4} \]
\[ y - (-4) = \frac{-3}{4} (x - 2) \]
\[ y + 4 = \frac{-3}{4} x + \frac{6}{2} \]
\[ y = \frac{-3}{4} x + \frac{3}{2} - 4 \]
\[ y = \frac{-3}{4} x - \frac{5}{2} \]
\[ \text{Inside the triangle: } y > \frac{-3}{4} x - \frac{5}{2} \]

\[ A \text{ to } C : \]
\[ m = \frac{10}{2} = 5 \]
\[ y + 4 = 5 (x - 2) \]
\[ y + 4 = 5x - 10 \]
\[ y = 5x - 14 \]
\[ y > 5x - 14 \]

\[ B \text{ to } C : \]
\[ m = \frac{7}{2} \]
\[ y + 1 = \frac{7}{2} (x + 2) \]
\[ 6y + 6 = 7x + 14 \]
\[ 6y - 7x = 8 \]
\[ 6y - 7x < 8 \]

\( (0, 0) \text{ passes through: } 6y - 7x < 8 \)
2. Graph each of the following sets of inequalities, and indicate whether the solution set is bounded or unbounded.

(a) \[2x - y \geq 4\]
\[4x - 2y < -2\]

- \[x=0 \Rightarrow y = -4\]
- \[y=0 \Rightarrow x=2\]
- \[x=0 \Rightarrow y = 1\]
- \[y=0 \Rightarrow x = -\frac{1}{2}\]

\[m = 2\]

_lines are parallel_

Solution set is empty (no solutions) (bounded)
(b) \[ 2x - y \leq 4 \]
\[ 4x - 2y > -2 \]
(c) \( x + y \geq 20 \)
\[ x + 2y \geq 40 \]
\[ x \geq 0 \text{ and } y \geq 0 \]

\[ x = 0 \Rightarrow y = 20 \]
\[ y = 0 \Rightarrow x = 40 \]

\( S: \text{unbounded} \)
(d) \[3x + 4y \geq 12\]
\[2x - y \geq -2\]
\[0 \leq y \leq 3 \text{ and } x \geq 0\]

\[x = 0 \Rightarrow y = 3\]
\[y = 0 \Rightarrow x = y\]
\[x = 0 \Rightarrow y = 2\]
\[y = 0 \Rightarrow x = -1\]

\[\text{S: unbounded}\]
3. Minimize $C = 10x + 15y$ subject to

- $x + y \leq 10$
- $3x + y \geq 12$
- $2x + 3y \geq 3$
- $x \geq 0$ and $y \geq 0$

Objective function

$x = 0, y = 12$

$y = 0, x = 4$

$3y = 2x + 3$

$x = 0, y = 1$

$y = 0 \Rightarrow x = -3/2$

$x = 2 \Rightarrow 3y = 7$

$y = 7/3$

$x = 2 \Rightarrow 3y = 7$

$y = 7/3$

Feasible region:

S: feasible region

$x + y = 10$

$3x + y = 12$

$x = 0 \Rightarrow y = 10$

$y = 0 \Rightarrow x = 10$

$-2x + 3y = 3$

$x + y = 10$

$-2x + 3y = 3$

$3x + y = 12$

A (3,3)

B (5.4, 4.6)

C (1, 9)

 Corner | $C = 10x + 15y$
|---|---
| (1,9) | 145 |
| (3,3) | 75 |
| (5.4,4.6) | 123 |

Conclusion:

Min $C$ occurs at (3,3)

$\frac{75}{75}$
4. Maximize $P = 2x + 5y$ subject to

\[
\begin{align*}
    x + y & \leq 10 \\
    3x + y & \geq 12 \\
    -2x + 3y & \geq 3 \\
    x & \geq 0 \text{ and } y \geq 0
\end{align*}
\]

\[\{ \text{Same} \}\]

<table>
<thead>
<tr>
<th></th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1,9)$</td>
<td>47</td>
</tr>
<tr>
<td>$(3,3)$</td>
<td>24</td>
</tr>
<tr>
<td>$(5,4,4,6)$</td>
<td>35.8</td>
</tr>
</tbody>
</table>

Max $P = 47 \in (1,9)$
5. A furniture company manufactures dining chairs and tables. Each table requires 40 board feet of wood and 4 labor hours, and each chair requires 16 board feet wood and 3 labor hours. The profit for each table is $45 and the profit for each chair is $20. In a certain week the company has 3200 board feet of wood available and 520 labor hours available. How many tables and chairs should the company manufacture to maximize the profit? What is the maximum profit? What is the leftover of each resource at the maximum level of profit?

\[
\begin{align*}
0 & \leq x, y \\
40x + 16y & \leq 3200 \\
3x + 4y & \leq 520 \\
x, y & \geq 0
\end{align*}
\]

Maximizing the objective function:

\[
P = 45x + 20y
\]

Maximizes at the corner points:

<table>
<thead>
<tr>
<th>Corner</th>
<th>( P = 45x + 20y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>0</td>
</tr>
<tr>
<td>(80,10)</td>
<td>3600</td>
</tr>
<tr>
<td>(40,100)</td>
<td>3800</td>
</tr>
<tr>
<td>(0,130)</td>
<td>2600</td>
</tr>
</tbody>
</table>

Max \( P = 3800 \) occurs at \((40,100)\).

Board feet leftover: \( 3200 - (40x + 16y) = 0 \)

Labor hour leftover: \( 520 - (3x + 4y) = 0 \)
6. Cheryl wants to invest up to $20,000 by purchasing two types of bonds, A and B. Bond type A yields 10% return and Bond type B yields 15% on the amount invested. She wants to invest at least as much in type A bond as in type B. She also wants to invest at least $5,000 in type A and no more than $8,000 in type B. How much should Cheryl invest in each type of bond to maximize the return?

\[
x = \text{amount invested in type A} \\
y = \text{amount invested in type B}
\]
\[
x \geq 0, y \geq 0 \\
x \geq 5,000 \\
y \leq 8,000 \\
x + y \leq 20,000
\]

Return: \[
R = 0.2x + 0.15y
\]

Objective function to maximize

<table>
<thead>
<tr>
<th>Corner</th>
<th>(R = 0.2x + 0.15y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((5000, 0))</td>
<td>1000</td>
</tr>
<tr>
<td>((5000, 5000))</td>
<td>1750</td>
</tr>
<tr>
<td>((8000, 3000))</td>
<td>2800</td>
</tr>
<tr>
<td>((12000, 5000))</td>
<td>3600</td>
</tr>
<tr>
<td>((20000, 0))</td>
<td>4000</td>
</tr>
</tbody>
</table>

Max \(R = 4000\) @ \((20000, 0)\)
7. A nutritionist has been asked to prepare a special diet for certain patients. She has decided that the meals are to be prepared from foods A and B. Each ounce of food A contains 30 mg of calcium, 1 mg of iron, 2 mg of vitamin C, and 2 mg of cholesterol. She has further decided that the meals should contain a minimum of 400 mg of calcium, 10 mg of iron and 40 mg of vitamin C. How many ounces of each type of food should be used so that the cholesterol content is minimized?
8. Perth Mining Company operates two mines for the purpose of extracting gold and silver. The Saddle Mine costs $14,000/day to operate, and it yields 50 oz of gold and 3000 oz of silver each day. The Horseshoe Mine costs $16,000/day to operate, and it yields 75 oz of gold and 1000 oz of silver each day. Company management has set a target of at least 650 oz of gold and 18,000 oz of silver. How many days should each mine be operated so that the target can be met at a minimum cost? What is the minimum cost?