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Let f be a function defined on $[0, \infty)$. Its *Laplace Transform* is defined as the integral:

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For which values of s does the integral

$$\int_{t=0}^{\infty} Ke^{at}e^{-st}dt$$

converge?

Based on this, we have the following theorem:

Theorem.

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Find $\mathcal{L}\{1\}(s)$.

Find $\mathcal{L}\{e^{at}\}(s)$.

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Find $F(s)$ if:

$$f(t) = \begin{cases} 1 & 0 \leq t \leq a \\ 0 & a < t \end{cases}.$$

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Find $\mathcal{L}\{\sin(at)\}(s)$.

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Find the Laplace transform of $3e^{2t} - \sin(5t)$. sol Observe the more general fact:

$$\begin{aligned}\mathcal{L}\{c_1 f_1 + c_2 f_2\}(s) &= \int_{t=0}^{\infty} (c_1 f_1(t) + c_2 f_2(t)) e^{-st} dt \\ &= c_1 \int_{t=0}^{\infty} f_1(t) e^{-st} dt + c_2 \int_{t=0}^{\infty} f_2(t) e^{-st} dt \\ &= c_1 \mathcal{L}\{f_1\}(s) + c_2 \mathcal{L}\{f_2\}(s).\end{aligned}$$

Thus, based on our previous work, the Laplace transform is:

$$\frac{3}{s-2} - \frac{5}{s^2+25},$$

for $s > 2$.

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If we know $\mathcal{L}\{f\}(s)$ and $f(0)$, find $\mathcal{L}\{f'\}(s)$.

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Find the solution to the IVP:

$$x''(t) - x'(t) - 2x(t) = 0, \qquad x(0) = 1, \quad x'(0) = 0.$$