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Find the solution to:

$$2x'' + x' + 2x = u_5(t) - u_{20}(t), \qquad x(0) = x'(0) = 0.$$

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Imagine a setting in which we want to model an external force that happens almost instantaneously (like striking a spring mass system with a hammer). We model this with something called a "unit impluse function". This is a "function" that isn't really a function. It's something that's called a *distribution* which is a "thing" that can be integrated but that doesn't make sense as a function. It is defined as:

$$\int_{t=-\varepsilon}^{\varepsilon} \delta_0 dt = 1,$$

for all $\varepsilon > 0$. Notice that if $a > 0$ then:

$$1 = \int_{t=-(a+h)}^{a+h} \delta_0 dt \geq \int_{t=-\frac{a}{2}}^{\frac{a}{2}} + \int_{t=a}^{a+h} \delta_0 dt = 1 + \int_{t=a}^{a+h} \delta_0 dt,$$

whence $\int_{t=a}^{a+h} \delta(t) dt = 0$ for all $a > 0$. And a similar argument shows this when $a < 0$.
Now, strictly speaking expressions such as $\delta(t)$ are not defined since δ isn't a function. However, a helpful idiom (which we use) is to define $\delta(t)$ as a function with the properties:

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ +\infty & t = 0 \end{cases}$$

$$\int_{t=-a}^{a} \delta(t) dt = 1.$$

Let $a \geq 0$. Find $\mathcal{L}\{\delta(t-a)\}(s)$.

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Solve the IVP:

$$x'' + x' + 2x = \delta(t-5).$$

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Solve

$$x'' + 4x = \sin t - u_{2\pi}(t)\sin(t - 2\pi), \qquad x(0) = x'(0) = 0.$$

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Solve

$$x'' + 3x' + 2x = \delta(t-5) + u_{10}(t), \qquad x(0) = 0, x'(0) = \frac{1}{2}.$$

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