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Let $F(y, x)$ be a function of two variables. If we assume that y, x are functions of t, then by the chain rule:

$$\frac{dF}{dt} =$$

Now, consider the case in which $y = t$, then this becomes:

Now, compare this with the differential equation:

$$M(t) + N(x)\frac{dx}{dt} = 0.$$

Observe that since $\partial_x M = \partial_t N = 0$, we can find a function F such that:

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To find F, observe that:

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Find the integral curves for the equation

$$\frac{dx}{dt} = \frac{t^2}{1 - x^2}.$$

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Consider an ODE written in this form:

$$f'(x)\frac{dx}{dt} = h(t).$$

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Solve the IVP:

$$\frac{dx}{dt} = \frac{3t^2 + 4t + 2}{2(x-1)} \qquad\qquad x(0) = -1.$$

For what values of t is this solution valid?

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The term ”separable” is motivated by the following idiomatic way to solve these equations. As above, consider the equation:

$$f'(x)\frac{dx}{dt} = h(t),$$

where as above f' and h are ”known” and ”$x(t)$” is the ”unknown”. By (formally) multiplying both sides by dt, this can be written in *differential form*:

Observe that this means we write the solution in the form:

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Solve the ODE

$$x' + x^2 \sin t = 0.$$

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We will now briefly look at some modeling. We have seen some models (object falling; basic population dynamics) and as the course goes on, we will see even more. The following is a classic.
At time $t = 0$, a tank contains Q_0 pounds of salt dissolved in 100 gallons of water. Assume that water containing .25 pounds of salt per gallon is entering the tank at a rate of r gallons per minute. The mixture is well–stirred and is leaving the tank from a pipe at the bottom at the same rate. Set up an IVP that models this situation and solve it.