Chapter 2 Sections 4 & 5

There are several important theorems regarding "uniqueness" and "existence" of solutions to differential equations. We discussed some of these ideas implicitly when dealing with exact equations and domains of validity. There are two theorems to discuss:

Theorem.

Example. Solve the IVP:

$$tx'(t) + (t\cos t)x(t) = e^{-\sin t}, \qquad\qquad x(\pi) = 1.$$

Using the equation, what is the largest t interval on which we can be sure a solution exists?

Chapter 2 Sections 4 & 5

The next theorem deals with nonlinear equations. Notice that the previous theorem is a special case of this one:

Theorem.

Example. Consider the IVP:

$$\frac{dx}{dt} = \frac{3t^2 + 4t + 2}{2(x-1)} \qquad x(0) = -1.$$

Find an interval on which there is a unique solution. What happens when the initial condition is changed to $x(0) = 1$?

Chapter 2 Sections 4 & 5

Autonomous Equations are an important class of equations that arise in many areas (e.g. population dynamics). An autonomous equation is an ODE $\frac{dx}{dt} = f(t, x)$ in which $\frac{\partial f}{\partial t} = 0$. That is, $f = f(x)$ is a function of x only. (Notice that we are considering x to be a function of t so f is implicitly or indirectly a function of t. That is why I made the definition by saying $f_x \equiv 0$). When dealing with autonomous equations, the idea is that we can get a lot of information from the equation without actually solving it.

Example. Sketch some solution curves of the equation $\frac{dx}{dt} = 2x(t)$.

Chapter 2 Sections 4 & 5

Example. Sketch some solution curves for solutions to the equation:

$$\frac{dx}{dt} = (1 - x(t))x(t).$$

Chapter 2 Sections 4 & 5

Example. Sketch some solution curves for solutions to the equation:

$$\frac{dx}{dt} = -r(1 - \frac{x(t)}{T})x(t).$$

Determine which solutions are equlibria and their stability.

Chapter 2 Sections 4 & 5

Example. Sketch some solution curves for solutions to the equation:

$$\frac{dx}{dt} = -r(1 - \frac{x(t)}{T})(1 - \frac{x(t)}{K})x(t).$$

Determine which solutions are equlibria and their stability. Assume $T < K$.