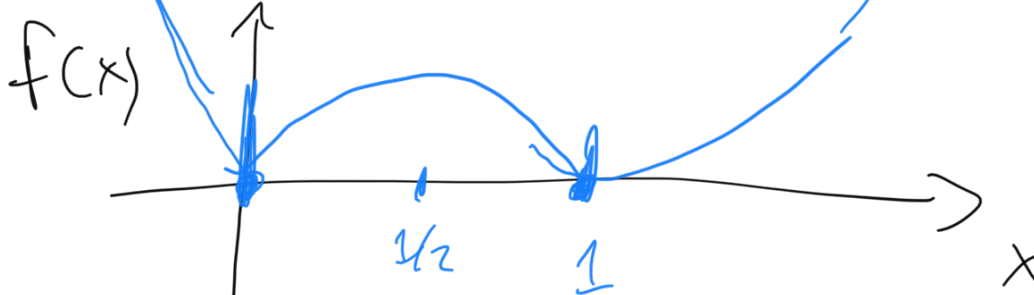


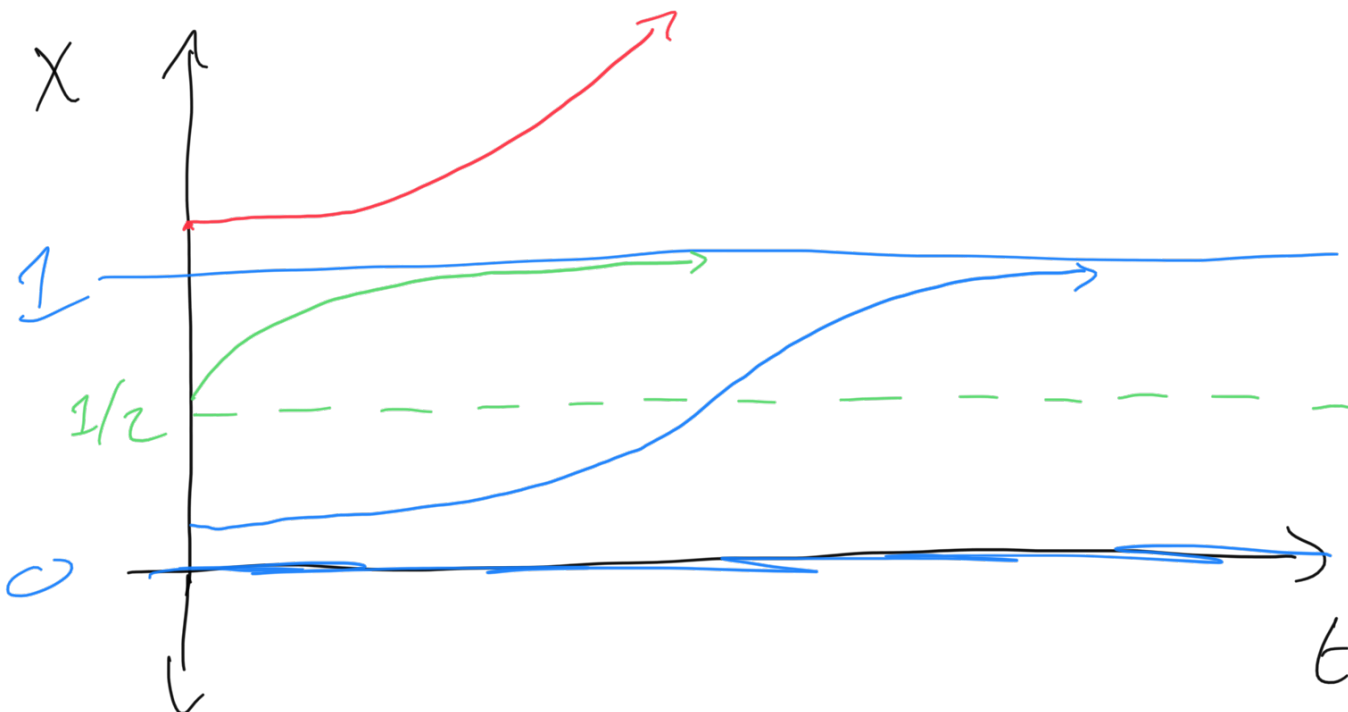
23. (15 points) Consider the following autonomous equation $\frac{dx}{dt} = f(x) = x^2(1-x)^2$.

(a) Draw the phase diagram for this ODE.



(b) Use the phase portrait to fill out the table below indicating whether the given function is positive or negative on the interval (there are more rows than you will need). You may assume that $x(t) \geq 0$ (that is, you don't need to worry about the interval $(-\infty, 0)$).

Interval	$x' = f(x)$	$f'(x)$	$x'' = f'(x)f(x)$	Concavity
$(0, 1/2)$	+	+	+	CU INC
$(1/2, 1)$	+	-	-	INC CD
$(1, +\infty)$	+	+	+	INC CU



$X(t) = 0$ is US

$X(t) = 7$ is SS

(c) Sketch some solution curves. Sketch at least one curve with an initial value in each of the intervals from the table above. (As in class, you only need to sketch solutions in the quadrant $t \geq 0$ and $x \geq 0$.)

(d) Determine the equilibrium solutions.

(e) Determine if each equilibrium solution is stable, unstable, or semi-stable.