Math 308 ODE **Name:** _____
Spring 2021
Exam 2 Review
April 8, 2021
Time Limit: 75 Minutes

This exam contains 14 pages (including this cover page), a table of Laplace transforms and 17 questions.
Total of points is 0.

Write your name on the line above. Each page contains one, two, or three questions. Write your solution on the pages that the problem is on. Use the back of the page if you need more room. Write clearly and neatly. Points may be deducted if your writing is unclear or not well organized. In each problem, you should provide enough justification for your answer; this means showing sufficient work to prove to that you know what you are doing or by providing an argument. Additionally, specific questions may require specific instructions that you must follow.

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1. Write down the definition of the Laplace Transform of a function $\mathrm{f}(\mathrm{t})$.

2. Find the Laplace Transform of $\mathrm{f}(\mathrm{t}) = \sin \mathrm{t}$. You must either do this from the definition (and show your work carefully and do the integration by parts and use the definition of improper integral) or you may use any lines on the Laplace transform table that don't have "$\sin$" in them.

3. Consider the Initial Value Problem:

$$x'' + 2x' + 3x = \sin t, \qquad x(0) = 1, x'(0) = 0.$$

Find $X(s)$, the Laplace transform of the solution of the IVP.

4. Find the inverse Laplace transform of:

$$X(s) = \frac{3s - 7}{s^2 + 2s + 3}.$$

5. Use Laplace Transforms to solve:

$$x''(t) + 5x'(t) + 6x(t) = 0, \qquad x(0) = 0, \quad x'(0) = 1.$$

6. Consider the Initial Value Problem:

$$x'' + 2x' + x = \begin{cases} 0 & 0 \le t < 1 \\ (t-1) & 1 \le t < 2 \\ 1 & 2 \le t \end{cases}, \qquad x(0) = 0, x'(0) = 0.$$

Find $X(s)$, the Laplace transform of the solution of the IVP.

7. Find the inverse Laplace transform of:

$$X(s) = \frac{(e^{-s} - e^{-2s})}{s^2 + 2s + 3}.$$

8. Consider a spring mass system where the mass, damping coefficient, and spring constant are all 1 (in the appropriate units). The system starts at rest at 1 meter below it's equilibrium position. At time $t = 1$, an event happens instantaneously (such as striking the mass with a hammer), that increases the mass's momentum by $1\frac{kg \cdot m}{s}$. The IVP that solves this equation is:

$$x'' + x' + x = f(t), \qquad x(0) = x_0, \quad x'(0) = x_0',$$

where:

$$f(t) =$$

$$x_0 =$$

$$x_0' =$$

9. Solve the following Initial Value Problem:

$$x'' + 2x' + 3x = \delta(t - 10), \qquad x(0) = 1, x'(0) = 0.$$

10. What is the definition of the convolution of $f(t)$ and $g(t)$?

11. Recall that $\mathcal{L}\{f * g\}(s) = F(s)G(s)$. Show that $(f * 1)(t) \neq f(t)$.

12. If $\delta_c(t) = \delta(t-c)$, what does $(f * \delta_c)(t)$ equal in terms of $f(t)$? (Your answer should not have an integral in it!)

13. The general solution to:

$$ax''(t) + bx'(t) + cx(t) = 0$$

is $x(t) = c_1 e^{2t}\cos t + c_2 e^{2t}\sin t$. Using convolution, write the solution to the IVP:

$$ax''(t) + bx'(t) + cx(t) = \cos t, \qquad x(0) = x'(0) = 0.$$

(Note: you may leave your answer as an an evaluated integral!)

14. Consider the ODE:

$$(t-1)x'(t)+x'(t)+x(t)=0.$$

Write the form that the solution will take if expanded in a power series at the point $t_0 = 2$ (don't take any steps to find the coefficients $\{a_k\}$, yet.)

15. Plug this series into the ODE to find a recurrence relation for the coefficients $\{a_k\}$.

16. (Continued from the previous page). Find a_2,a_3,a_4, and a_5 if $a_0 = a_1 = 1$.

17. Consider the ODE:

$$((t+3)^2+5)x''(t)+2x'(t)-x(t)=0.$$

Find a lower bound on the radius of convergence of the power series representation of the general solution expanded at $t=0$.