Math 308 ODE UPDATED **Name:** _____
Spring 2021
Exam 3
5/5/21
Time Limit: 120 Minutes

This exam contains 19 pages (including this cover page) and 18 questions.
Total of points is 0.

Write your name on the line above. Each page contains one, two, or three questions. Write your solution on the pages that the problem is on. Use the back of the page if you need more room. Write clearly and neatly. Points may be deducted if your writing is unclear or not well organized. In each problem, you should provide enough justification for your answer; this means showing sufficient work to prove to that you know what you are doing or by providing an argument. Additionally, specific questions may require specific instructions that you must follow.

No calculators or notes are allowed.

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1. [MC]Find the general solution to:

$$\mathbf{x}'(t) = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \mathbf{x}(t).$$

$\det \begin{pmatrix} 3-\lambda & -2 \\ 2 & -2-\lambda \end{pmatrix} = (\lambda+2)(\lambda-3)+4 = \lambda^2 - \lambda - 2$

$= (\lambda-2)(\lambda+1) = 0 \quad \lambda_1 = -1, \ \lambda_2 = 2$

$\begin{pmatrix} 3-(-1) & -2 \\ 2 & -2-(-1) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad 4x - 2y = 0, \ 2x - y = 0$

$y = 2x \quad \vec{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\begin{pmatrix} 3-2 & -2 \\ 2 & -2-2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad x - 2y = 0, \ 2x - 4y = 0$

$x = 2y \quad \vec{v}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

formula for this is time 1 real evals

$$\vec{x}(t) = \boxed{c_1 e^{\lambda_1 t} \vec{v}_1 + c_2 e^{\lambda_2 t} \vec{v}_2}$$

$$\vec{x}(t) = c_1 e^{-t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

2. (MC points) Find the general solution to $x' = rx$ where r is a constant.

(1) $x' = rx$ $\quad r = 1$ $\quad x(t) = ce^{t}$

$r \neq 1$? $\quad x(t) = ce^{rt}$

(2) $x' - rx = 0$ $\quad \mu(t) = \exp\{\int -r\}$

$= e^{-rt}$

$[xe^{-rt}]' = 0$

$x(t)e^{-rt} = c \Rightarrow x(t) = ce^{rt}$

3. [MC] Find the general solution to:

$$\mathbf{x}'(t) = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}(t).$$

4. [MC] Find the solution to:

$$\mathbf{x}'(t) = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}(t), \qquad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

5. [MC] Use the euler method to estimate $x(.2)$ where x is the solution to the IVP:

$$x' = 2x + 3t, \qquad x(0) = 1.$$

Use a step size of $h = .1$.

6. [MC]Draw the phase plane (that is, sketch some solution curves in the x_1, x_2 plane) for the ODE:

$$\begin{pmatrix} x_1'(t) \\ x_2'(t) \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}.$$

7. [MC]Draw the phase plane (that is, sketch some solution curves in the x_1, x_2 plane) for the ODE:

$$\begin{pmatrix} x_1'(t) \\ x_2'(t) \end{pmatrix} = \begin{pmatrix} 1 & -4 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}.$$

8. [MC]Draw the phase plane (that is, sketch some solution curves in the x_1, x_2 plane) for the ODE:

$$\begin{pmatrix} x_1'(t) \\ x_2'(t) \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}.$$

9. (MC points) When solving the ODE $x'' - 3x' + 2x = e^t$ using the method of undetermined coefficients, what is the form of the solution $x_P(t)$?

10. (MC points) Find the Laplace transform of the solution to the IVP $x' + 2x = e^t$, $x(0) = 1$.

11. [MC] Find the general solution to:

$$(3t^2 - 2tx + 2) + (6x^2 - t^2 + x)\frac{dx}{dt} = 0.$$

12. [MC] Find the general solution to:

$$\frac{dx}{dt} = \frac{2t(x+1)}{1+t^2}.$$

13. [MC] Find the general solution to:

$$x' + tx = t.$$

14. [MC] Find the Laplace Transform of the solution to:

$$x'' + 2x' + x = \delta(t-1) + e^t.$$

15. For the equation $x'(t) + tx = g(t)$, what is the integrating factor?

Work Out Problems

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. Solve the IVP:

$$x'' - 2x' + 2x = \delta(t-1), \qquad x(0) = x'(0) = 0.$$

17. Find the general solution to:

$$x'' - 2x' + x = te^t.$$

18. Find the general solution to:

$$\mathbf{x}'(t) = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \mathbf{x}(t).$$

Also, sketch the phase plane.