Q1. In the region $1 < |z| < 2$, find the Laurent series for

$$\frac{z - 2}{(z + 1)(z + 2)}.$$

Q2. Evaluate

$$\int_0^\infty \frac{\log x}{x^4 + 1} \, dx.$$

Q3. Find all poles and residues of

$$\frac{1}{z^2 \sin z}.$$

Q4. Evaluate

$$\oint_{|z|=1} \frac{z - 2}{\sin^2 z} \, dz.$$

Q5. Evaluate

$$\oint_{|z|=1} \frac{(z + 1)e^{1/z}}{z^3} \, dz.$$

Q6. Find the fourth roots of $1 + i$ in the form $re^{i\theta}$. 
Q7. Find all solutions of
\[ e^{2iz} = 1 - i. \]

Q8. Show that
\[ u(x, y) = x^2 - y^2 - y \]

is a harmonic function and find its harmonic conjugate.

Q9. Find the first three nonzero terms in the Taylor series of \( \tan z \) in powers of \( z \).

Q10.

(i) State the Cauchy-Riemann equations.

(ii) An analytic function \( f(z) = u(x, y) + iv(x, y) \) has the property that 
\( u(x, y) \) is a function \( g(x) \) of \( x \) only, and \( v(x, y) \) is a function \( h(y) \) of \( y \) only. Use the Cauchy-Riemann equations to show that \( f(z) = a + bz \) for some constants \( a \) and \( b \).

Q11. Evaluate
\[ \oint_{|z|=2} \frac{e^z}{z^2 + 1} \, dz. \]

Q12. Evaluate
\[ \oint_{|z|=2} \frac{e^z}{z^2(z + 1)} \, dz. \]