Name_____

There are 5 questions, for a total of 100 points. Point values are written beside each question. Calculators may be used only for basic arithmetic operations. *Show your work for full credit.*

1. (a) [7 points] Find a unit vector that has the same direction as $\langle 1, -2 \rangle$.

   (b) [7] Determine whether the following vectors parallel, perpendicular, or neither:

   $$2\mathbf{i} - 3\mathbf{j}, \quad 6\mathbf{i} + 4\mathbf{j}$$

   (c) [7] Given the vector equation $\mathbf{r}(t) = \langle 1 - t, \ 2t^2 + 1 \rangle$, find the corresponding Cartesian equation (i.e. involving only x and y).

2. (a) [5] State the definition of *limit*, that is $\lim_{x \to a} f(x) = L$ means

(b) [14] Prove $\lim_{x \to -2} (1 - 4x) = 9$ using the definition of limit.

3. Find the following limits. (You need not give proofs.)

(a) [10] $\lim\limits_{x \to 1} \left(4 - \dfrac{8x}{x+1} \right)$

(b) [10] $\lim\limits_{x \to \frac{1}{2}} \dfrac{2 - 4x}{\sqrt{2x} - 1}$

3. (continued)

(c) [10] $\displaystyle\lim_{x\to\infty} \frac{1 - 2x + 3x^2}{1 - \sqrt{2x^4 + 1}}$

4. [10] Use the Squeeze Theorem to find $\displaystyle\lim_{x\to 0} x^4 \cos\left(\frac{1}{x}\right)$. Justify your answer.

5. (a) [5] State the definition of continuity at a number a, that is, a function f is *continuous* at a means

(b) [5] Sketch the graph of $f(x) = \begin{cases} x^2 - 1 & , \quad \text{if } x \leq 0 \\ 1 & , \quad \text{if } 0 < x \leq 1 \\ 2 - x & , \quad \text{if } x > 1 \end{cases}$

(c) [5] Let f be the function from part (b). Is f continuous at 0? Explain why or why not.

(d) [5] Let f be the function from part (b). Is f continuous at 1? Explain why or why not.