Math 171 Final Exam
May 9, 2007
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Name_____

There are 9 questions, for a total of 200 points. Point values are written beside each question. Calculators may be used only for basic arithmetic operations. *Show your work for full credit.*

1. (a) [5] State the definition of *limit*, that is $\lim_{x \to a} f(x) = L$ means

(b) [15] Prove $\lim_{x \to 2}(-3x + 3) = -3$ using the definition of limit.

2. Find the following limits. Justify your answers.

(a) [10] $\lim_{x\to\infty} \dfrac{1-x+x^2}{2x^2+1}$

(b) [10] $\lim_{x\to 0^+} x^2 \cos\left(\dfrac{1}{\sqrt{x}}\right)$

3. (a) [5] State the definition of *derivative*, that is $f'(x) =$

(b) [15] If $f(x) = 1 - 3x^2$, find $f'(x)$ using the definition of derivative.

4. Differentiate the following functions.

(a) [10] $f(x) = x^3 \ln x$

(b) [10] $f(x) = \dfrac{\sin x}{1 + \cos x}$

4. (continued)
(c) [10] $f(x) = \tan^{-1}(2e^x)$

5. [20] Find an equation of the line tangent to the curve $\mathbf{r}(t) = \langle t^3 + t, 1 - t - 2t^2 \rangle$ at the point $\langle 0, 1 \rangle$.

6. [20] A ladder 5 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 m/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 3 m from the wall?

7. Let $f(x) = 2x^3 + 3x^2 - 12x + 6$.

(a) [10] Find all local maximum and minimum values of f.

(b) [10] Determine the intervals on which f is increasing or decreasing.

8. [20] If 500 cm^2 of cardboard is available to make a box with a square base and an open top, find the largest possible volume of the box.

9. Evaluate the following integrals.

(a) [10] $\int_{-2}^{0} \sqrt{4-x^2}\, dx$

(b) [10] $\int_{0}^{2} |2x-1|\, dx$

(c) [10] $\int_{1}^{e} \frac{\ln x}{x}\, dx$