

Math 171H Exam 1
September 26, 2014
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Name _____

There are 5 questions, and a total of 100 points. Point values are written beside each question. Calculators may be used only for basic arithmetic operations. *Show your work for full credit.*

1. (a) [5 points] State the definition of *limit*, that is $\lim_{x \rightarrow a} f(x) = L$ means

(b) [15] Prove $\lim_{x \rightarrow 3} (2x^2 - x) = 15$ using the definition of limit.

2. Find the following limits. (You need not give proofs.)

(a) [10] $\lim_{x \rightarrow 1} \frac{1 - \frac{1}{x^2}}{\frac{1}{x} - 1}$

(b) [10] $\lim_{x \rightarrow 0} \left(\frac{1}{x\sqrt{1+x}} - \frac{1}{x} \right)$

2. (continued)

$$(c) [10] \lim_{x \rightarrow \infty} \frac{1 + x - 5x^2}{\sqrt{4x^4 + 2} - 1}$$

3. [10] For each of the following functions, determine whether it is continuous at $x = 0$. Write “yes” in the blank if it is continuous, and “no” if it is not. You need not justify your answers.

$$\text{_____ (a) } f(x) = \begin{cases} x \sin(\frac{1}{x}), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

$$\text{_____ (b) } f(x) = \begin{cases} \frac{|x|}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

$$\text{_____ (c) } f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ -x, & \text{if } x \text{ is irrational} \end{cases}$$

4. (a) [5] State the definition of *derivative*, that is $f'(x) =$

(b) [15] If $f(x) = \sqrt{x} + 1$, find $f'(x)$ using the definition of derivative.

5. **True/False/Justify.** Determine whether each statement is true or false, and write “T” if true, and “F” if false, in the blank. Briefly **justify** each true statement with one or two sentences explaining why it is true. (For example, if it follows from a theorem we covered, give either a rough statement of the theorem or its name, if it has a name, and explain how it applies. If it follows from a definition or formula, give the definition or formula and show how it applies.) For each false statement, give a **counterexample** to show that it is false.

_____ (a) [5] For all vectors \mathbf{a} and \mathbf{b} , the vector $\text{proj}_{\mathbf{a}}\mathbf{b}$ is parallel to \mathbf{a} .

_____ (b) [5] If f is a function that is continuous on the interval $[0, 1]$, then f is continuous on the interval $[-1, 1]$.

_____ (c) [5] If f is a function for which $f(0) = -2$ and $f(1) = 2$, then there is a number c in the interval $(0, 1)$ for which $f(c) = 0$.

_____ (d) [5] If f and g are functions that are continuous on the interval $[0, 1]$ for which $f(0) < g(0)$ and $f(1) > g(1)$, then there is a number c in the interval $(0, 1)$ for which $f(c) = g(c)$.