MATH 171 EXAM 1 2014 SOLUTIONS

1. (a) orthogonal to $\langle 3, 2\rangle$ is $\langle -2, 3\rangle$

$r(t) = \langle -1, 1\rangle + t\langle -2, 3\rangle = \langle -1-2t, 1+3t\rangle$

$x = -1-2t$

$y = 1+3t$

(b) (i) Cartesian equation: $y = 2-x$; (ii) Cartesian equation: $y = -s = -(x-2) = -x+2$

The curve is the same, as the Cartesian equations are the same.

2. (a) For every $\varepsilon > 0$, there is a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x-a| < \delta$.

(b) Note that $\left|\frac{5}{2}x + 1 - 6\right| = \left|\frac{5}{2}x - 5\right| = \frac{5}{2}|x-2|$. So, given $\varepsilon > 0$, we may let $\delta = \frac{2}{5}\varepsilon$. Then whenever $0 < |x-2| < \delta$, we have $\left|\frac{5}{2}x+1-6\right| = \frac{5}{2}|x-2| < \frac{5}{2}\cdot\frac{2}{5}\varepsilon = \varepsilon$.

3. (a) $\displaystyle \lim_{x\to 1} \frac{x^3-1}{x^2-1} = \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{x^2+x+1}{x+1} = \frac{3}{2}$

(b) $\displaystyle \lim_{x\to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x-3} \cdot \frac{3x}{3x} = \lim_{x\to 3} \frac{3-x}{3x(x-3)} = \lim_{x\to 3} \frac{-1}{3x} = -\frac{1}{9}$

(c) $\displaystyle \lim_{x\to\infty} \frac{1-x-3x^2}{\sqrt{4x^4+1} - 1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\frac{1}{x^2} - \frac{1}{x} - 3}{\sqrt{4+\frac{1}{x^4}} - \frac{1}{x^2}} = -\frac{3}{\sqrt{4}} = -\frac{3}{2}$

4. Let $f(x) = x^3 - x - 1$. Then $f(1) = -1$ and $f(2) = 5$. By the Intermediate Value Theorem, there is a number c in the interval $(1, 2)$ such that $f(c) = 0$, so c is a solution of the equation.

5. (a) $\displaystyle \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ or $\displaystyle \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$

(b) $$f'(x) = \lim_{h\to 0} \frac{(2(x+h)^2 - 1) - (2x^2 - 1)}{h} = \lim_{h\to 0} \frac{2x^2 + 4xh + h^2 - 1 - 2x^2 + 1}{h}$$

$$= \lim_{h\to 0} \frac{4xh + h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(4x+h)}{h}$$

$$= \lim_{h\to 0} (4x+h)$$

$$= 4x$$