Math 365 Final Exam
December 10, 2012
S. Witherspoon

Name_____

There are 14 questions, for a total of 100 points. Point values are written beside each question. *No calculators allowed. Show your work for full credit.*

1. [8 points] Calculate the following in base 5. Show all work in base five (not just a conversion to base ten and back).

(a) $2013_{\text{five}} - 142_{\text{five}}$

(b) $21_{\text{five}} \cdot 34_{\text{five}}$

2. [6] *Without* computing each sum, find which is greater, S or T, and by how much:

$$\begin{aligned} S &= 1 + 4 + 9 + 16 + \cdots + 10,000 \\ T &= 2 + 5 + 10 + 17 + \cdots + 10,001 \end{aligned}$$

3. [8] Consider the following proposition about all whole numbers n.

$$p: \text{ If } n \text{ is a multiple of 4, then } n \text{ is even.}$$

(a) Is p true? If not, give a counterexample.

(b) State the *converse* of p. Is it true? If not, give a counterexample.

4. [6] How many one-to-one correspondences are there between the sets $\{1, 2, 3, 4, 5, 6\}$ and $\{a, b, c, d, e, f\}$ if in each correspondence, each multiple of 3 must correspond to a vowel?

5. [6] Find a digit to fill in the blank, if possible, so that the number

$$613 \_\_\_ 6$$

is divisible by

(a) 4

(b) 9

(c) 11

6. [8] Write each of the following in simplest form:

(a) $6^{-5} \cdot 6^7 \div 2^3$

(b) $3\frac{1}{3} \div \frac{2}{9}$

7. [7] Which of the following are rational numbers? Circle all those that are.

$$\frac{2}{5} \qquad \frac{10}{21} \qquad 3.14 \qquad \pi \qquad \sqrt{96} \qquad \sqrt{196} \qquad \frac{\sqrt{2}}{2} - \frac{1}{\sqrt{2}}$$

8. [10] Convert the following repeating decimal to a fraction (you need not simplify):

$$3.2\overline{15}$$

9. [6] Eighteen-karat gold contains 18 parts gold and 6 parts other metals. If a ring contains 12 parts gold and 3 parts other metals, is it 18-karat gold? Justify your answer.

10. [6] Find the sum $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$

11. [10] In an arithmetic sequence, the sum of the 11th and 21st terms is 94. The 21st term minus the 11th term is 30. Find the first term of the sequence.

12. [6] For a particular event, 250 tickets were sold, for a total of \$1,500. If students paid \$5 per ticket and nonstudents paid \$10 per ticket, how many student tickets were sold?

13. [6] For each of the following sequences (either arithmetic or geometric), find a function $f(n)$ whose domain is the set of natural numbers, and whose outputs are the terms of the sequence.

(a) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$

(b) $10,\ 3,\ -4,\ -11, \ldots$

14. [7] (**True/False.**) For each of the following statements, write "T" if it is true and "F" if it is false. (You need not give counterexamples for false statements.)

(a) _____ For all sets A, B, C, if $A \cup B = A \cup C$, then $B = C$.

(b) _____ For all integers a and b, if both a and b are even, then $\text{GCD}(a, b) = 2$.

(c) _____ For all integers a and b, if b divides a, then b divides $a + b$.

(d) _____ For all integers a, b, and prime numbers p, if p divides ab, then p divides a or p divides b.

(e) _____ For all integers a and b, $|a - b| = |b - a|$.

(f) _____ The sum of any two irrational numbers is an irrational number.

(g) _____ $0.\overline{3} = \dfrac{1}{3}$