ERRATUM TO “FINITE GENERATION OF THE COHOMOLOGY OF SOME SKEW GROUP ALGEBRAS”

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Abstract. For the class of examples considered in [3, Section 5], the proof of finite generation of cohomology is incomplete. We give here a proof of existence of a polynomial subalgebra needed there. The rest of the proof of finite generation in [3, Section 5] then applies.

Let \( k \) be a field of characteristic \( p > 2 \). Let \( A \) be the augmented \( k \)-algebra generated by \( a \) and \( b \), with relations

\[
a^p = 0, \quad b^p = 0, \quad ba = ab + \frac{1}{2}a^2,
\]

and augmentation \( \varepsilon : A \rightarrow k \) given by \( \varepsilon(a) = \varepsilon(b) = 0 \). Let \( G \) be a cyclic group of order \( p \) with generator \( g \), acting on \( A \) by

\[
g(a) = a, \quad g(b) = a + b.
\]

The corresponding skew group algebra \( A \#_kG \) is a pointed Hopf algebra described in [1, Corollary 3.14]. We remark that in [3, Section 4], we used the left \( G \)-module structure with \( g(a) = a \) and \( g(b) = b - a \), whereas the authors in [1, 2] used the right \( G \)-module structure given as above. We will apply the results in [2] to prove that the cohomology \( H^*(A \#_kG, k) := \text{Ext}^*_A(k, k) \) is finitely generated, and this will fill a gap in the proof in [3, Section 5]. Thus we will now also adopt the choices of group actions in [1, 2] instead of that in [3]. This change does not affect the results discussed in [3, Section 4].

Let \( k \) be an \( A \#_kG \)-module via the augmentation map \( \varepsilon \). To prove finite generation of \( H^*(A \#_kG, k) \), we wish to apply [3, Theorem 3.1]. We use results in [2], where the notation is slightly different, with \( x \) in place of \( a \) and \( y \) in place of \( b \). There it is shown that there are 2-cocycles \( \xi_a, \xi_b \) in \( H^*(A, k) \) generating a polynomial subring \( k[\xi_a, \xi_b] \). These 2-cocycles are not both \( G \)-invariant, as was claimed in [3]: Specifically, in [2] it is shown that \( \xi_a \) is \( G \)-invariant while \( \xi_b \) is not. The claimed \( G \)-invariance was used in [3, Section 5] to show that \( \xi_a \) and \( \xi_b \) are in the image \( \text{Im}(\text{res}_{A \#_kG, A}) \) of the restriction map from \( H^*(A \#_kG, k) \) to \( H^*(A, k) \). However, results in [2, Section 5.1] imply directly that \( \xi_a, \xi_b \) are in \( \text{Im}(\text{res}_{A \#_kG, A}) \); the
needed elements in $H^\ast(A \# kG, k)$ are constructed explicitly using a twisted tensor product resolution in [2, Section 3.3]. Now the rest of the finite generation proof in [3, Section 5] can proceed as before, since it is shown there that the rest of the hypotheses of [3, Theorem 3.1] are satisfied. An alternative proof is given in [2, Section 5.1].

References


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