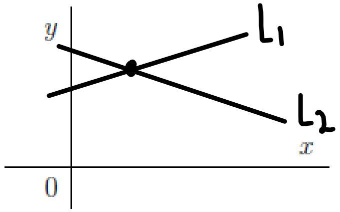
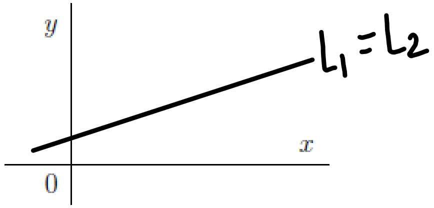
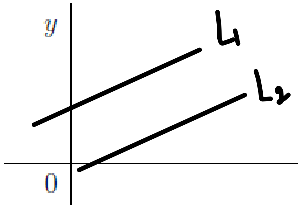
2: SYSTEMS OF LINEAR EQUATIONS AND MATRICES

2.1: Systems of Linear Equations - Introduction

Given two lines L_1 and L_2:

$$\begin{aligned} L_1 : & \left\{ \begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2 \end{aligned} \right. \\ L_2 : & \end{aligned}$$

One and only one of the following may occur:

Lines	System
L_1 and L_2 intersect at exactly one point 	has unique solution
L_1 and L_2 are parallel and coincident 	has infinitely many solutions
L_1 and L_2 are parallel and distinct 	has no solution

EXAMPLE 1. *Solve:*

(a) $\begin{aligned} x+y &= 1 \\ x-y &= 3 \end{aligned}$

$x+\cancel{y}+x-\cancel{y}=1+3$

$2x=4$

$x=4/2=2$

$y=1-x=1-2=-1$

$(x,y)=(2,-1)$

unique solution

(b) $\begin{aligned} x+y &= 1 \\ 2x+2y &= 2 \end{aligned}\ \left(\times\frac{1}{2}\right)$

$x+y=1$

$x+y=1$ infinitely many solutions

$x=t$ an arbitrary real number

$y=1-x \Rightarrow y=1-t$

Pattern $\underline{(x,y)=(t,1-t)}$

If you need to know one particular solution then

$t=3 \Rightarrow (x,y)=(3,1-3)=\boxed{(3,-2)}$

(c) $\left.\begin{aligned} x+y &= 1 \\ x+y &= 2 \end{aligned}\right\} \Rightarrow$

$\Rightarrow 1=2$

impossible

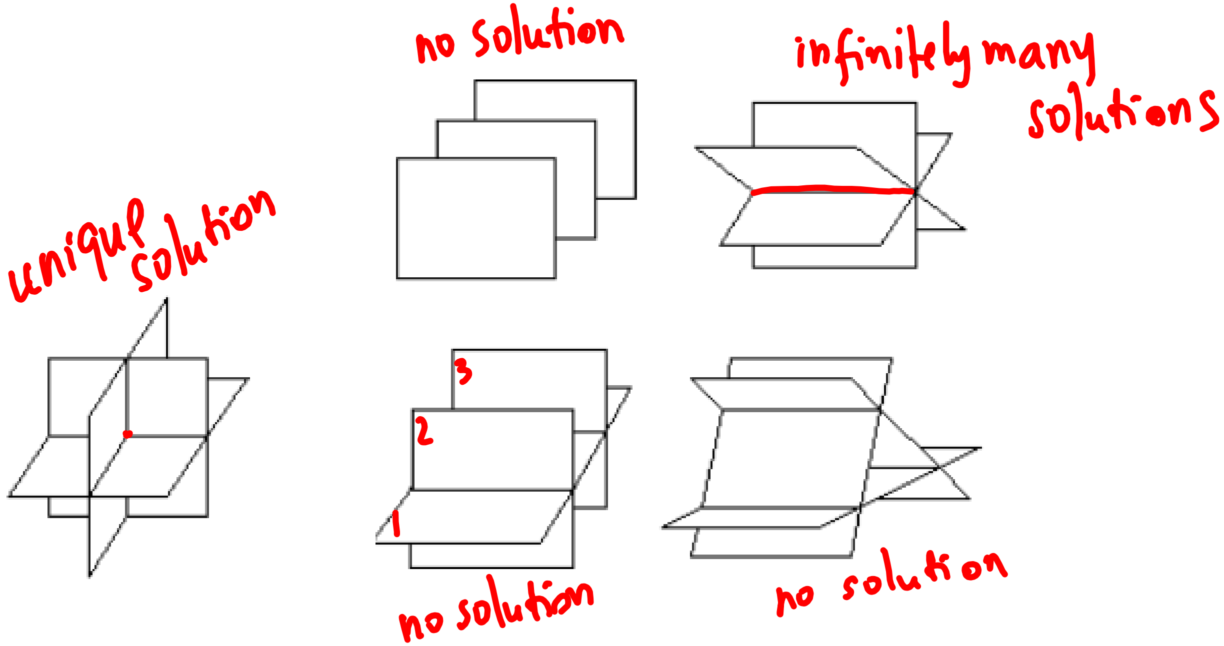
no solutions.

A linear equation in 3 variables has the form

$$ax + by + cz = d,$$

where a, b, c not all 0, and is a *plane* in the 3-dimensional space.

Three planes can (a) intersect at one point, (b) intersect along a line, or (c) be parallel.



Regardless of how many variables are involved, any system of linear equations has either one, no, or infinitely many solutions.

SET UP the following world problems. Do not solve (yet). Be sure that the VARIABLES ARE DEFINED.

EXAMPLE 2. *Mark has a total of \$30,000 invested in two municipal bonds that have yields of 8% and 10% interest per year, respectively. If the interest Mark receives from the bonds in a year is \$2640, how much does he have invested in each bond?*

x = money invested in 1st bond

y = " ——— 2nd bond

	interest
	$0.08x$
	$0.1y$
TOTAL 30000	2640

$$\begin{cases} x+y=30000 \\ 0.08x+0.1y=2640 \end{cases}$$

EXAMPLE 3. *A store sells 29 sodas one afternoon. Suppose that the prices for small, medium, and large soda are* \$1.00, \$1.25, *and* \$1.40, *respectively, and that the total sales were* \$37.25. *If they sold three times as many large sodas as they did small sodas, how many large sodas did the store sell that afternoon?*

Define variables	Price	
x = # small sodas	$1 \cdot x$	
y = # medium sodas	$1.25 \cdot y$	$z = 3x$
z = # large sodas	$1.4\, z$	
TOTAL 29	\$37.25	

$$\begin{cases} x + y + z = 29 \\ x + 1.25y + 1.4z = 37.25 \\ z = 3x \end{cases}$$

EXAMPLE 4. *A dietitian wishes to combine Baby Spinach, Baby Lettuces, and Radicchio. to make a 6-pound salad mixture. The caloric content per pound of Baby Spinach, Baby Lettuces and Radicchio is, respectively,* 105, 75 *and* 99 *calories. The units of Calcium per pound for each are* 14, 27 *and* 5 *units respectively. If the dietitian wants the total caloric content to be* 500 , *and the total calcium units to be* 120, *how many pounds of each type of food should be used?*

	Calories	Calcium
x = lbs. of Spinach	$105x$	$14x$
y = lbs. of Lettuce	$75y$	$27y$
z = lbs. of Radichio	$99z$	$5z$
6 lbs	500	120

$x + y + z = 6$

$105x + 75y + 99z = 500$

$14x + 27y + 5z = 120$

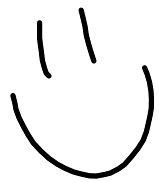
EXAMPLE 5. *Desmond Jewelry wishes to produce 3 types of pendants: A, B and C. To manufacture a type-A pendant requires 2 min on machines I and II and 3 min on machine III. A type-B pendant requires 2 min on machine I, 3 min on machine II and 4 min on machine III. A type-C pendant requires 3 min on machine I, 4 min on machine II and 3 min on machine III. There are $3\frac{1}{2}$hr available on machine I, $4\frac{1}{2}$hr available on machine II and 5hr on machine III. How many pendants of each type should Desmond make in order to use the available time?*

		Time I	Time II	Time III
X	Pend A	$2x$	$2x$	$3x$
Y	Pend. B	$2y$	$3y$	$4y$
Z	Pend.C	$3z$	$4z$	$3z$
	Total	$3\frac{1}{2}$ 210	$4\frac{1}{2}$ 270	5 300

$2x+2y+3z=210$

$2x+3y+4z=270$

$3x+4y+3z=300$



EXAMPLE 6. *A theater has a seating capacity of 900 and charges \$4 for children, \$6 for students, and \$8 for adult. At a certain screening with full attendance, there were half as many adults as children and students combined. The receipts totaled \$5600, How many children attended the show?*

		\$
	x=#of adults y=#of children z=#of students	8x 4y 6z
total	900	5600

$x=(y+z)/2$

$$\boxed{\begin{aligned} &x+y+z=900\\ &8x+4y+6z=5600\\ &x=(y+z)/2 \end{aligned}}$$