2.4: Matrices

- A matrix with m rows and n columns has the *dimension* (aka: *order* or *size*)
  $m \times n$ (ROW× COLUMN).

- If a matrix has the same number of rows and columns ($m = n$), it is called a **square matrix.** An $n \times n$ matrix is a square matrix of order n.

- A matrix consisting of only ONE row is called a **row matrix.**

- A matrix consisting of only ONE column is called a **column matrix.**

- A matrix is usually denoted by a capital letter and its elements by small letters : a_{ij} = entry in the ith row and jth column of A.

- A **Zero Matrix** is a matrix consisting of all zeros.

A matrix is a compact way to organize or display data.

EXAMPLE 1. *Mortality actuarial tables in the U.S. were revised in 2001, the fourth time since 1858. Based on the new life insurance mortality rates,* 1% *of* 60*-yr-old men,*2% *of* 70*-yr-old men,* 7% *of* 80*-yr-old men,* 18.8% *of* 90*-yr-old men and* 36.3% *of* 100*-yr-old men would die within a year. The corresponding rates for women are* 0.8%, 1.8%, 4.4%, 12.2% *and* 27.6%*, respectively. Express this information using a* 2×5 *matrix.*

• Two matrices are said to be **equal** if they are the same size and each corresponding entry is equal.

EXAMPLE 2. *Given matrices* A *and* B *below*

$$A = \begin{bmatrix} 2x-1 & 3 \\ y+5 & 2z \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & -3u \\ 0 & 1 \\ 0 & uv \end{bmatrix}$$

Solve the matrix equation $A = B$ *for* x, y, z, u, v.

• **ADDITION/SUBTRACTION** of matrices:

Two matrices can be added or subtracted ONLY if they are the SAME SIZE. This is done by adding (or subtracting) each entry with the corresponding entry and resulting in a solution matrix of the same size.

• **Scalar Multiplication**:

If A is a matrix and c ia a real number (or scalar), then the scalar product: cA is the matrix obtained by multiplying each individual entry of A by c. NOTE: Zero is NOT a scalar.

• **LAWS for Matrix Addition**:

If $A, B, and C$ are matrices of the *same* size, then

1. $A + B = B + A$ (Commutative Law)
2. $(A + B) + C = A + (B + C)$ (Associative Law)

EXAMPLE 3. *Given matrices A and B below. Find the matrix X satisfying the matrix equation* $\frac{1}{2}X + A = B$.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} -1 & 2 & -3 & 0 \\ -3 & 2 & -1 & 0 \end{bmatrix}$$

• **Transpose of a Matrix**

If A is an $m \times n$ matrix with entries a_{ij} , then A^T is the $n \times m$ matrix with entries a_{ji}.

A^T is obtained by interchanging rows and columns of A.

EXAMPLE 4. *Find A^T and $(A^T)^T$ given that*

$$A = \begin{bmatrix} -1 & 2 & 3 \\ -3 & 0 & 1 \end{bmatrix}$$

EXAMPLE 5. *Find the values for $a, b, c,$ and d for the matrix equation below:*

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + 3\begin{bmatrix} -2 & 1 \\ 4 & -1 \end{bmatrix}^T = \begin{bmatrix} 6 & 4 \\ 0 & 7 \end{bmatrix}$$