2.6: The Inverse of a Square Matrix

Let A be a square matrix of size n. A square matrix, A^{-1}, of size n, such that $AA^{-1} = I_n$ (or , equivalently, $A^{-1}A = I_n$) is called an **inverse matrix**.

EXAMPLE 1. *Are these matrices inverses?*

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.5 & 0.5 \\ 0.75 & -0.25 \end{bmatrix}$$

We shall use the calculator to find A^{-1} if it exists.

EXAMPLE 2. *If possible, find the inverse of the following matrices and express it with exact values (fractions).*

(a) $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ -4 & -2 & 7 \end{bmatrix}.$

(b) $B = \begin{bmatrix} 0 & 1 & 3 \\ -4 & -2 & 7 \end{bmatrix}.$

(c) $C = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 3 & 0 \\ -4 & -2 & 7 & 0 \\ 1 & 2 & -1 & 1 \end{bmatrix}.$

DEFINITION 3. *A matrix that does NOT have an inverse is called* **singular**.

EXAMPLE 4. *Solve the matrix equation for X. Assume all matrices are square and all inverses are possible.*

(a) $XA - 4B = D$

(b) $X + AX = B$

Solving Systems of Equations with Inverses.

Let $AX = B$ be a linear system of n equations in n unknowns and A^{-1} exists, then $X = A^{-1}B$ is the *unique* solution of the system.

EXAMPLE 5. *Suppose we have the following system of linear equations:*

$$\begin{array}{rcrcrcr} 2x & + & y & + & 2z & = & -1 \\ 3x & + & 2y & + & z & = & 2 \\ 2x & + & y & + & z & = & 1 \end{array}$$

(a) *write a matrix equation that is equivalent to the system of linear equations.*

(b) *solve the system of equations by using the inverse of the coefficient matrix.*

EXAMPLE 6. *Solve the following system of linear equations:*

$$\begin{array}{rcrcrcr} 2x & + & y & + & 2z & = & -1 \\ 3x & + & 2y & + & z & = & 2 \\ & & x & + & 3z & = & -4 \end{array}$$