# 3.1: Graphing Systems of Linear Inequalities in Two Variables

Definition. The **Feasible Region(FR)** *(or the solution set) for a system of inequalities are all the points (x, y) satisfying all of the inequalities at the same time.*

The feasible region is usually illustrated graphically with the xy-plane.

EXAMPLE 1. *Sketch the feasible region for these inequalities:*

(a) $x > 4$ **(b)** $y \leq 3$ **(c)** $x > y$
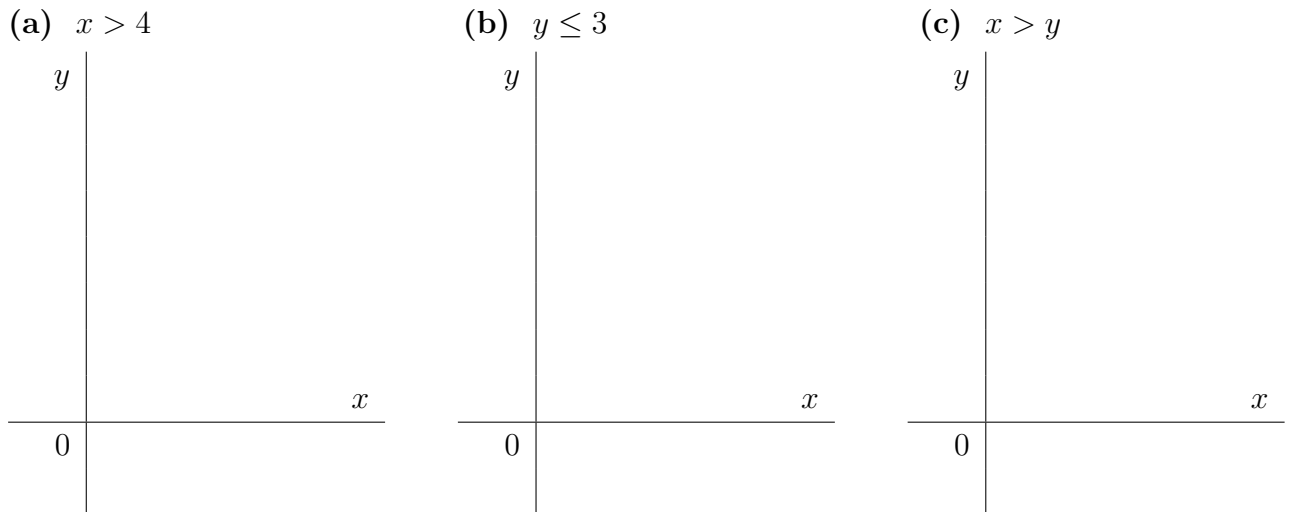
Procedure for graphing a linear inequality:

1. Replace the inequality by an equal sign, and graph it as a solid line if the original inequality is $\geq$ or $\leq$. Otherwise, graph it as a dashed/dotted line (for $>$, $<$).

2. Choose a test point not on the boundary line and substitute it into the inequality.

3. If the inequality is satisfied, shade the half-plane containing the test point. Otherwise, shade the other half-plane. The shaded region, including the boundary solid line, is the solution set.

EXAMPLE 2. *Find the graphical solution of the inequality $3x - 5y < 15$.*
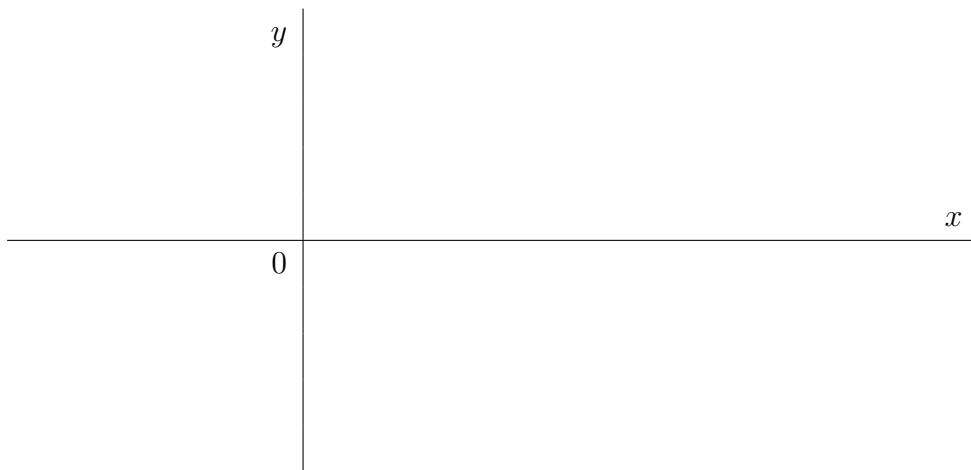
Note: **When you graph a feasible region, reverse shading is recommended, as the solution set will be the clean region and easier to see.**
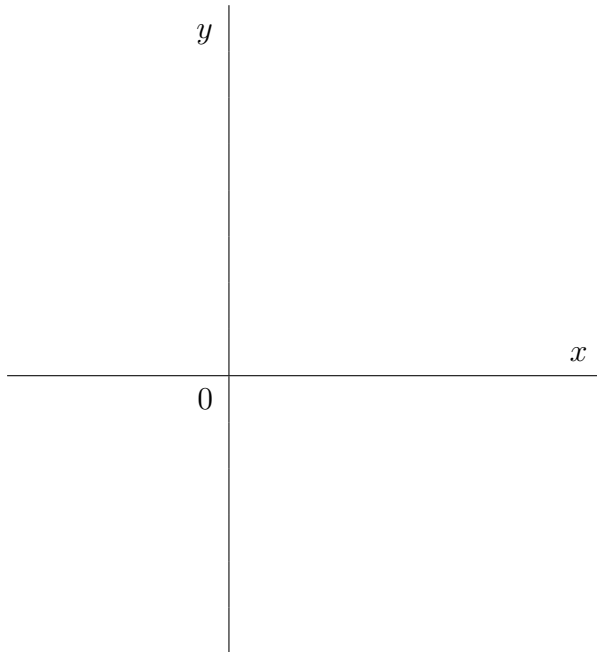
EXAMPLE 3. *Determine the feasible region for this system of inequalities:*

(a)
$$\begin{aligned}
x - y &\geq 0 \\
x - y &\leq 4 \\
x &\geq 0 \\
y &\geq 0
\end{aligned}$$

(b)
$$\begin{aligned}
x - y &\leq 0 \\
x - y &\geq 4 \\
x &\geq 0 \\
y &\geq 0
\end{aligned}$$
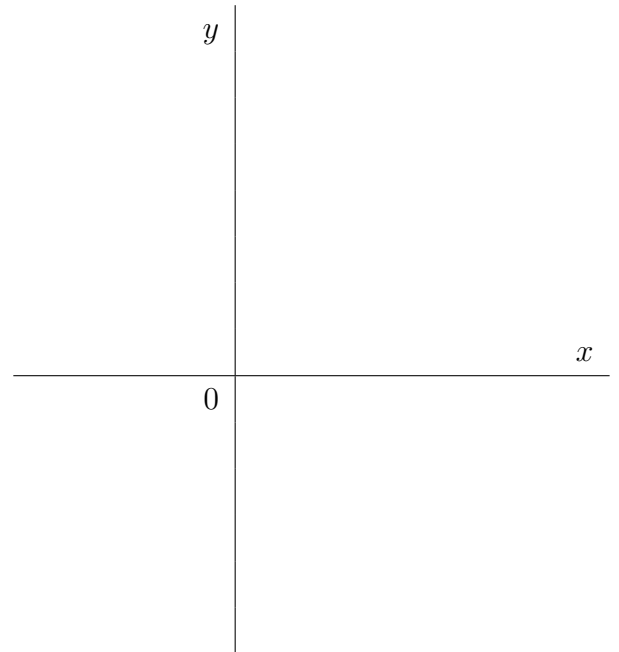
EXAMPLE 4. *Determine the feasible region for this system of inequalities:*

$$\begin{aligned}
4x - 3y &\leq 12 \\
5x + 2y &\leq 10 \\
x &\geq 0 \\
y &\geq 0
\end{aligned}$$
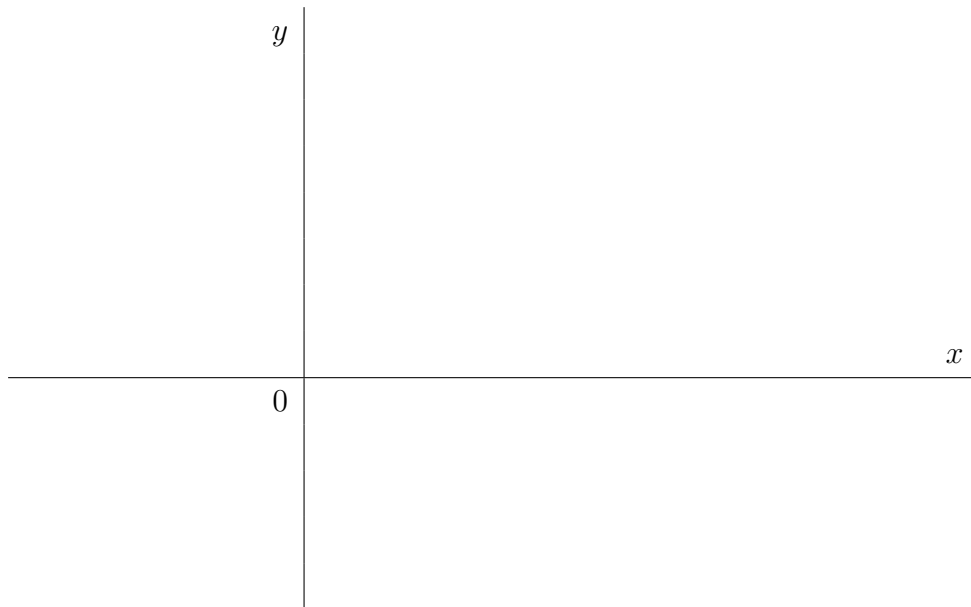
Definition. *A solution set of a system of linear inequalities is* **bounded** *if it can be enclosed by a circle. Otherwise, it is* **unbounded***.*

Definition. *The intersection of two boundary lines (if possible), is called a* **corner point** *of a feasible region provided that this point is part of the feasible region.*

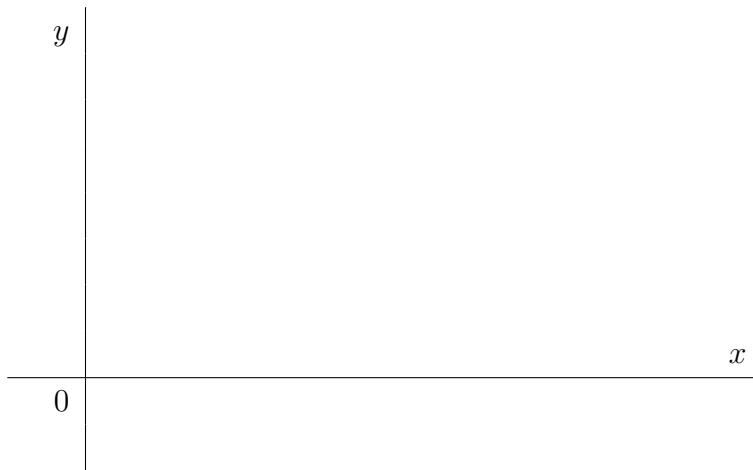EXAMPLE 5. *Find the corner points for Example 4.*

EXAMPLE 6. *Given:*
$$12x - 11y \leq 18$$
$$6x + 7y \leq 84$$
$$6x - 7y \leq 28$$
$$x \geq 0, y \geq 0.$$

(a) *Determine the feasible region for this system of inequalities.*

y

x

0

(b) *Find all corner points.*

(c) *Determine if the feasible region is bounded.*

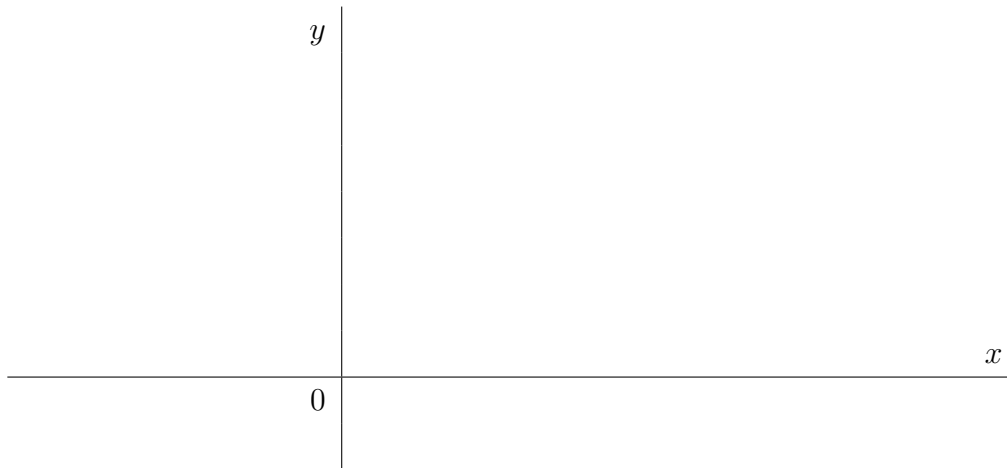**EXAMPLE 7.** *Given:*

$$x - 2y \leq -2$$
$$x - y \geq -6$$
$$x + 2y \geq 6$$
$$x + 2y \geq -14$$
$$x \geq 0, y \geq 0.$$

(a) *Determine the feasible region for this system of inequalities.*

(b) *Find all corner points.*

(c) *Determine if the feasible region is bounded.*