3.1: Graphing Systems of Linear Inequalities in Two Variables

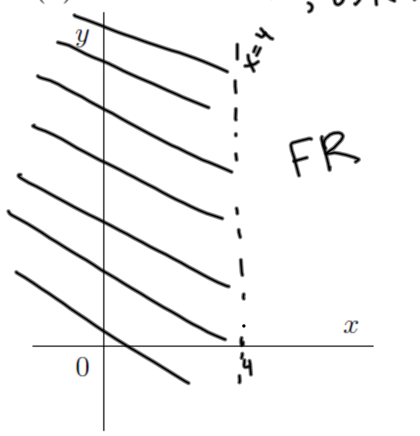
Reverse shading

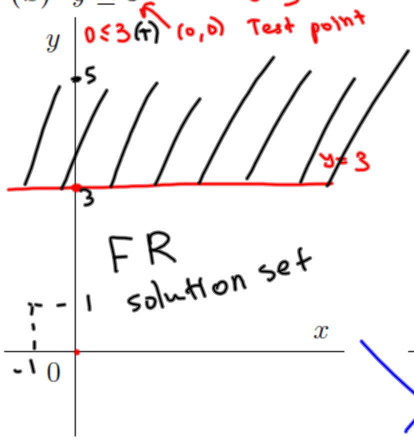
Definition. *The* **Feasible Region(FR)** *(or the solution set) for a system of inequalities are all the points (x, y) satisfying all of the inequalities at the same time.*

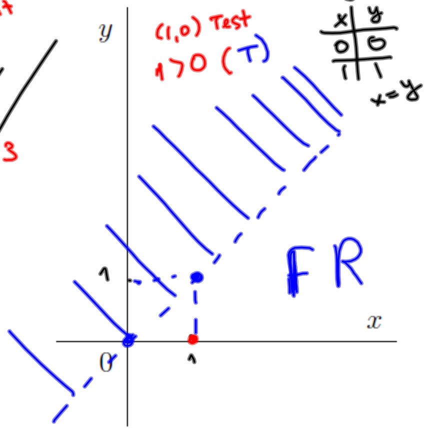
The feasible region is usually illustrated graphically with the xy-plane.

EXAMPLE 1. *Sketch the feasible region for these inequalities:*

(a) $x > 4$ $x=4$, $0>4$ (F) (b) $y \leq 3$ $y=3$ (c) $x > y$ $x=y$







Procedure for graphing a linear inequality:

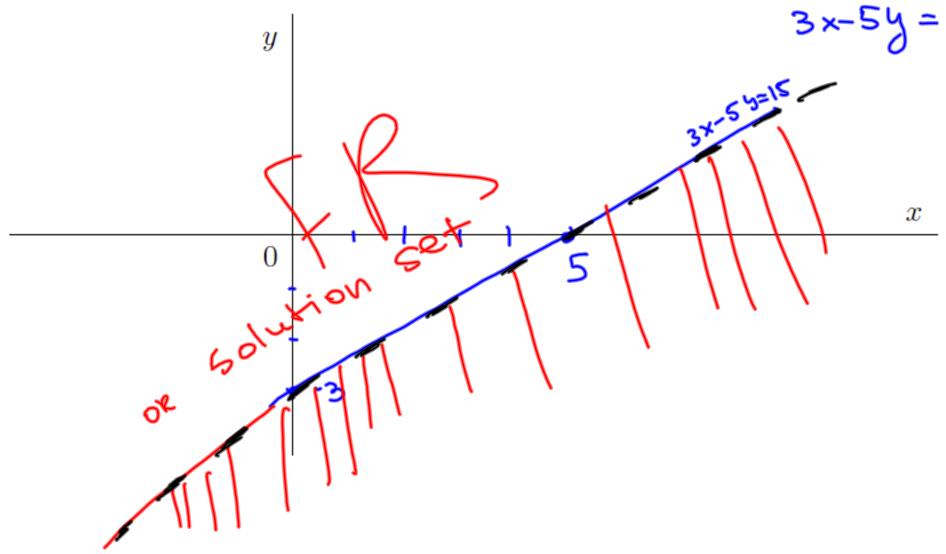
Reverse shading

1. Replace the inequality by an equal sign, and graph it as a solid line if the original inequality is $\geq$ or $\leq$. Otherwise, graph it as a dashed/dotted line (for $>$, $<$).
2. Choose a test point not on the boundary line and substitute it into the inequality.
3. If the inequality is satisfied (T), shade the half-plane non containing the test point. Otherwise, shade the other half-plane. The non-shaded region (clean region), including the boundary solid line, is the solution set.

EXAMPLE 2. *Find the graphical solution of the inequality* $3x - 5y < 15$. $(0,0) \Rightarrow 0 < 15\ (T)$

$3x - 5y = 15$

x	y
0	-3
5	0

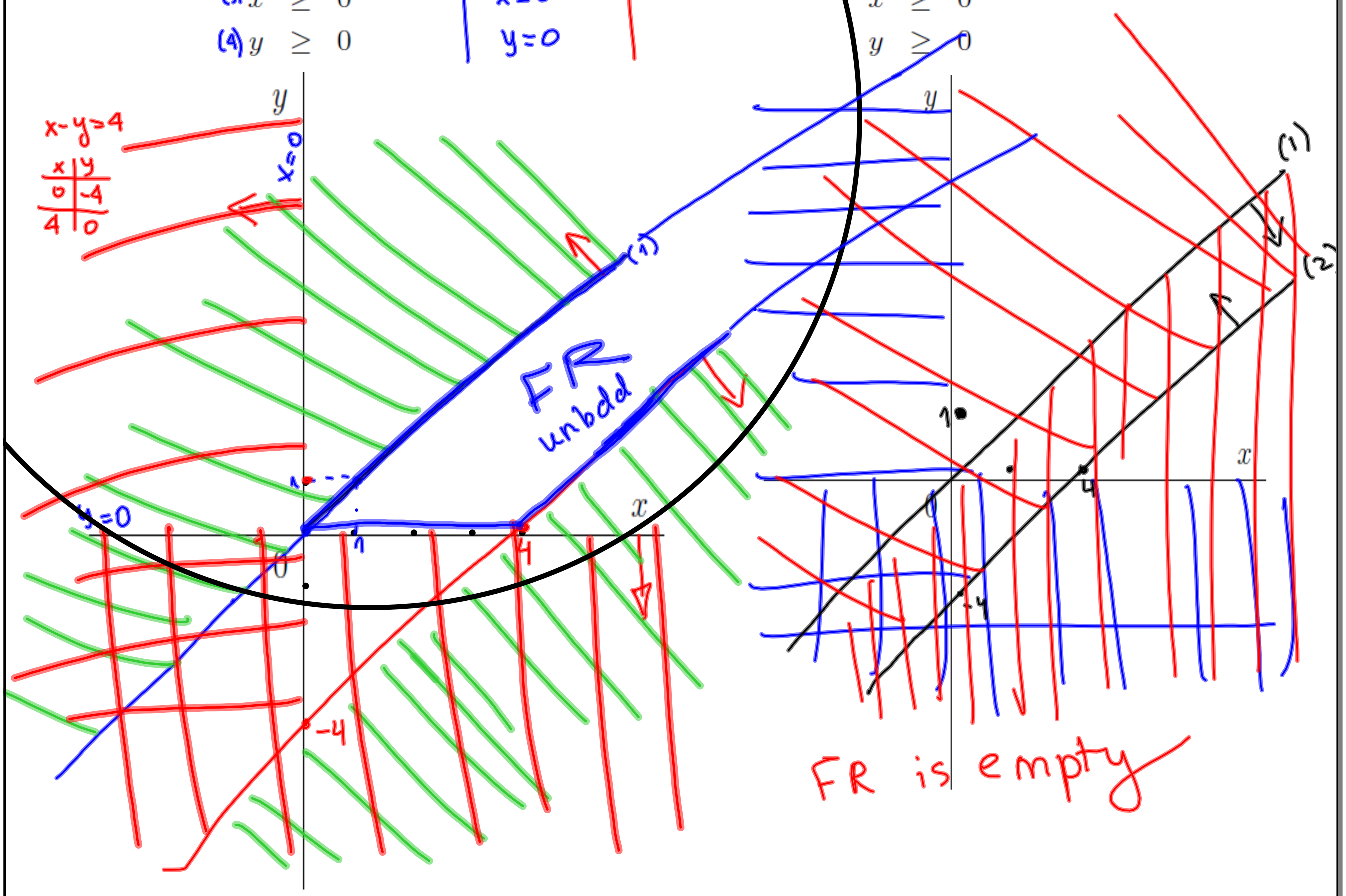


Note: ***When you graph a feasible region, reverse shading is recommended, as the solution set will be the clean region and easier to see.***

EXAMPLE 3. *Determine the feasible region for this system of inequalities:*

(a) (1) $x - y \geq 0$ | $x-y=0$ | Test $(0,1)$ $0-1\geq 0$ (F)
(2) $x - y \leq 4$ | $x-y=4$ | $(0,0)$ $0\leq 4$ (T)
(3) $x \geq 0$ | $x=0$
(4) $y \geq 0$ | $y=0$

(b) $x - y \leq 0$ Tes $(0,1)$ $0-1\leq 0$ (T)
$x - y \geq 4$ Test $(0,0)$ $0-0\geq 4$ (F)
$x \geq 0$
$y \geq 0$



EXAMPLE 4. *Determine the feasible region for this system of inequalities:*

(1) $4x - 3y \le 12$ Test (0,0) $0 \le 12$ (T)

(2) $5x + 2y \le 10$ $0 \le 10$ (T)

$x \ge 0$

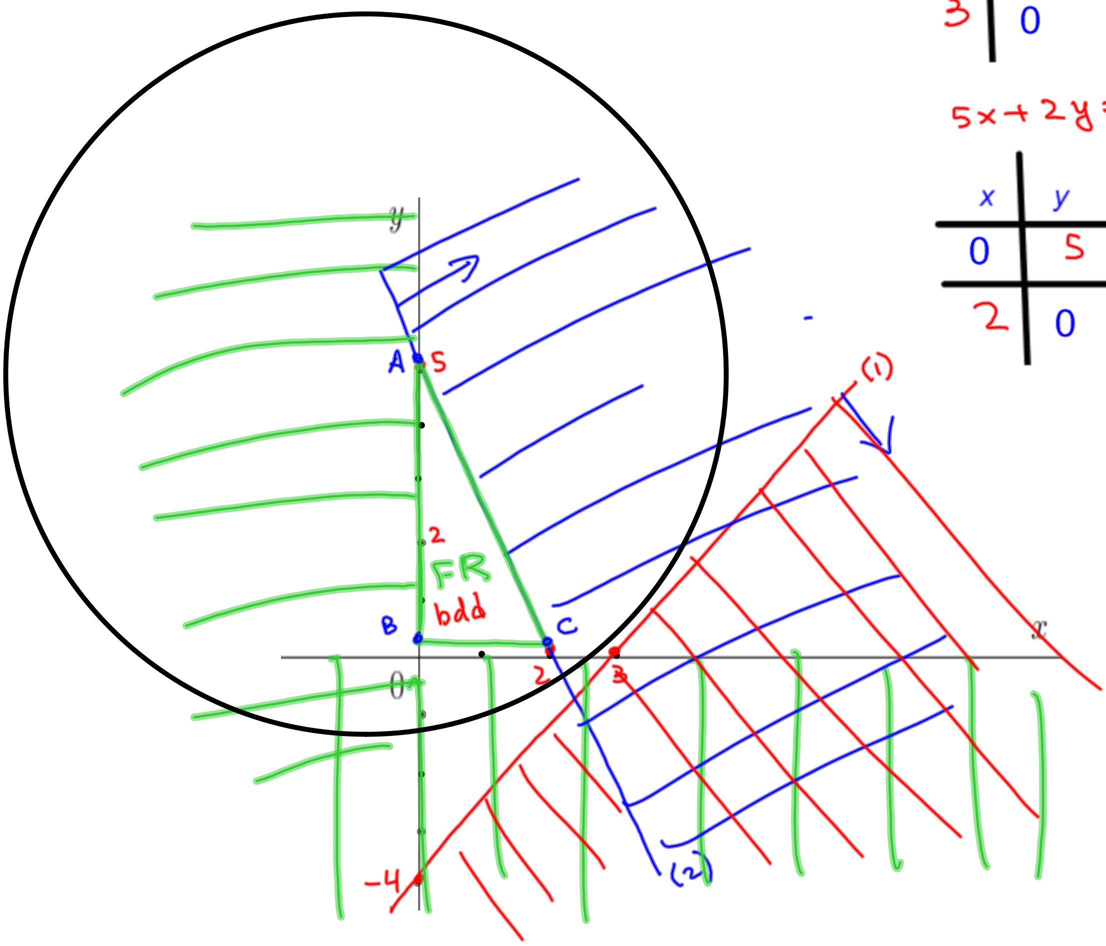
$y \ge 0$

$4x - 3y = 12$

x	y
0	-4
3	0

$5x + 2y = 10$

x	y
0	5
2	0



Definition. *A solution set of a system of linear inequalities is* **bounded** *if it can be enclosed by a circle. Otherwise, it is* **unbounded**.

Definition. *The intersection of two boundary lines (if possible), is called a* **corner point** *of a feasible region provided that this point is part of the feasible region.*

EXAMPLE 5. *Find the corner points for Example 4.*

A(0,5), B(0,0), C(2,0)

EXAMPLE 6. *Given:*

Test(0,0)

(T) $0 \le 18$ ⟵ (1) $12x - 11y \le 18$

(T) $0 \le 84$ ⟵ (2) $6x + 7y \le 84$

(T) $0 \le 28$ ⟵ (3) $6x - 7y \le 28$

$x \ge 0, y \ge 0.$

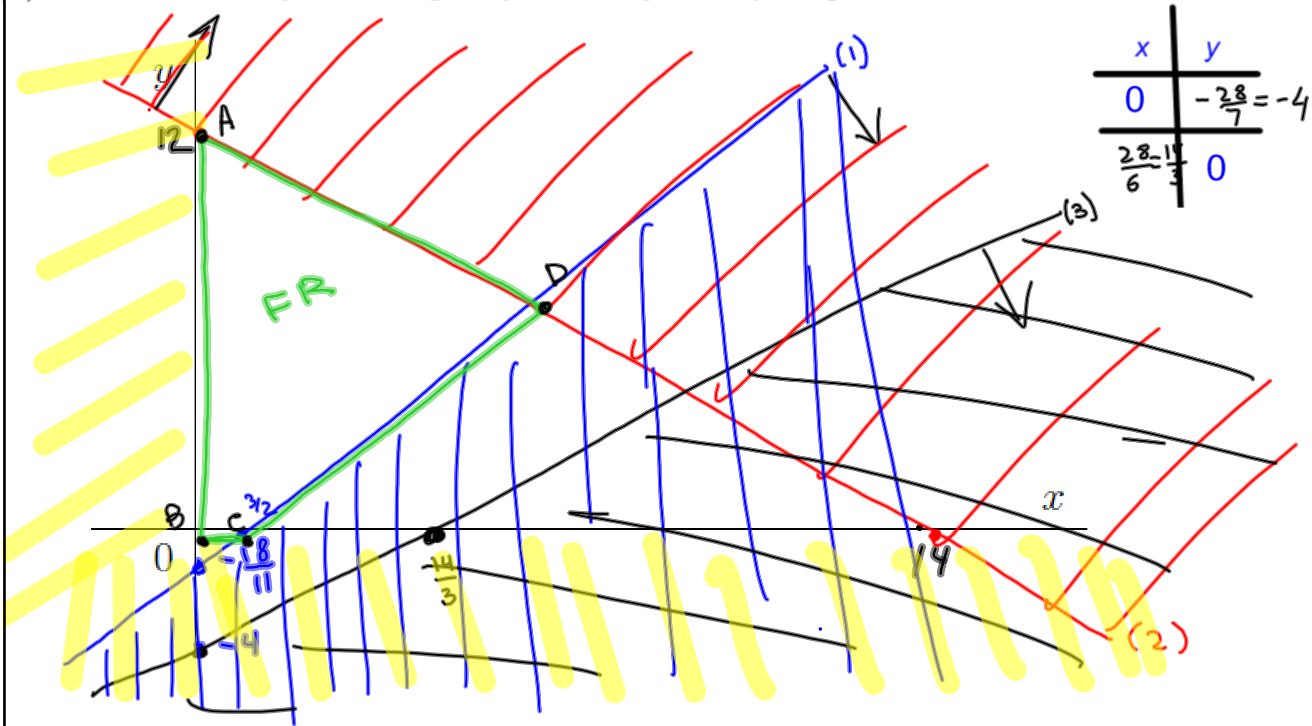
$12x - 11y = 18$

x	y
0	$-\frac{18}{11}$
$\frac{18}{12} = \frac{3}{2}$	0

$6x + 7y = 84$

x	y
0	$\frac{84}{7} = 12$
$\frac{84}{6} = 14$	0

(a) *Determine the feasible region for this system of inequalities.*



x	y
0	$-\frac{28}{7} = -4$
$\frac{28}{6} = \frac{14}{3}$	0

(b) *Find all corner points.*

$A(0,12)$, $B(0,0)$, $C(\frac{3}{2},0)$

$D = (1) \cap (2) = (7,6)$

$$\begin{cases} 12x - 11y = 18 \\ 6x + 7y = 84 \end{cases}$$

system of linear equations

$$\left[\begin{array}{cc|c} 12 & -11 & 18 \\ 6 & 7 & 84 \end{array}\right] \xrightarrow{\text{rref}} \begin{array}{cc} X & Y \end{array} \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 6 \end{array}\right] \Rightarrow$$

$\Rightarrow x = 7, \; y = 6$

(c) *Determine if the feasible region is bounded.*

YES

EXAMPLE 7. *Given:*

Test $(0,0)$

$0 \le -2$ (F)

$0 \ge -6$ (T)

$0 \ge 6$ (F)

$0 \ge -14$ (T)

(1) $x - 2y \le -2$

(2) $x - y \ge -6$

(3) $x + 2y \ge 6$

(4) $x + 2y \ge -14$

$x \ge 0, y \ge 0.$

$x - 2y = -2$

x	y
0	1
-2	0

$x - y = -6$

x	y
0	6
-6	0

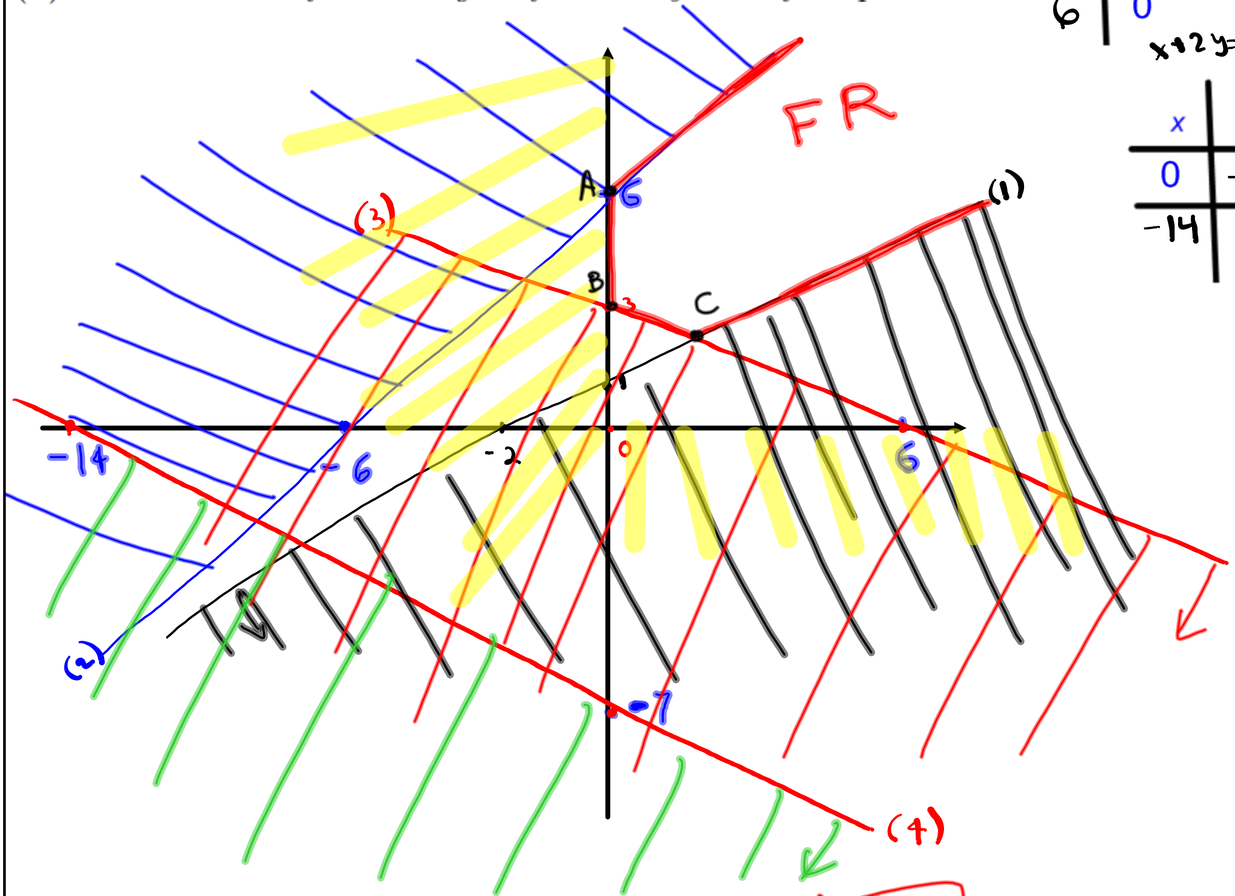
$x + 2y = 6$

x	y
0	3
6	0

$x + 2y = -14$

x	y
0	-7
-14	0

(a) *Determine the feasible region for this system of inequalities.*



(b) *Find all corner points.* $A(0,6)$, $B(0,3)$

$C = (1) \cap (3) = \boxed{(2,2)}$

$$\begin{cases} x - 2y = -2 \\ x + 2y = 6 \end{cases} \qquad \left[\begin{array}{cc|c} 1 & -2 & -2 \\ 1 & 2 & 6 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 2 \end{array}\right]$$

(c) *Determine if the feasible region is bounded.* NO