3.2: Linear Programming Problems

A **linear programming problem** consists of a linear objective function to be maximized or minimized subject to certain constraints in the form of linear equations or inequalities. (F R)

Set up the following linear programming problems (write the objective function and constraints). **Do not solve the problem.**

EXAMPLE 1. *The Acrosonic Company manufactures a model G loudspeaker system in plants I and II. The output at plant I is at most* (≤ 800) *800 systems per month, whereas the output at plant II is at most* (≤ 600) *600 per month.*
These loudspeaker systems are shipped to the three warehouses, A, B, and C, whose minimum monthly requirements are $\geq$ *500,* $\geq$ *400, and* $\geq$ *400, respectively.*
Shipping costs from plant I to warehouse A, warehouse B, and warehouse C are \$16, \$20, and \$22 per loudspeaker system, respectively, and shipping costs from plant II to each of these warehouses are \$18, \$16, and \$14, respectively.
What shipping schedule will enable Acrosonic to meet the warehouses' requirements and at the same time keep its shipping costs to a minimum?

Solution:

x - the number of loudspeakers shipped from plant I to warehouse A.
y - the number of loudspeakers shipped from plant I to warehouse B.
z - the number of loudspeakers shipped from plant I to warehouse C.
u - the number of loudspeakers shipped from plant II to warehouse A.
v - the number of loudspeakers shipped from plant II to warehouse B.
w - the number of loudspeakers shipped from plant II to warehouse C.

Plant \ Warehouse	A	B	C	TOTAL Output
Plant I	x \$16	y \$20	z \$22	≤ 800
Plant II	u \$18	v \$16	w \$14	≤ 600
min requirement	≥ 500	≥ 400	≥ 400	

Cost Function: $C = 16x + 20y + 22z + 18u + 16v + 14w \to \min$

Linear objective function

subject to
$$\begin{cases} x + y + z \le 800 \\ u + v + w \le 600 \\ x + u \geq 500 \\ y + v \geq 400 \\ z + w \geq 400 \\ x \geq 0, \quad y \geq 0, \quad z \geq 0 \\ u \geq 0 \quad v \geq 0 \quad w \geq 0 \end{cases}$$

EXAMPLE 2. *Kane Manufacturing has a division that produces two models of fireplace grates, x units of model A and y units of model B.*
To produce each model A requires 2 lb of cast iron and 8 min of labor.
To produce each model B grate requires 4 lb of cast iron and 5 min of labor.
The profit for each model A grate is \$1.00, and the profit for each model B grate is \$2.40.
If 740 lb of cast iron and 1420 min of labor are available for the production of grates per day, how many grates of each model should the division produce per day in order to maximize Kane's profits P?

	#units	IRON, lb	Labor, min	Profit \$
Model A	x	$2x$	$8x$	$\$1 \cdot x$
Model B	y	$4y$	$5y$	$\$2.4 \cdot y$
		≤ 740	≤ 1420	max

Lin. obj. fun. $P = x + 2.4y$

subject to $\begin{cases} 2x + 4y \leq 740 \\ 8x + 5y \leq 1420 \\ x \geq 0, \ y \geq 0 \end{cases}$

EXAMPLE 3. *A nutritionist at the Medical Center has been asked to prepare a special diet for certain patients.*
She has decided that the meals should contain a minimum of $\geq$ 308 *mg of calcium,* $\geq$ 20 *mg of iron, and* $\geq$ 41 *mg of vitamin* C.
She has further decided that the meals are to be prepared from x *ounces of food* A *and* y *ounces of food* B.
Each ounce of food A *contains* 26 *mg of calcium,* 2 *mg of iron,* 2 *mg of vitamin* C, *and* 3 *mg of cholesterol.*
Each ounce of food B *contains* 20 *mg of calcium,* 0.8 *mg of iron,* 5 *mg of vitamin* C, *and* 5 *mg of cholesterol.*
Find how many ounces of each type of food should be used in a meal so that the cholesterol, C, *content is minimized and the minimum requirements of calcium, iron, and vitamin* C *are met?*

	onces	Calcium	IRON	vitamin C	cholesterol
Food A	x	$26x$	$2x$	$2x$	$3x$
Food B	y	$20y$	$0.8y$	$5y$	$5y$
		≥ 308	≥ 20	≥ 41	min

$$C = 3x + 5y \to \min$$

subject to:

$$\begin{cases} 26x + 20y \geq 308 \\ 2x + 0.8y \geq 20 \\ 2x + 5y \geq 41 \\ x \geq 0, \quad y \geq 0 \end{cases}$$