

### 3.3: Graphical solution of Linear Programming Problems

PROBLEM: Find the **optimal** (maximum/minimum) solution for a linear programming problem with a feasible region  $FR$  and objective function  $P = ax + by$ .

- **FUNDAMENTAL THEOREM OF LINEAR PROGRAMMING:**

If the optimal value of the objective function exists, it must occur at one (or more) of the corner points of the feasible region.

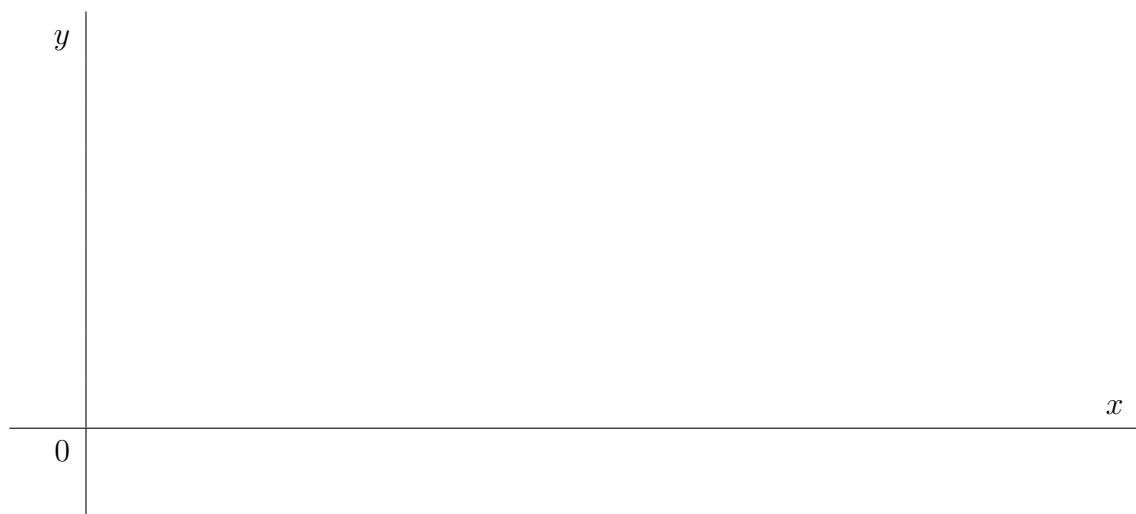
- **EXISTENCE OF A SOLUTION:**

For a linear programming problem with a feasible region  $FR$  and objective function  $P = ax + by$ ,

- (a) If  $FR$  is bounded, then  $P$  has both a maximum and a minimum value on  $FR$ .
- (b) If  $FR$  is unbounded you might have a max or min (or neither). In particular, if  $FR$  is unbounded and  $a, b > 0$ , then  $P$  has a minimum value on  $FR$  provided that  $x, y \geq 0$  are part of the constraints.  $P$  has no maximum value then.
- (c) If  $FR$  is the empty set, then  $P$  has no maximum or minimum value.

- **THE METHOD OF CORNERS:**

1. Graph the feasible region.
2. Find the coordinates of all the corner points (vertices).
3. Evaluate the objective function at each corner point.
4. Use the existence of solutions to determine the optimal solutions. If two adjacent corner points give the same optimal objective function value, all the points on the line segment joining the two corner points will be the optimal solutions.



EXAMPLE 1. (a) Set up the following linear programming problem:

The water-supply manager for a Midwest city needs to supply the city with at least 10 million gal of potable (drinkable) water per day.

The supply may be drawn from the local reservoir or from a pipeline to an adjacent town. The local reservoir has a maximum daily yield of 5 million gal of potable water, and the pipeline has a maximum daily yield of 10 million gallons.

By contract, the pipeline is required to supply a minimum of 6 million gal/day.

If the cost for 1 million gallons of reservoir water is \$300 and that for pipeline water is \$500, how much water should the manager get from each source to minimize daily water cost for the city. What is the minimum daily cost?

Solution:

- $x$  - the number of gallons of water in millions obtained from the local reservoir per day;  
 $y$  - the number of gallons of water in millions obtained from the pipeline.

$$C = 300x + 500y \longrightarrow \min$$

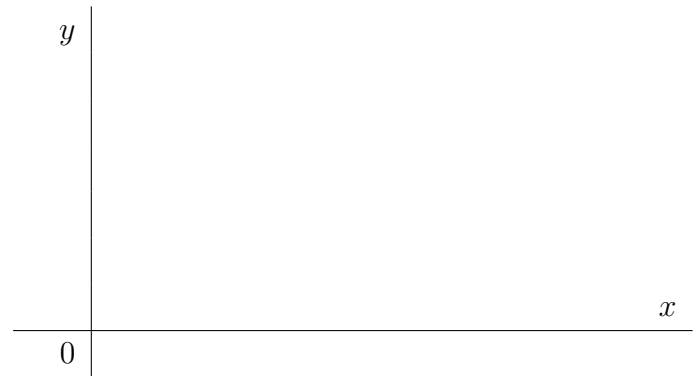
$$x + y \geq 10$$

$$x \leq 5$$

$$y \geq 6$$

$$y \leq 10$$

$$x \geq 0.$$

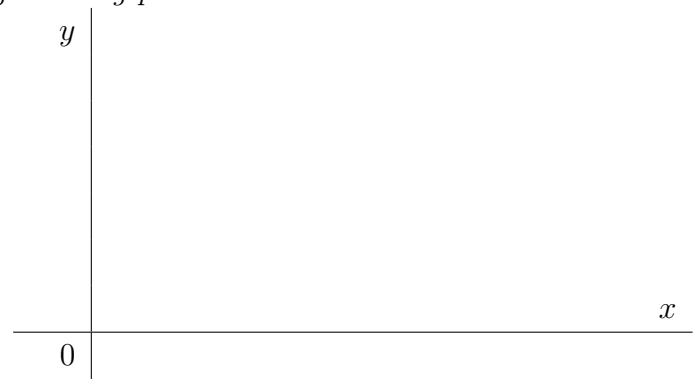


(b) Find all the corner points and the corresponding values of the objective function  $C$ .

(c) What is the minimum daily cost?

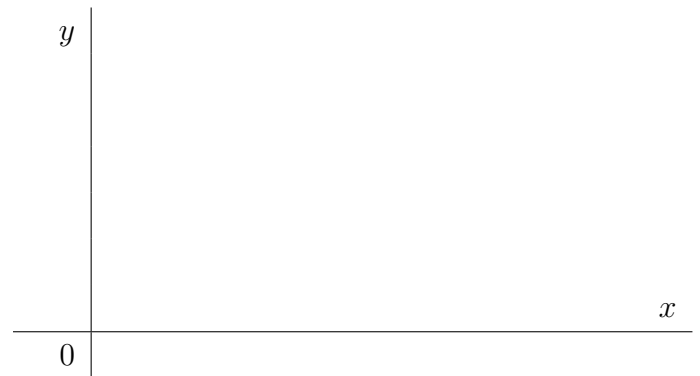
EXAMPLE 2. Solve the following linear programming problem:

$$\begin{aligned} \text{Maximize } & P = 4x + 2y \\ \text{subject to } & x + y \leq 8 \\ & 2x + y \leq 10 \\ & x \geq 0, y \geq 0 \end{aligned}$$



EXAMPLE 3. Solve the following linear programming problem:

(a) Minimize  $P = 3x + 4y$   
subject to  $x + y \geq 3$   
 $x + 2y \geq 4$   
 $x \geq 0, y \geq 0$



(b) Maximize  $P = 3x + 4y$   
subject to  $x + y \geq 3$   
 $x + 2y \geq 4$   
 $x \geq 0, y \geq 0$

EXAMPLE 4. Solve the following linear programming problem:

Maximize  $f = 4x - y$   
subject to  $x + 2y \geq 4$   
 $y \leq 4$   
 $x \geq 0, y \geq 0$

