5.2: Annuities $PMT \neq 0$

An **annuity** is an account with <u>payments made to it at regular time intervals.</u>

   Examples of annuity: regular deposits into a saving account, monthly home mortgage payments, monthly insurance payments.

   We use TVM Solver to study annuities that are

- <u>certain</u> (terms are given by fixed time periods)

- ordinary (payments made at the $\boxed{\text{END}}$ of the payment periods)

- simple (the payment periods coincide with the interest conversion periods: P/Y=C/Y)

- equal payments

The Future Value of an Annuity is where you are trying to find how much money will be in the account after depositing equal amounts over a fixed time period.

EXAMPLE 1. *Paul makes a* $\overset{m=4}{\text{quarterly}}$ *deposit of* $\overset{<0}{\$250}$ *in his saving account earning interest at the rate of 3.8% per year compounded quarterly.*

(a) *How much will he have in the account after 7 years?* $t = 7$

$$
\begin{aligned}
N &= 4 \cdot 7 = 28 \\
I\% &= 3.8 \\
PV &= 0 \\
PMT &= -250
\end{aligned}
\qquad
\begin{aligned}
? \ FV &= \boxed{\$7976.26} \\
P/Y &= 4 \\
C/Y &= 4
\end{aligned}
$$

(b) *How much interest will he earn over the 7 years?*

interest earned=accumulated amount (FV)-total deposit (PMT*N)=
7976.26-250*28=$976.26

EXAMPLE 2. *A young man puts* $100 *every month into an account for 3 years. If the interest is compounded monthly, what is the effective rate of interest if he has* $4084.27 *in the account at the end of 3 years?*

Find Eff(I,12). Calculate I using TVM solver:

PMT=-100, t=3, m=P/Y=C/Y=12, FV=4084.27. Then N=mt=12*3. Solve for I:

I=8.5. Thus Eff(8.5,12)=8.84%.

EXAMPLE 3. *Lauren's parents have decided to set up a college fund for her. They decided that* $75,000$ *should be enough for this goal. They also decided to open the account when Lauren was 6 years old and make monthly deposits every month until Lauren turns 18. If the account pays interest at a rate of 4.75% compounded monthly*

$$t = 18 - 6 = 12$$

(a) *what is the monthly deposit that would reach the Lauren's parents goal?*

$$N = 12 \cdot 12 = 144$$
$$I\% = 4.75\%$$
$$PV = 0$$
$$?\ PMT = \boxed{-\$387.42}$$

$$FV = 75000$$
$$P/Y = 12$$
$$C/Y = 12$$

$$\boxed{\$387.42}$$

(b) *How much interest did the account earn?*

interest=FV−PMT*N= 75000−387.42*144= $\boxed{\$19211.52}$

The Present Value of an Annuity is where you are making payments to zero out a loan (or reduce the amount) and are looking to find how much the loan was worth in the beginning.

EXAMPLE 4. *Laura made a down payment of* $3500 *toward the purchase of a new car. To pay the balance of the purchase price, she has secured a loan from her bank at the rate of 10%/year compounded monthly. Under the terms of her finance agreement she is required to make payments of* $200/month *for 40 months. What is the original price (cash price) of the car?*

$$N = 40$$
$$I\% = 10\%$$
$$?\ PV = \$6779.48$$
$$PMT = -200$$
$$FV = 0$$
$$P/Y = 12$$
$$C/Y = 12$$

car price=down payment+PV=
3500+6779.48=$\boxed{\$10279.48}$