

### 5.3: Amortization and Sinking Funds

Amortization is to pay back a debt with regular installments of money (when someone is gradually paying back a loan, or paying off money they owe, with regular installments of money. Interest is charged on the unpaid balance).

Mathematically amortization is the same as annuity. The big difference is that in an annuity the interest is bonus with a loan, while in amortization the interest is a penalty.

EXAMPLE 1. A young man has a credit card debt of \$5750. The credit card charges interest of 20% compounded monthly on the unpaid balance. If he wants to have this paid off in 1 year,

- (a) how much should he pay towards the balance each month?  $t = 1$
- $N = 12 \cdot 1 = 12$   
 $I\% = 20$   
 $PV = 5750$   
 $FV = 0$   
 $P/Y = 12$   
 $C/Y = 12$   
 $PMT = -532.65$
- \$ 532.65**

- (b) how much interest will he end up paying on the \$5750?

penalty = total deposit - principal value =  
 $= PMT \cdot N - PV = 532.65 \cdot 12 - 5750 = \$ 641.80$

EXAMPLE 2. Peter and Jessica borrowed \$50,000 from a bank. The bank charges interest of 8% per year on the unpaid balance, with interest computations made at the end of each year. Peter and Jessica have agreed to repay the loan in equal annuity installments over 5 years.

- (a) What should the payments be in order to have this loan amortized in 5 years?  $t = 5$
- $N = 5$   
 $I\% = 8$   
 $PV = 50,000$   
 $FV = 0$   
 $P/Y = 1$   
 $C/Y = 1$   
 $PMT = -12,522.82$

- (b) After making the first payment, how much money is still owed on this loan?

Use the previous screen with  $N=1$ .  
 Solve for  $FV = -\$41,477.17$

- (c) How much of that first payment went towards the principle, and how much went towards interest?

money towards interest =  $PV \cdot r = 50000 \cdot 0.08 = \$4000$

money towards the principle =  $PMT - 4000 = 12522.82 - 4000 = \$8522.82$

- (d) Will this be true for each payment? (after 2 years, 3 years) - ...

No, because the interest charged is calculated as 8% of outstanding principle, which is not constant for each payment (namely, it is decreasing.)

- (e) After making the second payment, how much money is still owed on this loan?

$FV|_{N=2} = -\$32,272.53$

- (f) How much of the second payment went towards the principle, and how much went towards interest?

toward interest =  $(FV \text{ with } N=1) \cdot r = 41477.17 \cdot 0.08 = \$3318.17$

toward principle =  $PMT - 3318.17 = \$9204.65$

- (g) Write an amortization schedule for the 5 years.

End of Period	Interest Charged	Payment made	Payment Toward Principal	Outstanding Principal
0	-----	-----		50000
1	4000	12,522.82	8,522.82	41477.18
2	3318.17	12,522.82	9,204.65	32272.53
3	$FV(N=2) \cdot r = 2581.80$	12,522.82	9,941.02	22331.51
4	$22331.51 \cdot 0.08 = 1786.52$	12,522.82	10,736.30	11595.21
5	927.62	12,522.82	11,595.20	0

$N$   $FV|_{N=1} \cdot r$   $PMT$   $- \text{interest charged}$   $FV|_{N=1}$   
 $PV = FV|_{N=0}$   $FV|_{N=1}$

EXAMPLE 3. The Hamiltons bought a house in 2005. By 2009, they still owed \$85,100 on it. It was being financed on a 30 year plan at a 6.7% monthly interest rate.

$$t = 30 - 4 = 26$$

(a) What were their monthly payments?

2009

$$\begin{aligned} N &= 12 * 26 \\ I\% &= 6.7 \\ PV &= 85100 \\ ? \text{ PMT} &= -576.64 \end{aligned} \quad \begin{aligned} FV &= 0 \\ P/Y &= 12 \\ C/Y &= 12 \end{aligned}$$

$$\boxed{\$576.64}$$

(b) How much interest will they end up paying on their house?

$$\text{Total payments} - \text{Original loan} = \text{PMT} * 30 * 12 - 89,363.43 = \$118,226.97$$

Find original loan

2005

$$\begin{aligned} N &= 12 * 30 \\ I\% &= 6.7 \\ ? \text{ PV} &= 89,363.43 \\ \text{PMT} &= -576.64 \end{aligned} \quad \begin{aligned} FV &= 0 \\ P/Y &= 12 \\ C/Y &= 12 \end{aligned}$$

(d) In 2009 Hamiltons decided to refinance their house on a 15-year plan at a 4.525% monthly interest rate. What are their monthly payments now?

$$t = 15$$

$$\begin{aligned} N &= 12 * 15 \\ I\% &= 4.525 \\ ? \text{ PV} &= 85100 \\ \text{PMT} &= -652.10 \end{aligned} \quad \begin{aligned} FV &= 0 \\ P/Y &= 12 \\ C/Y &= 12 \end{aligned}$$

$$\boxed{\$652.10}$$

(e) Why would they want to do this?

To decrease the interest they end up paying on their house.

Indeed, new interest = total payments - original loan =

$$= 576.64 * 4 * 12 + 652.10 * 15 * 12 - 89363.43 = \$55693.29 \text{ less than}$$

the interest paid for 30 years loan, which is \$118226.97

$$PMT \neq 0 \quad PMT < 0 \quad \cdot \quad PV = 0 \text{ (usually)}$$

A sinking fund is an account set up for a specific purpose at some future date (when someone makes periodic payments to so that he will have a certain amount at a future date.)

EXAMPLE 4. A corporation wishes to set up a sinking fund in order to have the funds necessary to replace a current machine. It is estimated that the machine will need to be replaced in 6 years and cost \$80,000.

$$t = 6$$

(a) How much per quarter should be deposited into an account with an annual rate of 7.8% compounded quarterly to meet this future obligation?

$$\begin{aligned}
 N &= 4 * 6 = 24 \\
 I\% &= 7.8 & FV &= 80\,000 \\
 PV &= 0 & P/Y &= 4 \\
 ? \quad PMT &= \boxed{-2645.77} & C/Y &= 4
 \end{aligned}$$

$\$2645.77$

(b) What will be the total amount of payments in 6 years?

$$PMT * N = 2645.77 * 24 = \$63498.48$$

(c) What will be the interest earned in 6 years?

$$\begin{aligned}
 \text{interest earned} &= FV - PMT * N = 80,000 - 63,498.48 = \\
 &= \$16,501.52
 \end{aligned}$$

(d) Determine the equity of the sinking fund if the corporation decides to use the accumulated amount at the end of 2 years, 4 years, and 6 years and fill in the sinking fund schedule below.

$t$	$N = mt$	$PMT * N$	Equity - Interest Earned	Total payment	Equity
2	$\frac{2 \cdot 4}{8}$	$2645.77 * 8$ 21,166.16	1502.31		22,668.47
4	$\frac{4 \cdot 4}{16}$	42,332.32	6791.90		49,124.22
6	$\frac{6 \cdot 4}{24}$	63,498.48	16501.52		\$80,000.00

$FV|_N$   
 $FV|_{N=8}$   
 $FV|_{N=16}$   
 $FV|_{24}$