6.1: Sets and Set Operations

- **Set Terminology and Notation**

DEFINITIONS:

Set is well-defined collection of objects.

Elements are objects or members of the set.

Roster notation:

$A = \{a, b, c, d, e\}$ Read: Set A with elements a, b, c, d, e.

$B = \{a, b, c, ..., z\}$ Read: Set B with elements being the letters of the alphabet.

Set-builder notation:

$B = \{x|x$ is a letter of the English alphabet$\}$

$C = \{x|x$ is a student in this classroom$\}$

If a is an element of a set A, we write $a \in A$ that read "a belong to A." However, if a does not belong to A, we write $a \notin A$.

Two sets, A and B, are **equal**, written $A = B$, if and only if they have exactly the same elements. (NOTE: they do not have to be in the same order!). If every element in set A is also an element in set B, then A is a subset of B, written $A \subseteq B$.. If $A \subseteq B$, but $A \neq B$, then A is a **proper** subset of B, written $A \subset B$.

EXAMPLE 1. *Given*

$$\begin{array}{l} A = \{a, e, i, o, u\} \\ B = \{u, i, e, a, o\} \\ C = \{a, e, i, o\} \\ D = \{e, i, o, a\} \end{array} . \textit{ Then}$$

The **empty set** is the set that doesn't have any elements, denoted by $\emptyset$ or $\{\}$. The **universal set** is the set that contains all of the elements for a problem, denoted by U.

EXAMPLE 2. *Give all the subsets for these sets.*

(a) $A = \{0, 1\}$ **(b)**$X = \{a, b, y\}$

EXAMPLE 3. *Given* $A = \{0, 1, 2, \ldots, 8\}$, $B = \{1, 3, 5, 7\}$, $C = \{3, 5, 1, 7\}$, *and* $D = \{5, 3, 1\}$, $E = \emptyset$ *then which of the following are TRUE?*

(**a**)$B = C$ (**b**)$B \subseteq C$ (**c**)$B \subset C$ (**d**)$C \subseteq B$ (**e**)$D \subset B$

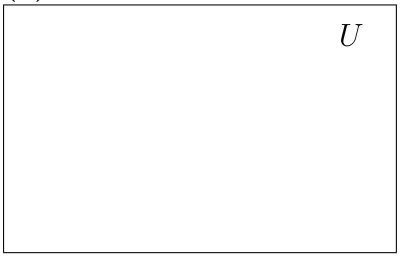
(**f**)$D \subseteq B$ (**g**)$B \subset D$ (**h**)$8 \in A$ (**i**) $\{4, 6\} \subset A$ (**j**)$1, 5 \subset A$

(**k**)$9 \notin C$ (**l**)$D \subseteq D$ (**m**)$\emptyset = 0$ (**n**)$0 \in E$ (**o**)$A \in A$

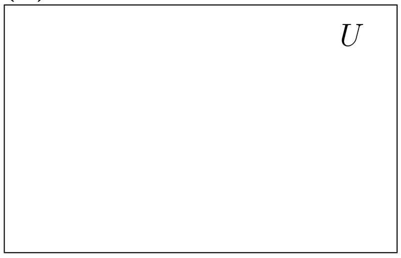
VENN DIAGRAMS - a visual representation of sets (the universal set U is represented by a rectangle, and subsets of U are represented by regions lying inside the rectangle).

EXAMPLE 4. *Use Venn diagrams to illustrate the following statements:*

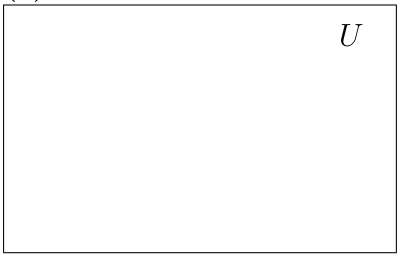
(a) $A = B$



(b) $A \subset B$



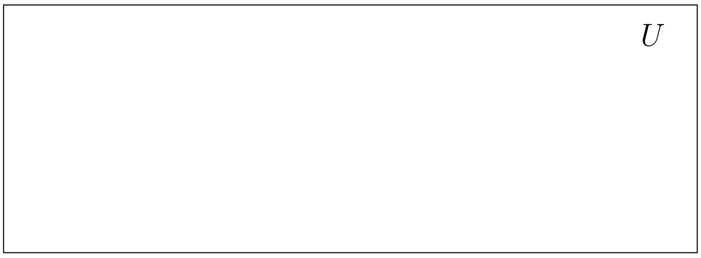
(c) *A and B are not subsets of each other.*



• OPERATIONS OF SETS

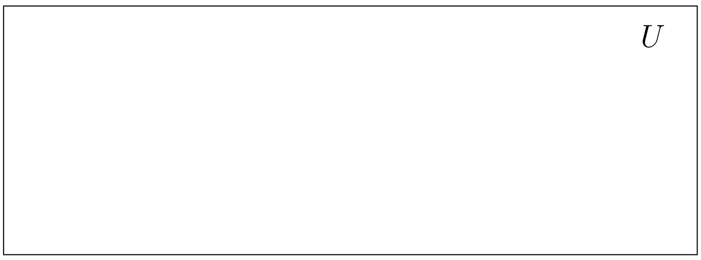
Let A and B be sets. The **union** of A and B, written $A \cup B$, is the set of all elements that belong to either A or B or both. Symbolically:

$$A \cup B = \{x | x \in A \ \text{ or } \ x \in B \ \text{ or both}\}.$$



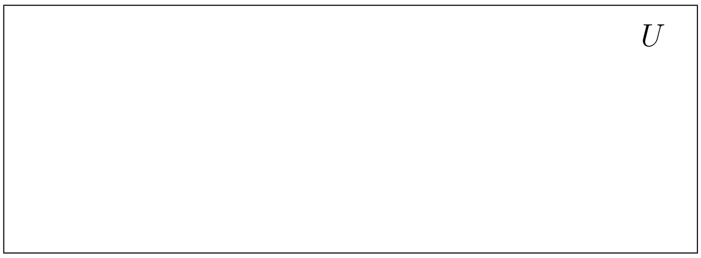
Let A and B be sets. The **intersection** of A and B, written $A \cap B$, is the set of all elements in common with A and B. Symbolically:

$$A \cap B = \{x | x \in A \ \text{ and } \ x \in B\}.$$



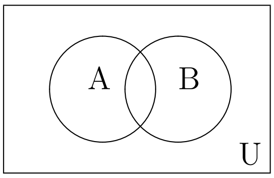
If U is a universal set and A is a subset in U, then the set of all elements in U that are NOT in A is called the **complement** of A, denoted A^c. Symbolically:

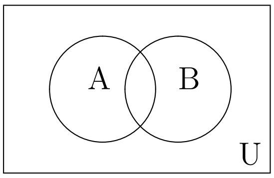
$$A^c = \{x | x \in U, x \notin A\}$$

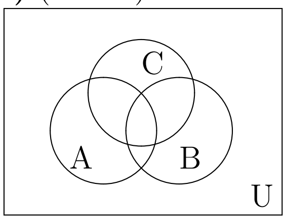


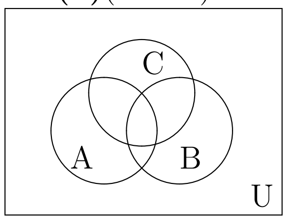
EXAMPLE 5. *Shade the Venn diagrams below to represent the following sets*

(a) $A \cup B^c$ **(b)** $A^c \cap B$ **(c)** $(A \cap B) \cup C$ **(d)** $(A \cup B)^c \cap C$









EXAMPLE 6. *Use these sets to find the following:* $U = \{0, 1, 2, ..., 9, 10\}$, $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{1, 3, 5, 7, 9\}$, $C = \{0, 3, 4, 5, 7\}$, $D = \{4, 5, 6, 7, 8, 9, 10\}$, $E = \{0, 6, 7, 9\}$, $F = \{1, 3, 8, 9\}$

(a) $B \cup E$

(b) $A \cup B$

(c) F^c

(d) A^c

(e) $(A \cap C) \cup E^c$

(f) $(D \cup E)^c \cap (C \cup F)$

EXAMPLE 7. *Let U denote the set of all students at Texas A&M. Let*

$A = \{x \in U | x \text{ lives in College Station}\}$,

$B = \{x \in U | x \text{ lives in Bryan}\}$,

$S = \{x \in U | x \text{ is a sophomore}\}$.

Write the set (using the set notation of $\cup, \cap$, and ^{c}) that represents each of the given statements:

1. *The sophomores that do not live in College Station .*

2. *The sophomores that live in Bryan or College Station.*