6.1: Sets and Set Operations

- **Set Terminology and Notation**

  DEFINITIONS:

  Set is well-defined collection of objects.

  Elements are objects or members of the set.

$\boxed{\{ \quad \}}$

( ) NO

[ ] NO

Roster notation:

$A = \{a, b, c, d, e\}$ Read: Set A with elements a, b, c, d, e.

$D = \{e, d, c, a, b\}$

$A = D$

$B = \{a, b, c, ..., z\}$ Read: Set B with elements being the letters of the alphabet.

$A \neq B$	$A = D$
$A \subset B$	$A \subseteq D$
$A \subseteq B$	$A \subset D$ (False)

Set-builder notation:

$B = \{x | x$ is a letter of the English alphabet$\}$ ("|": such that)

$C = \{x | x$ is a student in this classroom$\}$

If a is an element of a set A, we write $a \in A$ (belongs) that read "a belongs to A." However, if a does not belong to A, we write $a \notin A$.

element $\in$ set

Two sets, A and B, are **equal**, written $A = B$, if and only if they have exactly the same elements. (NOTE: they do not have to be in the same order!). If every element in set A is also an element in set B, then A is a subset of B, written $A \subseteq B$.. If $A \subseteq B$, but $A \neq B$, then A is a **proper** subset of B, written $A \subset B$.

EXAMPLE 1. *Given*

$$A = \{a, e, i, o, u\}$$
$$B = \{u, i, e, a, o\}$$
$$C = \{a, e, i, o\}$$
$$D = \{e, i, o, a\}$$

. *Then*

TRUE	FALSE
$A = B$	$B \subset A$
$B \subseteq A$	$B \subset C$
$C \subset A$	$a \notin C$
$C \subseteq A$	$D \in A$ (not element)
$D \subset B$	$i \subset B$ (elem. / set)
$a \in A$, $u \notin D$	
$D \subset A$	

The **empty set** is the set that doesn't have any elements, denoted by $\emptyset$ or $\{\}$. The **universal set** is the set that contains all of the elements for a problem, denoted by U.

$\emptyset \quad \{ \}$

$$\boxed{\begin{array}{l} D = \{a, b, c, d, e, f, g\} \\ A = \{a, z, x, y\} \end{array}}$$

$U = \{a, b, c, ..., z\}$

EXAMPLE 2. *Give all the subsets for these sets.*

(a) $A = \{0, 1\}$

$\underbrace{\emptyset, \{0\}, \{1\},}_{\text{proper}}$ A

4 subsets

(b) $X = \{a, b, y\}$

$\emptyset$, $\{a\}$, $\{b\}$, $\{y\}$

$\{a, b\}$, $\{a, y\}$, $\{b, y\}$, X

8 subsets.

EXAMPLE 3. *Given* $A = \{0, 1, 2, \ldots, 8\}$, $B = \{1, 3, 5, 7\}$, $C = \{3, 5, 1, 7\}$, *and* $D = \{5, 3, 1\}$ $E = \emptyset$ *then which of the following are TRUE?*

(a) $B = C$ (T) (b) $B \subseteq C$ (T) (c) $B \subset C$ (F) (d) $C \subseteq B$ (T) (e) $D \subset B$ (T)

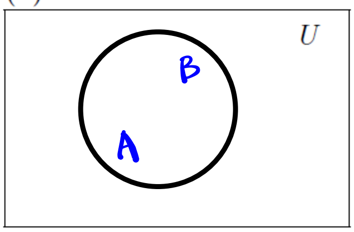
(f) $D \subseteq B$ (T) (g) $B \subset D$ (F) (h) $8 \in A$ (T) (i) $\{4, 6\} \subset A$ (T) (j) $\underbrace{1, 5}_{\text{elem.}} \subset \underbrace{A}_{\text{set}}$ (F)

(k) $9 \notin C$ (T) (l) $D \subseteq D$ (T) (m) $\emptyset = 0$ (F) (n) $0 \in E$ (F) (o) $\underbrace{A}_{\text{set}} \in \underbrace{A}_{\text{set}}$ (F)

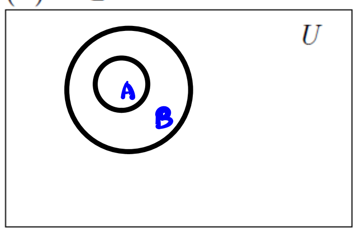
VENN DIAGRAMS - a visual representation of sets (the universal set U is represented by a rectangle, and subsets of U are represented by regions lying inside the rectangle).

EXAMPLE 4. *Use Venn diagrams to illustrate the following statements:*

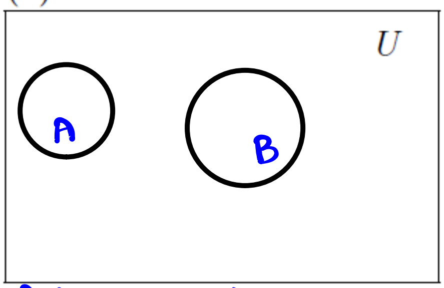
(a) $A = B$



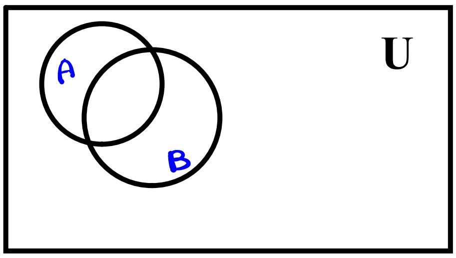
(b) $A \subset B$



(c) *A and B are not subsets of each other.*



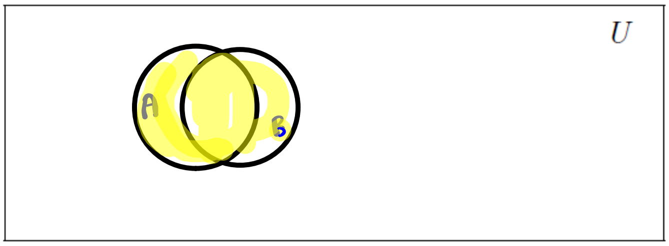
A & B are disjoint



• OPERATIONS OF SETS

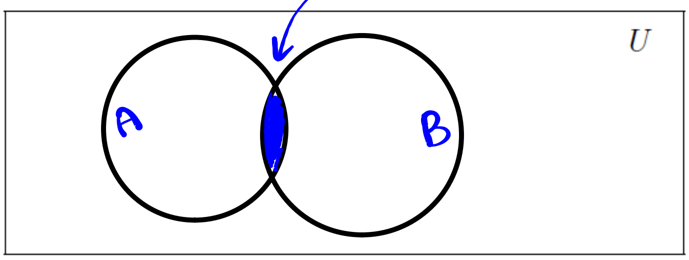
Let A and B be sets. The **union** of A and B, written $A \cup B$, is the set of all elements tha belong to either A or B or both. Symbolically:

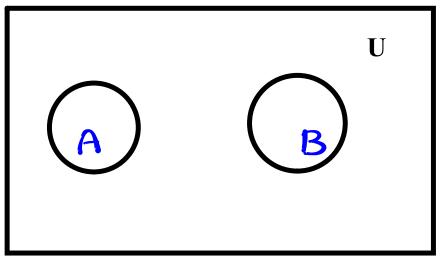
$$A \cup B = \{x | x \in A \text{ or } x \in B \text{ or both}\} .$$



Let A and B be sets. The **intersection** of A and B, written $A \cap B$, is the set of all element in common with A and B. Symbolically:

$$A \cap B = \{x | x \in A \text{ and } x \in B\} .$$

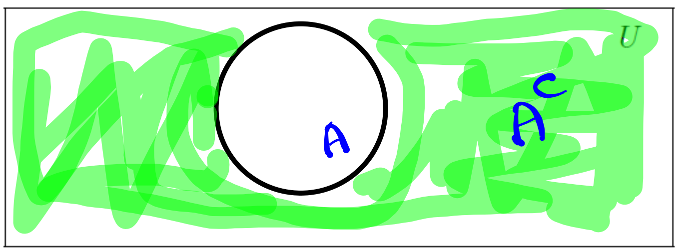




$A \cap B = \varnothing$

If U is a universal set and A is a subset in U, then the set of all elements in U that are NOT in A is called the **complement** of A, denoted A^c. Symbolically:

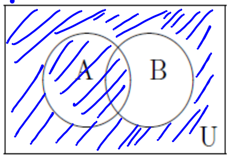
$$A^c = \{x | x \in U, x \notin A\} = U - A$$



EXAMPLE 5. *Shade the Venn diagrams below to represent the following sets*

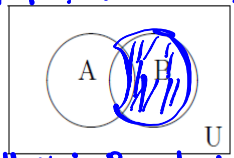
(a) $A \cup B^c$

$\{x \mid x \in A$ or $x \notin B\}$



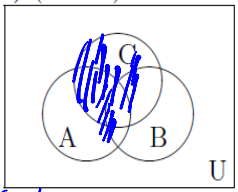
(b) $A^c \cap B$

$\{x \mid x \notin A$ and $x \in B\}$



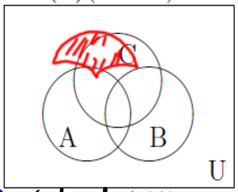
all x in B which are not in A

(c) $(A \cap B) \cup C$



$\{x \mid x \in A \cap B$ or $x \in C\}$

(d) $(A \cup B)^c \cap C$



$\{x \mid x \notin A \cup B$ and $x \in C\}$

all x in C which are not in $A \cup B$

EXAMPLE 6. *Use these sets to find the following:*

$U = \{0, 1, 2, ..., 9, 10\}$ $\quad A = \{0, 2, 4, 6, 8, 10\}$ $\quad B = \{1, 3, 5, 7, 9\}$

$C = \{0, 3, 4, 5, 7\}$ $\quad D = \{4, 5, 6, 7, 8, 9, 10\}$ $\quad E = \{0, 6, 7, 9\}$ $\quad F = \{1, 3, 8, 9\}$

(a) $B \cup E = \{1,3,5,7,9\} \cup \{0,6,7,9\} = \{1,3,5,0,6,7,9\}$

(b) $A \cup B = \{0,2,4,6,8,10\} \cup \{1,3,5,7,9\} = U$

(c) $F^c = U - F = \{0,\cancel{1},2,\cancel{3},4,5,6,7,\cancel{8},\cancel{9},10\} = \{0,2,4,5,6,7,10\}$

(d) $A^c = U - A = B$

$= \{0,1,2,3,4,5,8,10\}$

(e) $(A \cap C) \cup E^c$

$A \cap C = \{0,2,4,6,8,10\} \cap \{0,3,4,5,7\} = \{0,4\}$

$E^c = U - E = \{\cancel{0},1,2,3,4,5,\cancel{6},\cancel{7},8,\cancel{9},10\}$

$= \boxed{\{1,3\}}$ Final answer

(f) $(D \cup E)^c \cap (C \cup F)$

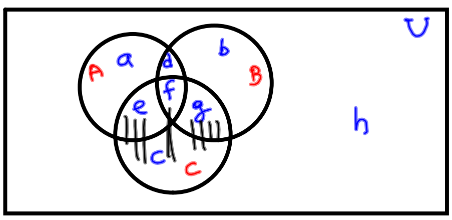
$D \cup E = \{4,5,6,7,8,9,10\} \cup \{0,6,7,9\} = \{0,4,5,6,7,8,9,10\}$

$(D \cup E)^c = U - (D \cup E) = \{\cancel{0},1,2,3,\cancel{4},\cancel{5},\cancel{6},\cancel{7},\cancel{8},\cancel{9},\cancel{10}\} = \{1,2,3\}$

$C \cup F = \{0,3,4,5,7\} \cup \{1,3,8,9\} = \{0,3,4,5,7,1,8,9\}$

Remark: Another way to solve Example 5.

(d)



$(A \cup B)^c \cap C = \{c\}$

$A \cup B = \{a, d, b, e, f, g\}$

$U = \{a, b, c, d, e, f, g, h\}$

$(A \cup B)^c = U \setminus (A \cup B) = \{c, h\}$

$C = \{c, e, f, g\}$

EXAMPLE 7. *Let U denote the set of all students at Texas A&M. Let*

$A = \{x \in U | x$ *lives in College Station*$\}$,

$B = \{x \in U | x$ *lives in Bryan*$\}$,

$S = \{x \in U | x$ *is a sophomore*$\}$.

Write the set (using the set notation of $\cup, \cap$, *and* ^{c}*) that represents each of the given statements:*

1. $\underbrace{\text{The sophomores}}_{S}$ $\overset{\text{and } \cap}{\text{that}}$ $\underbrace{\text{do not } \underbrace{\text{live in College Station}}_{A}}_{A^c}$. $= S \cap A^c$

2. $\underbrace{\text{The sophomores}}_{S}$ $\underbrace{\text{that}}_{\cap}$ live in Bryan $\overset{\cup}{\text{or}}$ College Station. $S \cap (B \cup A)$

$B \cup A$